The Brown-Hoek stress-depth relation revisited

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ABSTRACT: In 1978 Brown and Hoek published their well-known paper on the relationship between measured in-situ stress and depth. The key figure of this paper can be found in many textbooks related to rock mechanics and rock engineering and the given relationships are used frequently. In this paper we use the World Stress Map data to check if the more than 40 years old interpretation of the limited dataset at that time is still valid. The presented analyses clearly shows that the general trends are, but that some details need to be tuned in future. In an ongoing initiative more measured stress data is included into the analysis.

Keywords: in-situ stress, horizontal stress, depth-related, stress gradients.

1 INTRODUCTION

In 1978 E. T. Brown and E. Hoek published their paper on ‘Trends in Relationships between Measured In-Situ Stresses and Depth’ (Brown & Hoek 1978). Their figure showing the average horizontal stresses normalized by the vertical stress as a function of depth (Figure 1) can ever since be found in many rock mechanics, rock engineering, engineering geology or related disciplines textbooks (e.g., Hudson & Harrison 1993, Brady & Brown 2004, Zang & Stephansson 2010). Based on 120 datasets of measured in-situ stresses it was shown, that the horizontal stresses may be several times larger than the vertical stress at shallow depths, and that the vertical stress follows a gradient of 27 MPa/km.

Knowledge of the primary in-situ stress is a pre-requisite for any underground engineering application, e.g., mining, tunnelling, (radioactive) waste disposal, underground (energy) storage, or hydrocarbon and geothermal energy extraction. In many cases in-situ stress measurements are scarce and a green field stress modelling needs to be performed to approximate the in-situ primary stress tensor components. Such a stress modelling is based on rock mechanics principles reflecting the strength of rock and rock mass, structural geology and seismological indicators, the geological history of the area as well as existing in-situ data. In addition, empirical relationships may help limiting the uncertainty of such a stress modelling.

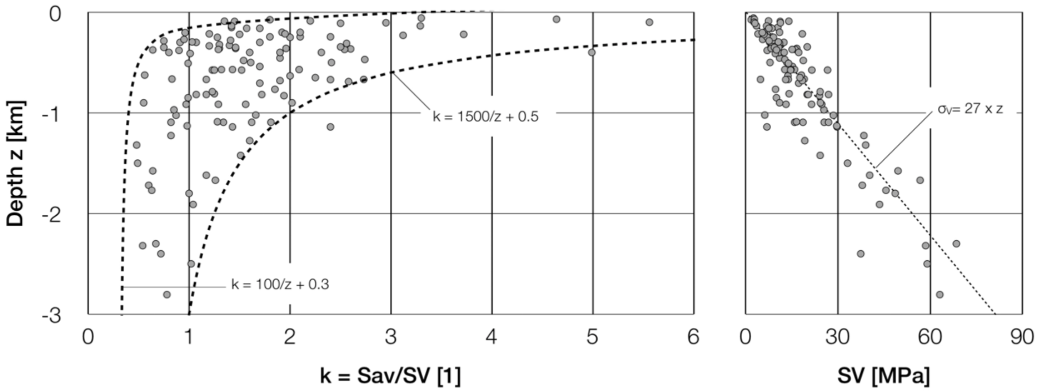

Figure 1. Original data from Brown & Hoek (1978). SV: vertical stress, Sav = (SH+Sh)/2, SH: maximum horizontal stress, Sh minimum horizontal stress.

Because a large amount of in-situ stress data has been gained since Brown and Hoek published their paper, we revisited their stress-depth relation to evaluate its validity. In a first step we analyzed the data compiled in the World Stress Map database (Heidbach et al. 2016), showing that the trends reported by Brown & Hoek (op. cit.) are, in principle, valid, but the equations describing the relationships may be refined. In addition, some data suggests that at shallow depth also very low horizontal stresses may exist. We will continue compiling data from different sources in the future and refine the empirical relation for stress vs depth to further contribute to a data-driven approach for in-situ stress modelling.

2 DATA BASIS AND DATA HANDLING

For this study we used the data published in the World Stress Map Project database (Heidbach et al. 2016). By the end of 2022 the open-access database comprised about 42,870 stress information entries for the upper 40 km of the earth's crust. The information provides stress orientations for all listed locations, but magnitudes only if measured by over-coring or hydraulic methods. For 2,038 locations (data subsets) worldwide, 5,604 reported stress values were extracted from the database. Most of the data is reported for Europe, and least for Africa (Figure 2a). Not all locations list the three principal stresses, and the data quality is ranked from A (high) to E (low).

In this paper we follow the geomechanical stress conventions, i.e., compressive stresses are positive. The maximum principal stress is denoted S1, the intermediate principal stress is S2 and the minimum principal stress is S3, hence S1 > S2 > S3. Vertical stress is indicated by SV, whereas the larger horizontal principal stress is SH and the smaller horizontal principal stress is Sh. Following structural geology convention, a normal faulting stress regime is defined as SV = S1 and Sh = S3, thrust faulting is characterized by SV = S3 and SH = S1, and strike slip faulting is evident with SH = S1 and Sh = S3. The average of the two horizontal stresses is indicated by Sav.

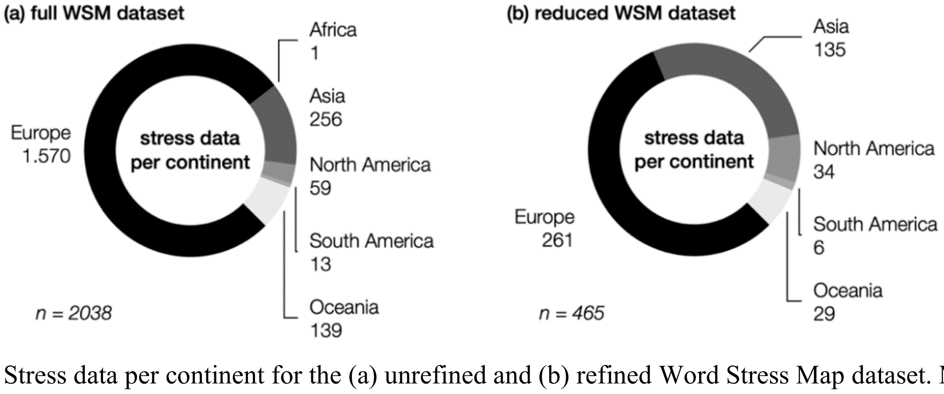

Figure 2. Stress data per continent for the (a) unrefined and (b) refined Word Stress Map dataset. Most data of the reduced data subset are reported for Europe, followed by Asia (see text for consolidation procedure).

The unrefined World Stress Map dataset without any consolidation is shown in Figure 3. For the upper approximately 1.5 km data shows large scatter, and a more defined trend may be described for greater depths. The vertical stress SV is represented best by an average stress gradient grad_SV = 25 MPa/km up to a depth of 7 km. SH varies significantly but suggests that grad_SH > grad_SV. Averaged grad_Sh is about 16 MPa/km. This suggests a world-wide homogenized strike-slip stress regime at depth below 2.5 km. In a Sav/Sv-depth-diagram analogue to that of Brown & Hoek (1978) their proposed general trend still reflects the data, but at shallow depths Sav/SV ratios well below 1 are reported and the range of Sav/SV ratios at greater depths is larger.

The World Stress Map dataset was consolidated such that (a) only entries comprising all principal stresses were considered for further analysis, (b) entries with low WSM quality ranking (D and below) were excluded, and (c) locations with a gradient of vertical stress lower than hydrostatic or larger than 35 MPa/km were excluded, as this would suggest an unrealistically high rock density or extremely unusual geological settings. After this procedure 465 of the initial 2,038 location entries remained. About half of the data subset is linked to Europe, and more than one quarter to Asia (Figure 2b). The refined dataset is used in the following for further analyses.

3 IN-SITU STRESS AS A FUNCTION OF DEPTH – ANALYSIS AND INTERPRETATION

Based on the measured in-situ stress data available in the World Stress Map Project database in the following stresses and derived measures of stress are depicted as a function of depth. The original WSM Project dataset was consolidated by omitting incomplete and low-quality ranking data subsets as well as data subsets showing extreme vertical stress gradients. The consolidated dataset is depicted in Figure 4 along with additional derivations.

Data density decreases with depth due to the limited number of deep wellbores and advanced in-situ stress measurements therein. Up to a depth of 1.5 km data density is high and shows large scatter. The scatter reduces considerably for SV but also for SH and Sh at greater depths, for which the contribution of decreasing data density has not been statistically quantified in this study. For 0 – 1.5 km SV is mostly plotting between gradients of 16 MPa/km < grad_SV < 34 MPa/km; some data at lower depths suggest lower or higher gradients. The vertical stress gradient down to 6 km is on average approximately 25 MPa/km.

While the maximum horizontal stress SH tends to show a larger gradient than SV, the minimum horizontal stress Sh shows a smaller gradient than SV. The vertical stress is controlled by rock mass density, if no extreme geological settings are present, and the horizontal stresses are the sum of vertical stress induced transverse deformation and tectonic stress. Neglecting the tectonic contribution and assuming isotropy, the horizontal stress can be calculated from elastic theory from SV according to

$$\mathrm{SH} = \mathrm{Sh} = \mathrm{SV} \cdot \frac{\nu}{1-\nu} \qquad (1)$$

where ν is the Poisson's ratio for the rock mass. In Figure 4 resulting horizontal stress gradients for grad_SV = 25 MPa/km are plotted for selected ν.

If not otherwise known, an often constant, i.e., depth-independent, Poisson's ratio of 0.25 is usually assumed for rock when using eq. (1), whereas lab data suggests ν to range from < 0.1 to 0.5. Assuming hydrostatic water pressures, an average vertical stress gradient grad_SV = 25 MPa/km and no tectonic stresses, ν is required to exceed 0.3 to have resulting horizontal stress larger than the pore pressure and thus reflecting a condition not allowing for hydraulic opening/fracturing of the rock mass. Alternatively, tectonic stresses need to increase both horizontal stresses above the hydraulic pressure within the rock mass.

Majority of SH data plots above the elastic stress gradient for ν = 0.4, while Sh mostly plots above the elastic stress gradient for ν = 0.3. This is in particular true for shallow depths and suggests pronounced tectonic contribution to the magnitude of horizontal stress. The average of SH and Sh, Sav, also plots above the elastic stress gradients for ν = 0.3 for shallow depths, and above the elastic stress gradients for ν = 0.4 at greater depths below 2.5 km.

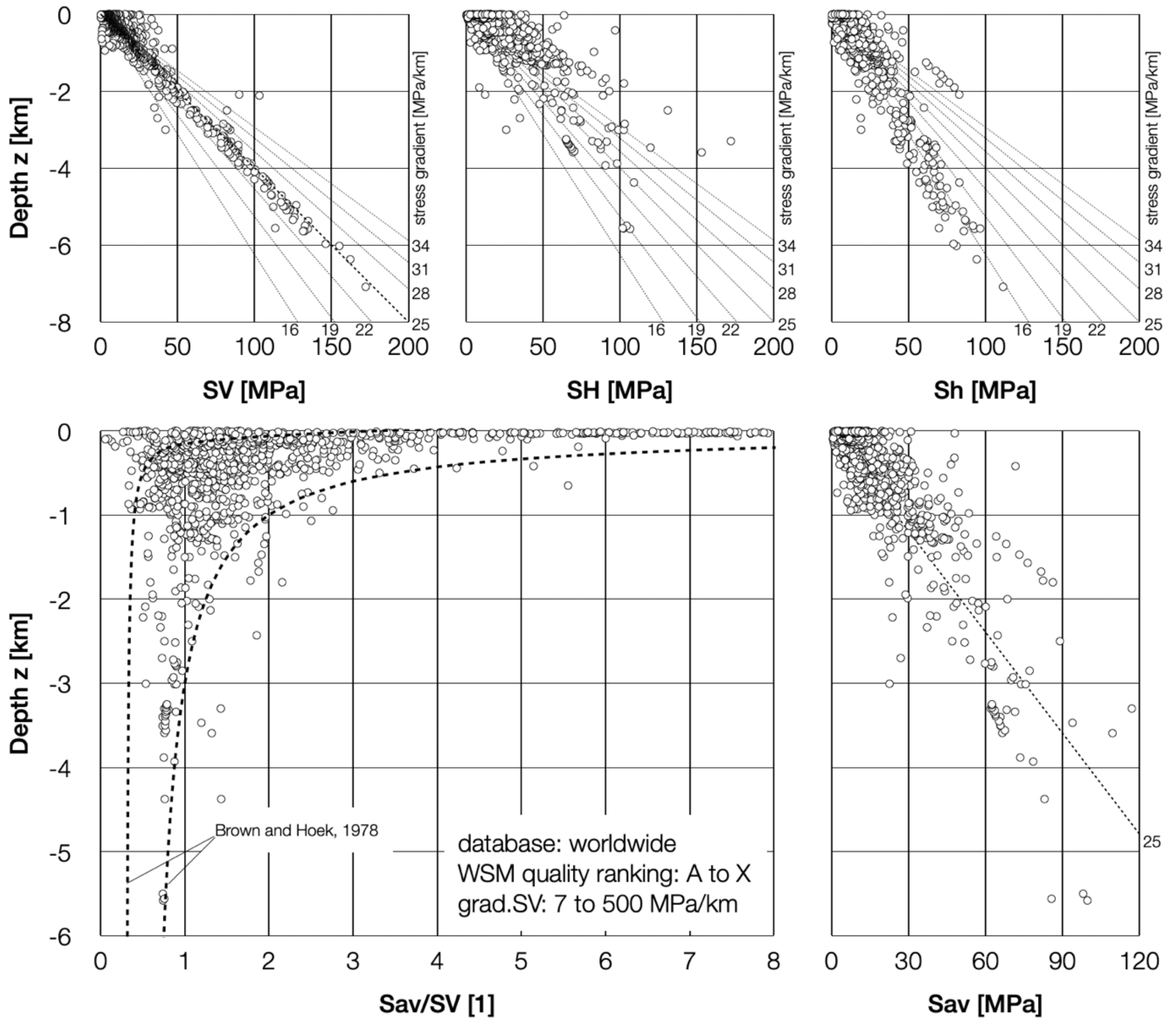

Figure 3. Unrefined stress data from the World Stress Map database plotted vs depth. Vertical stress SV follows a 25 MPa/km gradient, SH suggests a larger gradient, whereas Sh increases with 16 MPa/km at depths below 2.5 km. The Sav/SV-depth relation in principle confirms the 1978 Brown & Hoek equations with few exceptions. The three principal stress orientations are not provided for all locations, hence Sav/SV vs depth is not shown for all entries.

The Sav/SV-depth relation qualitatively shows the same trend as proposed by Brown and Hoek (op. cit.) with only few exceptions. At shallow depths high differential stresses frequently result in Sav/SV well above unity, but also below. With increasing depth Sav/SV does not show as pronounced high values and low ratios vanish. A convergence of the ratio to unity at greater depth may be assumed from the principle of time-depended elimination of shear stresses and anisotropy in rock; viscoplastic flow should result in a lithostatic state at depth (Heim's rule) as also confirmed by the data which suggest a shift towards slightly higher rations than proposed by Brown and Hoek. Further separating the refined data based on their location for the represented continents (Figure 5) does not show distinct differences in the stress - depth relationships except for Oceania. The afore-described trends hold in general for the continentally separated data, but further constraints on the tectonic contribution or anisotropy of the rock mass may be derived in future analysis.

4 DISCUSSION AND CONCLUSION

Compared to the original publication by Brown & Hoek (1978) the amount of data has been increased by a factor of 4 and the depth range was doubled. It was shown that the proposed formulation for the upper limit of Sav/SV vs depth is in principle a sufficient envelop to the new data. However, the formulation limiting the lower Sav/SV and the constraints at greater depths may need a revision in future. Low Sav/SV in European and Asian data demands for a more detailed analysis of the individual data subsets. If Sav/SV is well below unity, stress regimes with very low Sh must be evident, which requires extreme geological settings. Also, the original trends do not reflect the implications of time- and temperature- dependent elimination of shear stresses.

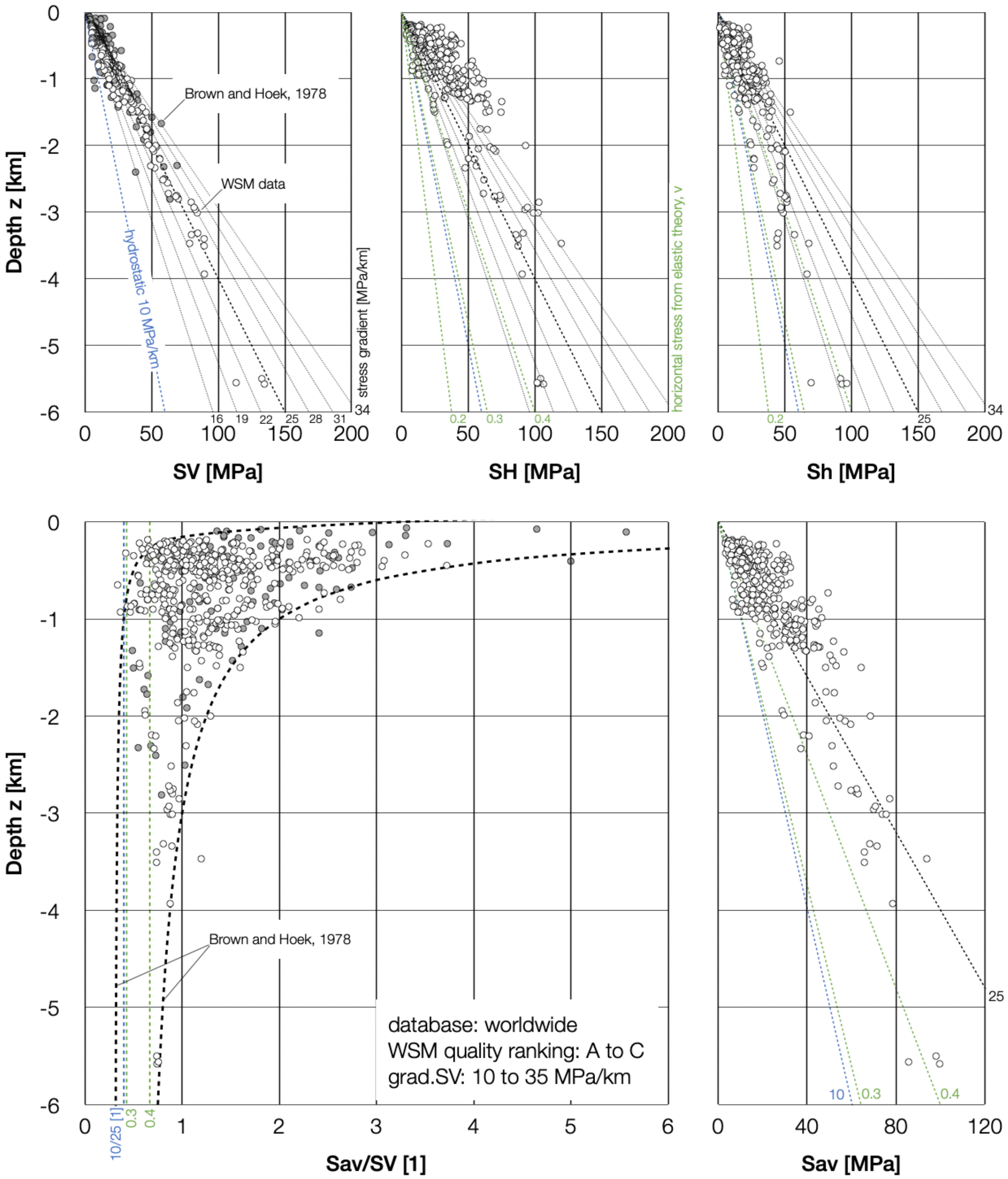

Figure 4. In-situ principal stresses SV, SH and Sh, and Sav and Sav/SV as a function of depth for a refined dataset based on WSM (white dots) and Brown & Hoek original (grey dots) data. Stress gradients are indicated by thin black dashed lines, green dashed lines show stress gradients from elastic theory for given Poisson's ratios and grad_Sv = 25 MPa/km, the blue dashed line is the hydrostatic gradient. The 1978 Brown & Hoek relations are plotted as thick black dashed lines.

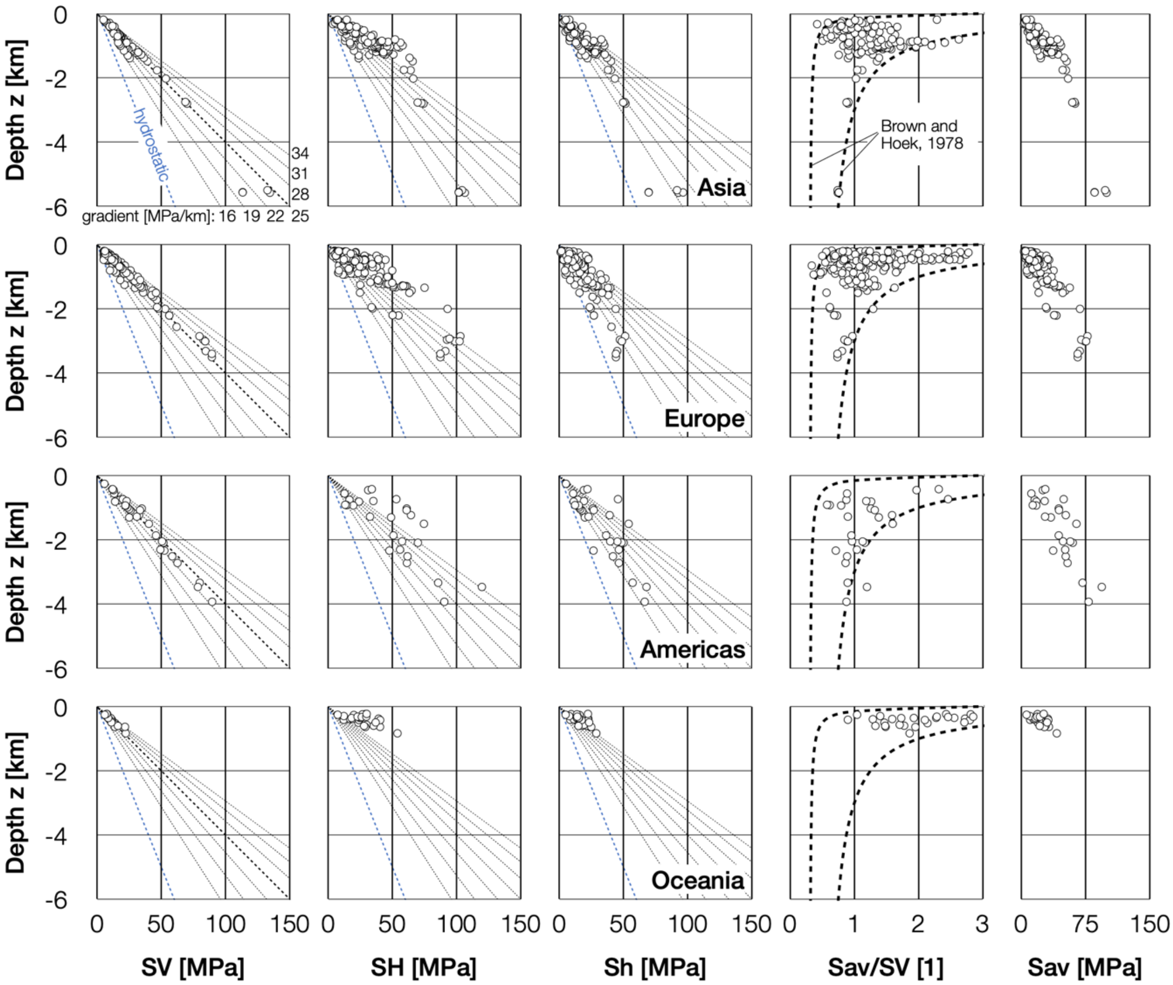

Figure 5. In-situ stress as a function of depth (c.f. Figure 4) of the refined dataset separated with respect to their locations.

The vertical stress gradient as proposed by the discussed original publication may be corrected to a lower gradient of grad_SV = 25 MPa/km.

The revisitation of the trends of measured in-situ stress with depth clearly shows that the primary state of stress in a rock mass is not amenable to calculation by any known method but requires careful location-based geomechanical stress modelling along with experimental determination considering the change in mechanical properties with depth and orientation if depth-dependent in-situ stress measurements are not available. In ongoing and future work, the database to this amendment of the stress vs depth relationship will be further extended with the aim to develop an equation framework that can be used for constraining stress states in primary stress field modelling. The framework shall -in a mature stage- reflect regional and geological settings.

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