Stability analysis of a rock slope: Fully-probabilistic approach

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ABSTRACT: Stability analysis of rock slopes is an exercise that poses a variety of challenges to a rock engineer. Field information is often lacking and subjective modelling of the rock mass is required. Expert judgment also influences the quantification of the targeted safety level, reflecting the confidence of the designer in his own assumptions and hypothesis. With the introduction of the EN1997 in Europe, structural reliability concepts were brought to the design of geotechnical structures, now also including rock masses after its current revision. Yet, fully-probabilistic methods are still considered valid alternatives to the partial safety method recommended there. Understanding how to address rock engineering problems from a probabilistic perspective becomes essential to move beyond the traditional practice. In this study, a simple problem relating to the stability of a rock slope is explored. Limit states for foreseeable failure mechanisms are defined and the system reliability problem is formulated and solved.

Keywords: Rock slope, wedge stability, probabilistic approach, system reliability, failure mode analysis.

1 INTRODUCTION

Fully-probabilistic methods are recognized in EN 1997 (2004) as valid alternatives to the application of the partial safety method. They are also considered as advanced approaches given the increasing complexity involved. However, they have known advantages in supporting design and achieving economical optimization under uncertain conditions. In fully-probabilistic methods, uncertainties are modelled by random variables and the structural safety is quantified by the probability of failure. The boundary between acceptable and non-acceptable structural performance is defined as a limit state $G(\boldsymbol{x})$, given in terms of random variables $\boldsymbol{x}$. Probability theory is then used to compute the probability of failure as,

$$p_f = \int_{G(x)<0} f_x(\boldsymbol{x})\, d\boldsymbol{x} \cong \frac{1}{m} \cdot \sum_{j=1}^{m} I(\boldsymbol{x}) \tag{1}$$

where $f_x(\boldsymbol{x})$ represents the joint probability density distribution function of the random variables $\boldsymbol{x}$. The right-hand side of the equation is an approximation of the probability of failure using the Monte Carlo method, which consists in performing m realizations of the limit state and determining how many failures actually occur using a failure indicator function I. For large enough values of m, the failure frequency provides an estimate of the probability of failure.

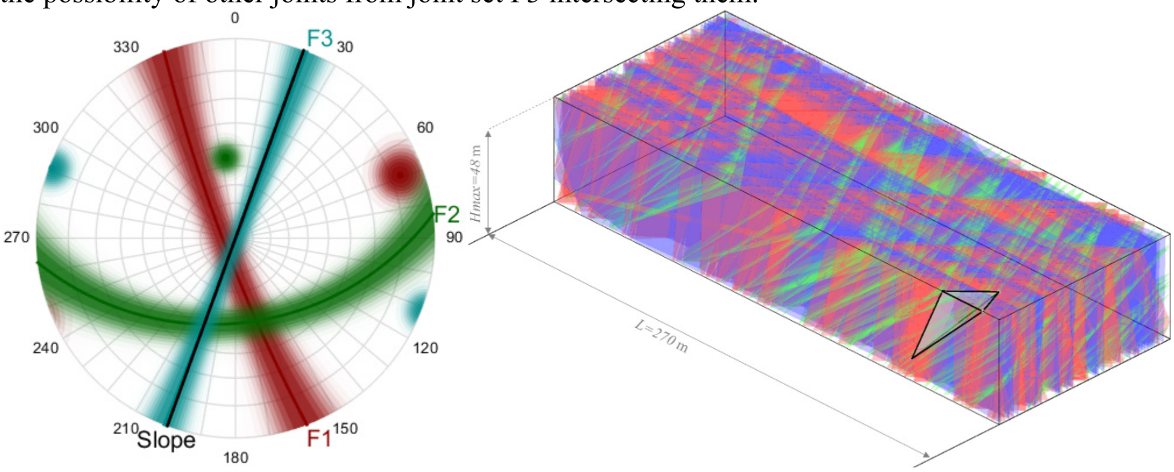
In rock engineering, probabilistic approaches are still seldom performed. Instead, expert judgement usually governs all the design stages of a project, including geotechnical surveys, failure mode analysis and ground modelling (Muralha 1991). The high uncertainty often involved in many rock engineering problems and the complex and cumbersome models sometimes needed, contribute for that. However, with EN1997 now also covering rock engineering, applying probabilistic methods to simple problems that a rock engineer often faces is a first step to formulate and deal with more complex problems in the future.

This work follows a study aiming at comparing the application of deterministic, semi-probabilistic and probabilistic methods to the sliding stability analysis of a rock slope (Pereira *et al.* 2021). In this paper, the probabilistic analysis of the stability of a rock slope accounting for the geometry uncertainty is addressed using an extension of the procedure followed in Jimenez-Rodriguez *et al.* (2006) to a tri-dimensional wedge that may be split in several blocks by a third joint set.

2 CASE STUDY AND FAILURE MODE ANALYSIS

The case study used in this paper to test probabilistic approaches to the stability analysis of rock slopes comes from a real project in which large cuttings in jointed gneiss were carried out as part of the construction of a bypass in the French Pyrenees (Gasc-Barbier *et al.* 2008). This specific cutting is 270 m long and 48 m high, and is nearly vertical, striking N20°E.

Statistical analysis of joint data, obtained from outcrops measurements and scanlines, allowed the distinction of three joint sets (F1, F2 and F3), and the estimation of the respective average orientations (dip and dip direction), the dispersion parameters k (Fisher distribution) and the spacing parameters λ (negative exponential distribution). Graphical analysis of the lower hemisphere projection of joint planes (Figure 1a) reveals a predominant failure mechanism characterized by the formation of rock wedges from the intersection of joint sets F1 and F2. Joint set F3, sub-vertical and sub-parallel to the slope face, might cut the rock slope at multiple distances from the slope face, also enabling the occurrence of toppling mechanisms. A randomly generated model of the fracture network is shown in Figure 1b, where an example of a rock wedge is highlighted. Without loss of generality, this study focuses on the stability of the rock wedges defined by joints from sets F1 and F2, and considering the possibility of other joints from joint set F3 intersecting them.



a) Lower hemisphere projection b) Realization of a discrete fracture network

Figure 1. Geometric data of the rock slope.

Wedge stability, either a part of it or the whole wedge, is quantified through a factor of safety commonly given by the quotient between resistance and loads (or, more generically, between stabilizing and destabilizing effects of actions). A closed-form solution for the factor of safety can be obtained from block theory and vector analysis detailed in Goodman & Shi (1985), which encompasses the possibility of sliding along one or both joints, depending on the relative orientation of the joints and the direction of the total force. The corresponding limit state for a specific stability problem j (G_j) is simply set as,

$$G_j(\boldsymbol{x}) = FS(\boldsymbol{x}) - 1 \tag{2}$$

where $\boldsymbol{x}$ is the vector of random variables involved in the problem. $FS=1$ (or $G=0$) then sets the boundary between stable ($FS>1$ or $G>0$) and unstable conditions ($FS<1$ or $G<0$).

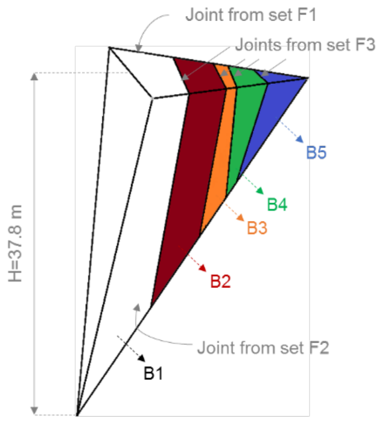
3 SYSTEM RELIABILITY FORMULATION

Since the wedge instability is the only failure mechanism being considered, a geotechnical design model, *i.e.* a conceptual representation of the media for the verification of the limit state, can be simply given by an isolated wedge limited by any two joints from sets F1 and F2 (Figure 2a). Other joints from joint set F3 intersecting the wedge, that may, in theory, behave as tension cracks since no tensile strength is considered for these joints, can also be accounted for, making the wedge an assembly or a system of blocks. Conservatively, all joints are modelled as planar and persistent.

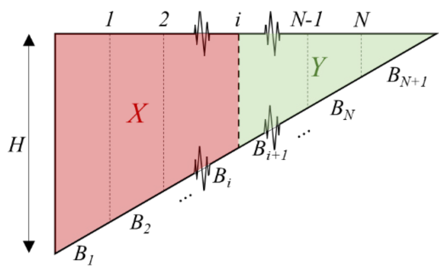
The state of the system can then be inferred from the state (stable or unstable) of each block. However, since an inner block cannot be unstable without the instability of all the outer blocks, this problem, that would originally require testing the stability of each block individually, can be simplified. In turn, the system is considered to fail if any set of blocks, from the slope face to any joint from set F3, is unstable. Therefore, a rock wedge intersected by N other joints (Figure 2b) can be idealized as a series system with $N+1$ alternative failure modes ($i = 1, \ldots, N+1$), corresponding to the instability of any key set of blocks X (from block B_1 up to block B_i) and the stability of the remaining set of blocks Y (from block B_{i+1} to the block B_{N+1}), i.e.,

$$p_{f,sys} = P\left(\bigcup_{i=1}^{N+1} \left[G_{X(i)} < 0 \cap G_{Y(i)} > 0\right]\right) \tag{3}$$

where the $X(i)$ is the key set of blocks ($B_1,\ldots, B_i$), and $Y(i)$ is the set of the remaining blocks ($B_{i+1},\ldots, B_{N+1}$). The failure mode with $i=N+1$ corresponds to the instability of the whole wedge.



a) Wedge highlighted in Figure 1b



b) Two-dimensional equivalent representation

Figure 2. Idealization of the rock wedge as a system of blocks.

4 RANDOM VARIABLES

The relevant sources of uncertainty involved in reliability problems are considered in the analysis as random variables modelled by probability distributions. Their definition is sometimes subjective, especially in the absence of objective data. At the end, mathematical features of probability distributions should at least comply with the theoretical constraints of those variables.

The following random variables, other than the geometric proprieties already mentioned in Gasc-Barbier *et al.* (2008), are included in this study, namely: the rock density, the groundwater level, the joint water pressure, and the joint shear strength. Their statistical properties are synthesized in Table 1. Special references to some of them must, however, be made:

- Being the only variable load considered, the groundwater level determines the reference period related to the probability of failure. A normal distribution with mean value 40 m above the slope toe and a standard deviation of 3 m, corresponding to the point-in-time (quasi-permanent) groundwater level, is assumed. As a consequence, calculated probabilities of failure correspond to an estimate of its instantaneous value.
- The joint water pressure depends on the groundwater level and varies along the joints. Two hypothetical extreme conditions are conservatively assumed: the joint pressure distribution is idealized as a bi-linear diagram where the peak value is located at mid-height ($k_w=0$) and near the slope face ($k_w \approx 1$). Without further information, any situation in-between is considered equiprobable, *i.e.* the joint water pressure factor k_w is considered a random variable following a uniform distribution ranging from 0 to 1. Figure 3a illustrates the water load effects on the joints.
- The joint shear strength is presumed to be given by the Mohr-Coulomb envelope ($\bar{\tau}$) with cohesion $\bar{c}=40kPa$ and friction angle $\bar{\varphi}=35°$, after test data analysis. The associated uncertainty can be taken into using a random variable θ, which multiplies the adjusted envelope ($\tau=\bar{\tau}.\theta$). A lognormal distribution with unit mean value and a coefficient of variation of 20%, which appears to be a realistic value (Muralha 1995), is assumed for the random variable θ considered for each joint.
- The location of the *i*-th joint from set F3 is also uncertain. However, from the field survey, it is known that the spacing between consecutive joints is 3 m on average, and shall be modelled by a negative exponential distribution. Therefore, the location of *i*-th joint, counted from the slope face inwards, is given by the sum of *i* negative exponentially distributed random variables with parameter λ=3 m.

Table 1. Probability distribution of random variables included in the reliability problem.

Random variable	Description	Probability distribution	Parameters p_1	p_2
ρ_r (kg/m^3)	Rock density	Normal[a]	2600	50
H_w (m)	Groundwater level above slope toe	Normal[a]	40	3
k_w	Joint water pressure factor	Uniform[b]	0	1
θ_1	Shear strength model uncertainty (set F1)	Lognormal[c]	1	0.2
θ_2	Shear strength model uncertainty (set F2)	Lognormal[c]	1	0.2
ΔL (m)	Spacing between joints (set F3)	Exponential[d]	3	-
$jdip_1$ /jdd_1 (°/°)	Joint 1 orientation (dip/dip direction)	Fisher[e]	83/249	86
$jdip_2$ /jdd_2 (°/°)	Joint 2 orientation (dip/dip direction)	Fisher[e]	44/173	105
$jdip_3$ /jdd_3 (°/°)	Joint 3 orientation (dip/dip direction)	Fisher[e]	89/111	196.5

[a] $p_1=\mu$, $p_2=\sigma$. [b] $p_1=a$, $p_2=b$. [c] $p_1=\mu$, $p_2=\sigma$. [d] $p_1=\lambda$. [e] $p_1=\mu$, $p_2=k$.

5 RELIABILITY ANALYSIS AND RESULTS

System reliability problems can be addressed following a component-based approach in which the probability of occurrence of each failure mode (parallel sub-system) is computed separately using first-order approximations or simulation methods. The system probability of failure can later be bounded considering that different failure modes may display some degree of correlation, ranging from fully independent to fully dependent. Alternatively, the overall system probability of failure can be computed directly using simulation methods. The latter is attempted in this study, although the computational cost may be higher.

The approximation of the system probability of failure via simulation techniques consists, in its simplest form, in using the Monte Carlo method (Eq. 1) with the indicator function $I(\boldsymbol{x})$ now counting the occurrence of any failure mode of the system. This procedure can become cumbersome, since for each realization of the random variables, all failure modes must be tested (or at least, until the system fails for the first time). This option is only practical in this case because the failure mode verification relies on solving limit states with explicit, analytical formulas. For a wedge with height H, the following algorithm is used:

Step 1. Generate values of the random variables involved in the definition of the rock wedge (ρ_R, Hw, kw, θ_1, θ_2, $jdip_1$ /jdd_1 and $jdip_2$ /jdd_2).

Step 2. Compute the length of the rock wedge (L) from H, $jdip_1$ /jdd_1 and $jdip_2$ /jdd_2.

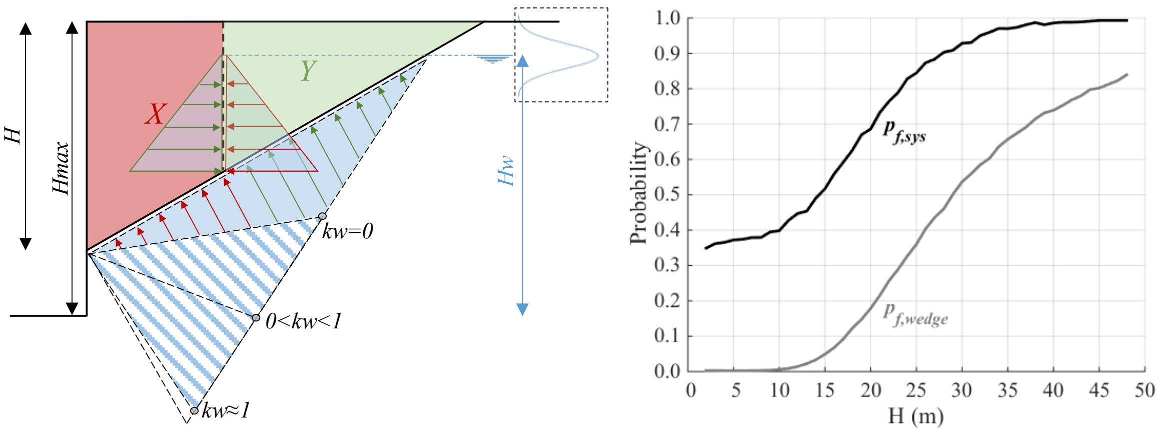
Step 3. Generate N joints from set F3, with orientation $jdip_3$ /jdd_3 and the distance to the previous one, ΔL_3. Note that only the joints intersecting the wedge are relevant, i.e. $\sum_{i=1}^{N} \Delta L_{3,i} < L$.

Step 4. Check the occurrence of failure for each N+1 failure modes (see section 3).

Step 5. Update the estimation of the system probability of failure and its coefficient of variation.

Step 6. Repeat the previous steps until a small coefficient of variation (set here to 2%) is achieved.

This procedure was first performed for $H=48$ m and then repeated considering gradually smaller rock wedges, since it is expected that several wedges with variable dimensions exist throughout the rock slope. Figure 3b shows the system probability of failure versus the rock wedge height. The outcome of the failure mode corresponding to the whole wedge being unstable was isolated from the remaining failure modes for comparison purposes.



a) Water load effects on joints b) Output from reliability analysis

Figure 3. Representation of the water load effects on joints and output from reliability analysis.

Given, on one hand, the unfavourable geometry of the wedge and, on the other hand, the statistical properties considered for the other random variables, especially the shear strength and the groundwater level, the system probability of failure of higher wedges is close to 1.0. This means that it is almost certain that failure of large volumes will occur if no reinforcement measures are implemented. The extent of those measures would yet be dependent on the maximum allowed probability of failure.

It is also clear the contribution of the failure modes involving a joint from set F3 splitting the wedge in two and causing the instability of the key set of blocks. The difference for the case only considering the failure mode corresponding to the instability of the whole wedge proves how much important was, in this case, to contemplate the existence of the third joint set that may intersect the wedge in various locations. This difference becomes even more evident for smaller wedges. In that case, the probability of the whole wedge being unstable is lower but, owing to the existence of other joints from joint set 3, where groundwater pressure may be present with important effects regarding the wedge stability, the system probability of failure is still significant.

6 CONCLUSION

As an alternative to the partial safety method proposed in EN1997 (2004), fully-probabilistic approaches, involving reliability analyses, were followed in this case study. The difficulty of performing reliability analyses usually lies on the formulation of the problem, modelling both failure and uncertainty. To avoid using complex numerical models with high computation efforts, simplifications of the failure mechanisms are required.

Although other failure mechanisms are possible, the stability of a simple rock wedge was tested. Complexity is added to this rather simple stability problem as the presence of joints from a third joint set, that may intersect the rock wedge, is introduced. In this case, rock wedges possibly created by joints from sets F1 and F2, and intersected by other joints from set F3, form a system of blocks. Together with the instability of the whole wedge, one shall also consider the failure of any set of blocks from the slope face to any of these joints. For N of these joints intersecting the wedge, it was showed that $N+1$ failure modes actually exist. The system reliability analysis shall then take into account that each of these failure modes represents the wedge failure and must be checked. An algorithm to estimate the system probability of failure was developed.

The system probability of failure estimated for wedges of variable height was compared to the probability of failure of the whole wedge, taken as the reference study to be made. The significance of addressing the problem as presented became clear. In fact, the contribution of the failure modes involving a joint from set F3, splitting the wedge in two and triggering a failure mechanism characterized by the instability of the key set of blocks, is relevant. Even for smaller rock wedges, whose probability of failure of the whole wedge is very low, the existence of other joints serving as tensions cracks, where groundwater pressure may be present with important effects regarding the wedge stability, makes the system probability of failure still significant.

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