An improved calibration and uncertainty analysis for a ground subsidence using physics-informed neural network (PINN) approach

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ABSTRACT: Accurately estimating surface settlement requires assessing under-bed layer properties. However, limited information necessitates the use of computational methods to address uncertainties. Recent advancements in machine learning and physics informed neural networks (PINN) have allowed researchers to incorporate physics into neural network models, enabling the forecasting of necessary parameters using inverse analysis algorithms. In this study, we employ the PINN approach to estimate unknown parameters associated with under-lying clay layer thickness and consolidation-aided material constants in an offshore vicinity of a river delta area. We implemented and compared Mikasa's one-dimensional approach, and also compared the outcomes of PINN techniques with Terzaghi's one-dimensional analytical solution and PEST back analysis. Additionally, our work introduces a methodology that utilizes PINN to identify these attributes and explores potential future applications of this framework.

Keywords: PINN, Consolidation, Subsidence, Uncertainty.

1 PROBLEM STATEMENT

It is common that the location where field measurements of ground subsidence are taken and the boreholes used to gather information about subsurface geological layers do not align with each other. As a result of the discrepancy between the measurement location and the borehole, estimating subsidence at the measurement point may exhibit a significant level of divergence. This divergence can be attributed to errors in determining the thickness of the clay layer, as well as its properties such as the compression index and consolidation coefficient. Figure 1 illustrates the conditions of a field in which "Observation points" can be utilized to estimate surface subsidence at specific locations, while other points of borehole locations are employed to obtain geological layer data and sampling of clay material. In practice, we need to estimate the potential long-term deformation at a regional scale, which is referred to as the "Interested Area", and it would be necessary to have prior knowledge of the properties of the clay and their spatial distribution within the "Interested Area" to achieve that.

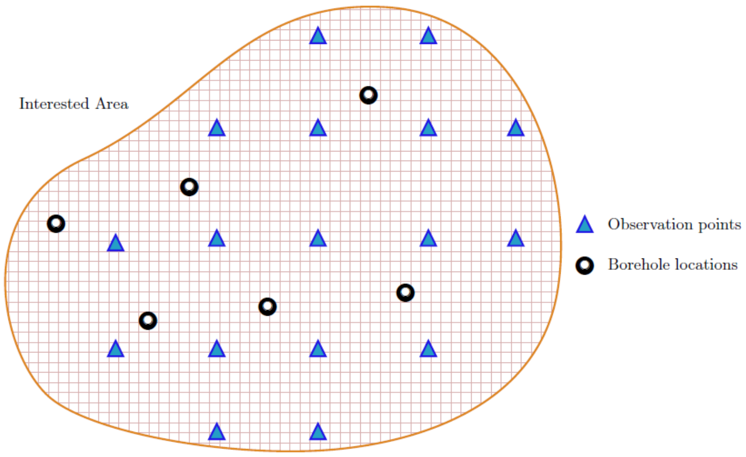

Figure 1. Conceptual domain of study with observation and measurement locations.

Figure 2 illustrates a typical uncertain parameter, specifically the thickness of the clay layer, which may not remain constant throughout the domain. The information regarding the soil layer is solely available at the borehole locations. At other location, the data is uncertain, leading to inaccurate estimations of the eventual surface settlement. Furthermore, the material parameters, including the modulus of elasticity, Poisson's ratio, void ratio, and intrinsic permeability, are also unknown at these locations, resulting in inaccurate estimations.

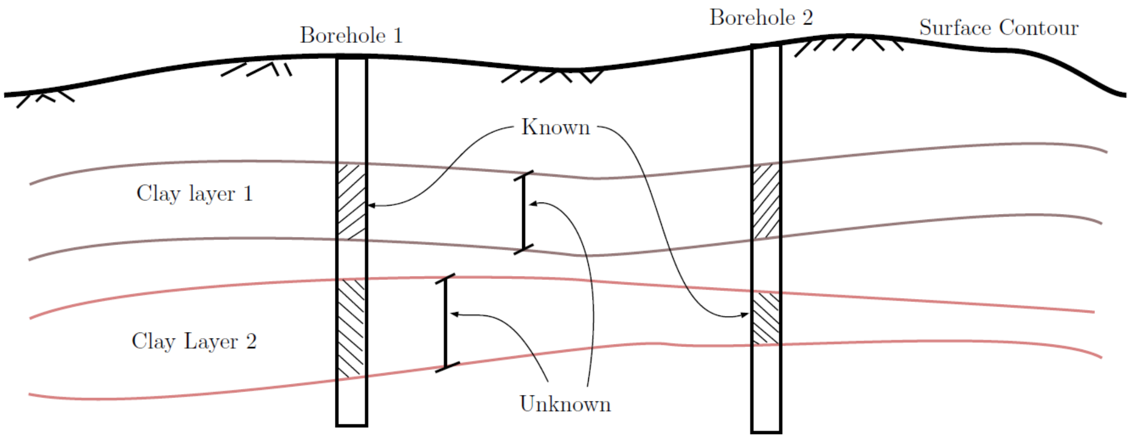

Figure 2. Unknown clay layer thickness at the settlement measurement locations.

2 THEORY OF PHYSICS INFORMED NEURAL NETWORK (PINN)

Physics-informed neural networks (PINNs) are a class of universal function approximators that incorporate the knowledge of governing physical laws for a given dataset in the learning process. They are typically used for datasets that can be described by partial differential equations (PDEs). The PINN approach can be effective in scenarios where the state-of-the-art machine learning techniques lack robustness due to low data availability and provide more accurate results. The incorporation of prior knowledge regarding general physical laws acts as a regularization agent during the training of neural networks (NNs), restricting the space of feasible solutions and leading to more accurate function approximation. By embedding this prior information into a neural network, the available data is enriched, making it easier for the learning algorithm to identify the correct solution and achieve good generalization performance, even with limited training examples (Karniadakis et al., 2021).

Within the PINN framework, initial and boundary conditions are not inherently satisfied analytically, and must therefore be incorporated into the loss function of the network in order to be learned simultaneously with the unknown functions of the differential equation (DE). If we express a surrogated model without physics in neural network as $y = f(x, \theta)$ then the loss function to be optimized in PINN can be written as follows.

$$Loss(\theta) = [f(x,\theta) - y_{obs}]^2 + G.E + B.C \;\rightarrow argmin\;\; Loss(\theta) \tag{1}$$

The loss function in the conventional neural network approach corresponds to the first term on the right-hand side, while the other two terms are typical components of the physics-informed approach, representing the governing equation and boundary conditions, respectively. In this study, the above architecture of PINN was established in PyTorch modules.

3 FUNDAMENTALS OF 1D PINN APPROACH WITH MIKASA'S EQUATION

Terzaghi (1943) introduced a one-dimensional consolidation equation for a single layer of homogeneous material. The following assumptions are made for the solution that the coefficient of consolidation remains constant, the impact of self-weight is disregarded, the material is fully saturated, and the deformation is infinitesimal.

In the Terzaghi's solution, the surface settlement (S_t) with time and uniform thickness (H) of clay layer is expressed as

$$S_t = \mathrm{U}_t m_v \Delta\sigma \tag{2}$$

$$U_t = 1.0 - \sum_{i=1}^{\infty} \frac{2}{M^2} exp^{-M^2 T_v} \tag{3}$$

$$M = 0.5\pi(2i + 1) \tag{4}$$

$$T_v = \frac{C_v t}{H^2}, \quad C_v = \frac{K}{\mu m_v}, \quad m_v = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \tag{5}$$

In this study, we developed a Python script to create solution procedures for the equations mentioned above. We then utilized PEST (http://www.pesthomepage.org, accessed on Feb. 27, 2023), an inverse analysis code that estimates uncertain parameters, to determine the coefficient of consolidation of the clay layer.

Mikasa (1963) derived a comparable equation with the assumption of infinitesimal strain and a constant coefficient of consolidation, as shown below:

$$\frac{\partial \varepsilon_z}{\partial t} = C_v \frac{\partial^2 \varepsilon_z}{\partial z^2} \tag{6}$$

where, C_v is the coefficient of consolidation and ϵ_z is the strain in the z−direction.

According to literature, normalizing the parameters prior to employing a neural approach has been discovered to enhance both convergence and result accuracy. As such, to implement Mikasa's algorithm, we normalize the dimensional parameters, which will establish boundary conditions on the model. The dimensional parameters are normalized as

$$z_{sd} = \frac{z_i - z_{min}}{z_{max} - z_{min}}, t_{sd} = \frac{t_i - t_{min}}{t_{max} - t_{min}} \tag{7}$$

and the normalized governing equation for the problem is defined as

$$f_{ge} = \frac{1}{t_{max} - t_{min}} \cdot \frac{\partial \varepsilon_z}{\partial t} - \frac{1}{(z_{max} - z_{min})^2} \cdot C_v \frac{\partial^2 \varepsilon_z}{\partial z^2} \tag{8}$$

The exponential function of coefficient of consolidation is to compel the positivity of the value of consolidation coefficient. After the data processing and the variables which are trained based on normalized values, original values can be estimated by multiplying with the norms.

In solving the function using PINN approach, we utilized an Adam optimizer and implemented 8 hidden layers with 20 nodes in each layer.

4 TEST APPLICATION OF 1D PINN APPROACH WITH FIELD SETTLEMENT

To validate the algorithm and to test applicability of the approach, we use the surface settlement in a river delta area of land reclamation to train the PINN model and estimate the coefficient of consolidation based on predefined clay layer thickness. Considering the distance between the measurement location and investigation borehole, the thickness of this clay layer can be estimated by calculating a weighted average based on the measurements taken at nearby borehole locations. Figure 3 shows all the 2,000 collocation points in the domain with time and boundary conditions prescribed in the analysis. Figure 4 compares the field measurement (dot), the outcomes of PEST analysis for specific parameters (line-red), and the results obtained through PINN analysis (line-black). Our observations indicate that the PINN analysis was able to replicate settlement behavior quite closely to the actual measurements, whereas the PEST analysis demonstrated a more ideal (; theoretical) evolution from an overall perspective. The estimated coefficients of consolidation estimated from both methods are quite similar, at 9.97E-04 cm^2/sec and 1.23E-3 cm^2/sec, respectively.

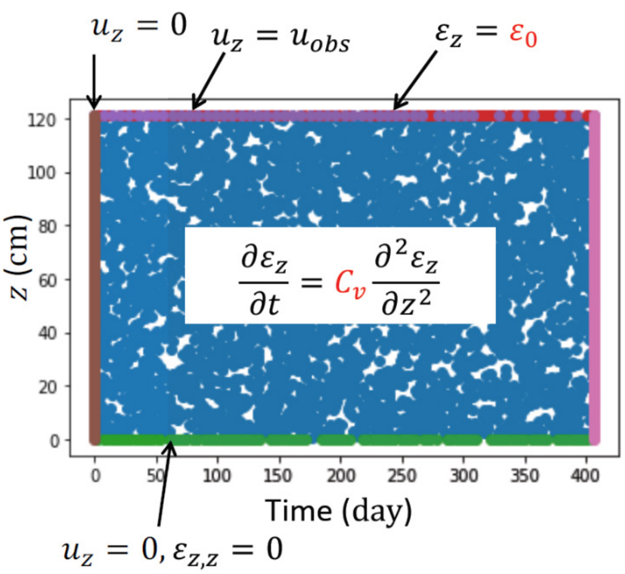

Figure 3. Collocation points and boundary conditions in the PINN analysis.

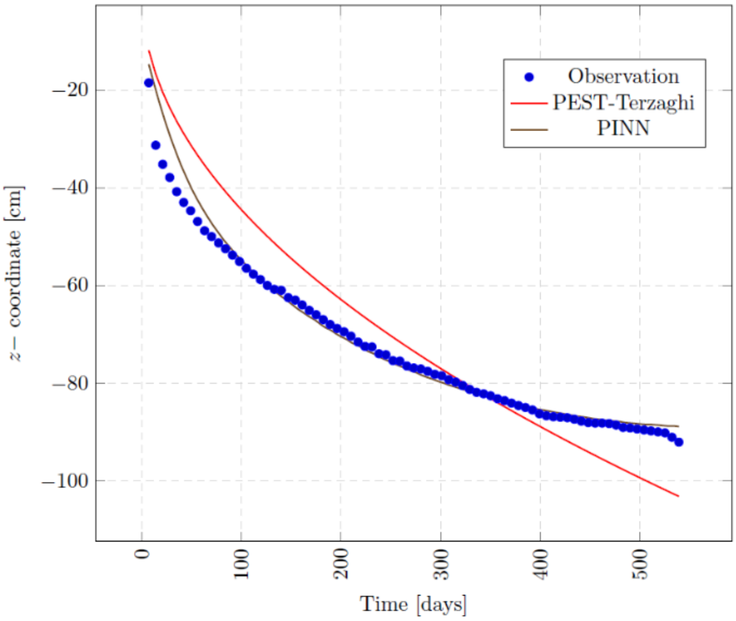

Figure 4. Comparison of the surface settlement measurements (dots), PEST (line-red) and PINN analysis (line-black).

5 APPLICATION TO ESTIMATION OF UNCERTAIN CLAY LAYER THICKNESS

The two primary uncertain factors affecting subsidence phenomena are the thickness of the clay layer and the coefficient of consolidation. This instance illustrates the utilization of the 1D PINN approach to estimate the thickness of an unknown clay layer. The fundamental premise of this application is that the property (; coefficient of consolidation) of the clay layer is uniform and scale independent.

The blue circular markers depicted in the Figure 5 represent the coefficient of consolidation of the clay sample acquired from borehole (IBS-144) under various loading conditions, while the red diamond signifies their mean value. A 1D PINN analysis of consolidation is conducted for various clay thickness conditions using field settlement measurements (MBS-18), which is the closest location to IBS-144. Through the scale-independent assumption of clay properties, we can determine the thickness of the clay layer that closely matches the laboratory oedometer test results by examining the estimated coefficient of consolidation. In this particular instance, we can infer that the thickness of the clay layer falls between 2.42 (H/2=1.21 m) meters and 6 meters (H/2 = 3 m), with the laboratory coefficient of consolidation falling within this range. It is noted that the clay thickness observed from the nearest borehole (IBS-144) is approximately 15 m, which differs by a factor of 2.5.

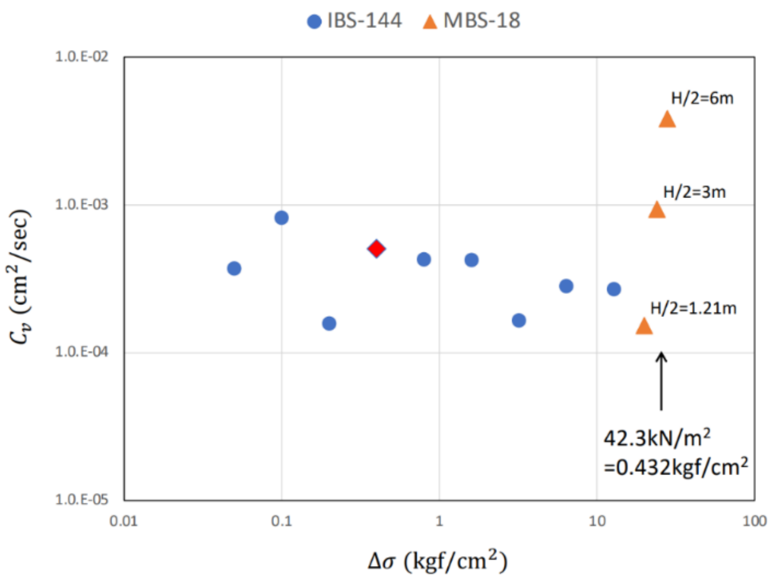

Figure 5. The estimated coefficient of consolidation (C_v) for different layer thicknesses.

6 CONCLUDING REMARKS

In this study, we developed a PINN method to analyze a ground subsidence and verified it by comparing to Terzaghi's theoretical solution and the field measurement of surface settlement from a river delta region with a wide distribution of clay layers. The PINN approach has demonstrated its ability to reproduce field measurements and produce results comparable to theoretical solutions. Furthermore, it is a faster and more efficient alternative to numerical simulations and has potential for estimating uncertain properties of clay layers, which often contribute to large errors in final settlement assessments. While the current implementation is limited to 1D analysis, the approach can be extended to multi-dimensional analysis. Further research will be conducted to explore its potential in regional subsidence analysis.

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