Development of failure criterion for extensile fracturing of Kannur limestone under triaxial stresses

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ABSTRACT: The classic rock mechanics is limited to evenly fractured rock mass at shallow depths wherein works of Mohr, Coulomb, Hoek and Brown, Drucker-Prager, Wiebols and Cook and more consider isotropic structural effects of joints seen at shallow depths. However, hardly any failure criterion is applicable to massive rocks available at deep depths. At deep tunnels, as one moves from the excavation surface into the rock, the fracture mechanism changes from extensional to shear cracking. The representative Kannur limestone is observed to exhibit exquisite extensile fractures as columnar shards are blown away in a manner much similar to strainburst in deep tunnels. The aim of the paper is to discuss the confinement dependency of limestone when subjected to triaxial stresses. Also, an improvement is suggested to Mogi criterion. This will further help to decide upon the multistep unloading of confining pressure observed during an excavation process in tunnels.

Keywords: Brittle, triaxial, extensile, principal stress, crack.

1 INTRODUCTION

High geo-stress and complex geological conditions such as varying rockmass properties, geological structures, discontinuities, and rock types, as well as some induced seismic and blasting activities may trigger rockburst, spalling, caving, or buckling on the side walls and excavation faces of the tunnels or vertical boreholes. This paper discusses a special case of polyaxial stress state i.e., triaxial stress state in deep tunnels, wherein massive limestone (say Kannur limestone) could be present. The rock type under consideration is ideal to strainburst as it exhibits exquisite extensile fractures at the exposed surface of the specimens, independent of applied confinement. The term ‘extensile’ is used as it produces splendid columnar fragments under the compressive stresses acting on all sides of the specimens. These ‘shards’ are aligned parallel to the major principal stress. The shards are of the length of the 54 mm diameter specimen, tested for uniaxial compressive strength, which produces columnar structure.

The initiation of tensile cracks was first studied by Griffith (1924) and then modified by McClintock & Walsh (1963) to account for shear mechanism by considering the friction acting at the surface of the closed cracks. However, since Griffith criterion does not consider the extension of the tensile crack, it needed to be modified as explained in Hoek & Martin (2015). Herein the dimensions of the columnar fractures were found to be related to modulus of elasticity and peak stress, applicable to brittle rocks.

In contrast to popular linear failure criteria such as Mohr-Coulomb criterion, Hoek & Brown (1980) suggested a non-linear failure criterion for brittle rocks. As explained by Zuo et al. (2008) and Hoek & Brown (1988), the Hoek-Brown criterion agrees well with Griffith criterion. This can also be proved mathematically by estimating the parameters such as uniaxial tensile strength using both the criteria. Based on the shear sliding observed in most rocks at the crack surface, Wiebols & Cook (1968) theorized effective shear strain energy in most prominent loading combinations based on directional cosines, including for polyaxial stress state. Later on, Colmenares & Zoback (2002) modified the Wiebols and Cook criterion, for which constants can be calculated from conventional triaxial tests. This proved to be yet the most versatile theorized modified criterion.

Ultimately Mogi (1967) using a durable and effective high confinement true triaxial test setup found that the major principal stress, σ_1 is dependent on intermediate and minor principal stresses (σ_2, σ_3) as

$$\frac{(\sigma_1 - \sigma_3)}{2} = f_1\left(\frac{\sigma_1 + \beta\sigma_2 + \sigma_3}{2}\right) \tag{1}$$

where β is less than 1. Then Mogi (1971) based on a new triaxial compression method proposed a failure criterion based only on $\sigma_2 + \sigma_3$. As observed later in section 2, equation (1) holds true where β is unity. Hence, Rao (1984) criterion also may not apply to the observations in section 2 due to the dependency of 2D deviatoric stress and first invariant of stress tensor.

2 ROCK BACKGROUND

2.1 Geological Properties

Kannur limestone has been collected from Kannur, Kerala quarry, belonging to Quilon formation , Neogene geologic period which lies between unconformably laid Archean granites at bottom and the late Miocene Wakalay beds at the top (Eames, 1950; Mallikarjuna et al., 1995). From the optical microscopy horizontal mineral lineation can be observed with dark pigmentation distributed along the deposition direction. The prime minerals from the FESEM image can be observed to be of length 4-6 μm. The main composition of limestone is calcite, dolomite, quartz and iron oxide, as read from EDX and XRD data.

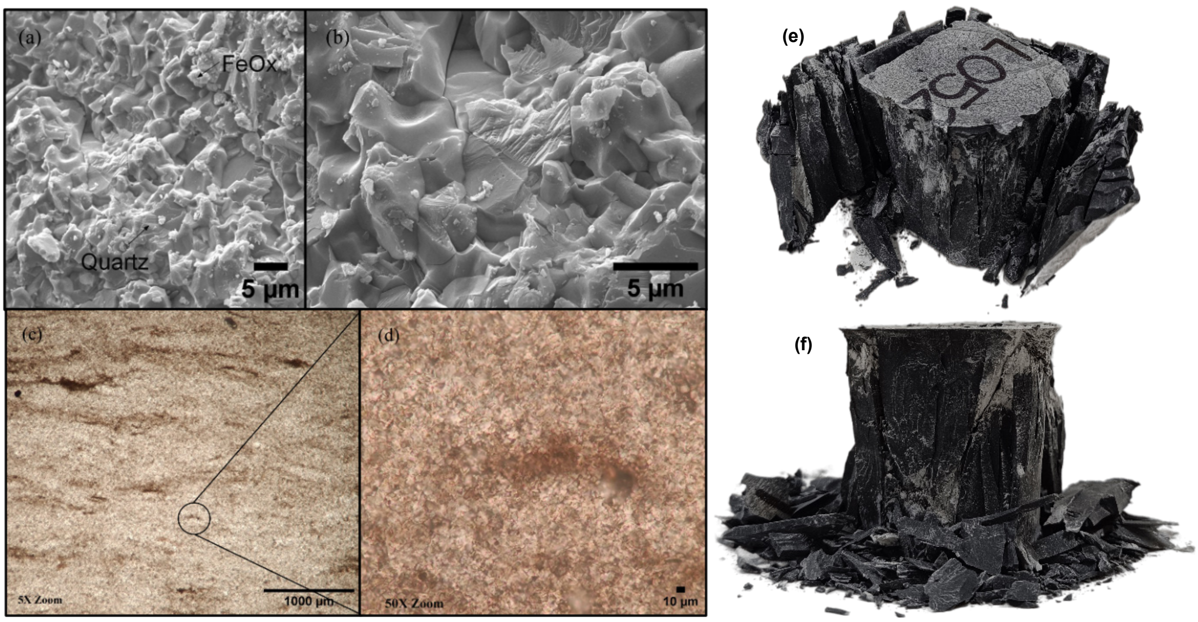

Figure 1. (a)-(b) Surface microstructure image using FE-SEM of Kannur limestone sample, respectively; (c)-(d) Optical microscopic image of Kannur limestone with 5X and 50X magnifications, respectively; (e) Surface spalling and (f) internal extensile fracturing of failed limestone specimen.

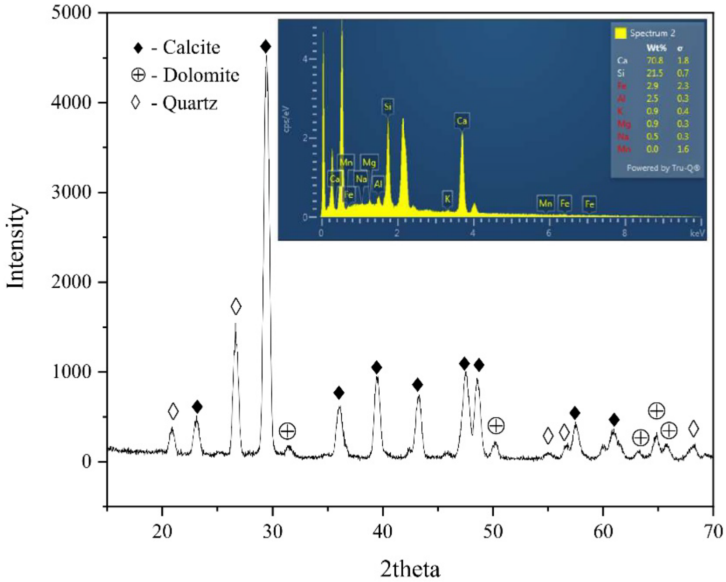

Figure 2. XRD pattern and EDS spectrum of Kannur limestone.

2.2 Physico-Mechanical Characterization of Rock

Table 1. Physical and mechanical properties of Kannur limestone.

Properties	Value	Properties	Value
Specific gravity	2.70 g/cc	$E_{t50\%}$ (UCS, 54 mm dia)	69.89 MPa
Porosity	0.07 %	ν	0.3
Slake durability (I_{di}) (5 cycle)	99.91	UCS (60 mm cube)	146.53 MPa
Brazilian tensile strength (dry)	21.8 MPa	c (60 mm cube)	26.11 MPa
UTS calculated (54 mm dia)	18.63 MPa	φ (60 mm cube)	50.43˚
P-wave velocity (54 mm dia)	6053 m/s	Dilation angle (60 mm cube)	37.63˚

2.3 Conventional Triaxial Test on Cubical Specimens

In addition to the properties gained from physical and mechanical characterization of the rock (Table 1), conventional triaxial test ($\sigma_1 > \sigma_2 = \sigma_3$; $\sigma_3 > 0$) was performed for 60 mm standard size of cubical specimens of Kannur limestone as shown in Table 2 along with failed specimens in Figure 1 (e) – (f). The tests on cube are performed using the rigid platen type polyaxial testing machine (PTM), IIT Delhi established by Rao & Tiwari (2008). The uniaxial tensile strength was calculated using the factor $\sigma_{t\_BTS}/\sigma_{t\_UTS} = 1.17$ (Fuenkajorn & Klanphumeesri, 2011). It can be seen that since the Griffith criterion does not account for crack propagation, the coefficient of determination is the lowest for Modified Griffith (MG) criterion among the three failure criteria. Since the Modified Wiebols-Cook (MWC) criterion considers the shear strain energy, which is observed to be in increasing order with the increase in confinement, the coefficient of determination proves it to the closest among the three. As the Hoek-Brown (HB) criterion back calculates the uniaxial compressive strength (σ_{ci}) from the triaxial test data, it underestimates the tested UCS (σ_c).

Table 2. Combinations of confinement pressure and tested and estimated peak strength based on three failure criteria.

σ_3 (MPa)	σ_1_test (MPa)	σ_1 (HB)	R^2	σ_1 (MWC)	R^2	σ_1 (MG)	R^2
0	146.53	141.70	0.94	146.53	0.96	146.53	0.89
4	180.25	181.64		177.45		177.10	

6	208.49	199.12	192.90	190.96
10	215.61	230.85	223.82	216.65
14	267.36	259.47	254.73	240.27
16	271.62	272.89	270.19	251.47

2.4 Comparative Analysis of Failure Criteria

As observed from Figure 3, although the HB and MWC criteria predict σ_1 well, whereas the MG criterion vastly underestimates it at higher confinement. Following limitations have been observed from the plots.

- Inability of the HB criterion to account for tested UCS value.
- Since the total failure of the specimens is driven by the inability of the fractured specimen to take further load after the extensile fractures, leading to final shear failure at the core of the specimens. The MWC criterion is improvement to inculcate tensile strength.
- The MG criterion needs to be improved upon to account for extensile and shear crack propagation separately. Zuo et al. (2015) already has worked out the inclusion of shear crack propagation, however, extensile crack propagation is still unestablished.

3 NEW EXTENSILE FAILURE CRITERION

Upon carrying out extensive mathematical variations of (σ_1, σ_3, σ_c), the linear plot of (σ_3- σ_3) vs (σ_3 +2σ_3) has been concluded to produce the best fit. Also, it was observed that no matter the value of constant, any of its simple four mathematically operations with (σ_3 +2σ_3) did not produce any difference in the plot.

Thus, based on equation (1), if β is substituted with unity, it estimates the value of σ_1 well. However, the exact relation is ambiguous. Also, comparison of failure criteria with the test data on the plot as described in section 3, suggests that all the three failure criteria functions well in very low confinement and significantly overestimates the value of σ_1 at higher confinements.

As observed from Figure 4, the relation can be simply expressed as

$$(\sigma_1 - \sigma_3) = A(\sigma_1 + 2\sigma_3) + B \quad (2)$$

Using σ_c = 146.53 MPa, B ≈ (1-A)* σ_c = 43.77 MPa ≈ 44.19 = B. Thus, simplifying equation (2)

$$(\sigma_1 - \sigma_3) = \sigma_c + A(\sigma_1 + 2\sigma_3 - \sigma_c) \quad (3)$$

For a tensile test, σ_1 = 0 and σ_t = -σ_3. Substituting the same in equation (3),

$$A = \frac{(\sigma_c - \sigma_t)}{(\sigma_c + 2\sigma_t)} \quad (4)$$

Knowing A = 0.7013 gives σ_t = 18.217 ≈ UTS calculated.

4 CONCLUSION

As explained earlier constant A does not vary with confinement. It can be seen that the equation (3) satisfies the Mogi (1967) criterion. As the criterion is derived for extensile fractures, the criterion can be called an extensile failure criterion. The equation will work wherein the material cannot be loaded further after macroscopic extensile fracturing. For triaxial test wherein as the confinement increases, the area of the core of the specimen under shear fracture increases, Modified Wiebols-Cook criterion is shown to perform the best. However, for the triaxial tests of low confinement or unloading triaxial test, Hoek-Brown criterion is experienced to perform better. Thus, for extensile fractures dominating the triaxial tests, the new extensile failure criterion is encouraged.

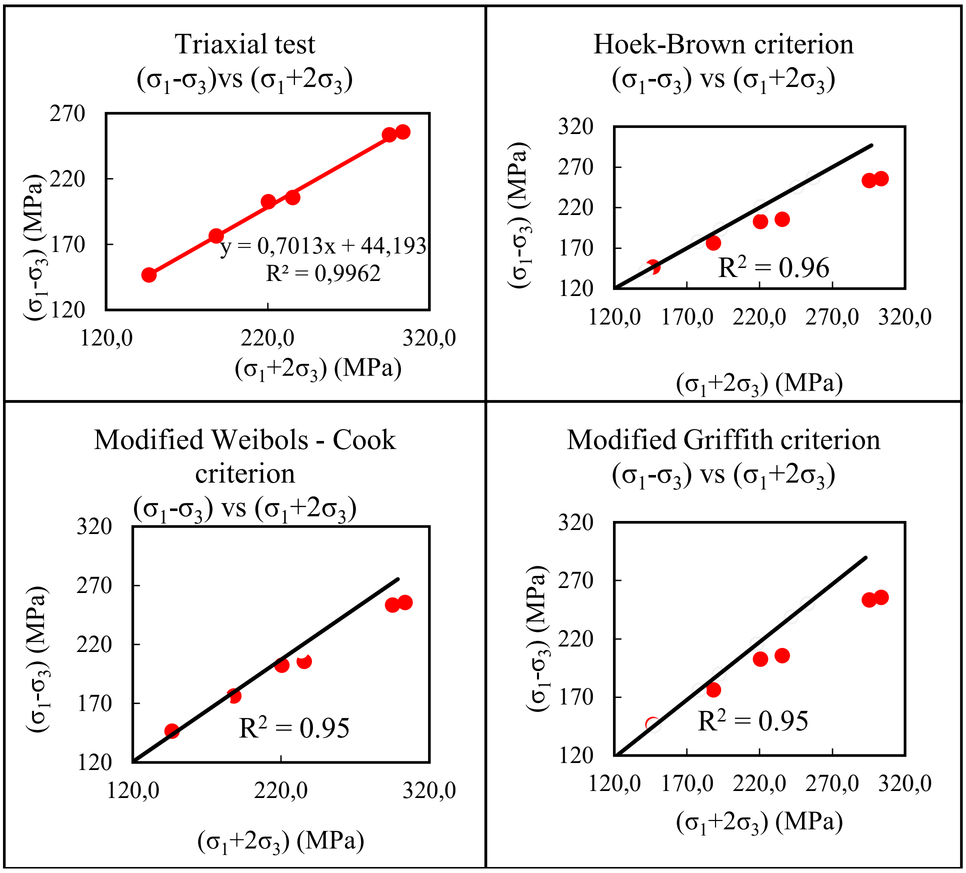

Figure 3. Overestimation of major principal stress by the failure criteria on the plot of new extensile criterion.

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