Regression analysis of slope instabilities evolution for time of failure estimation

Alberto Michelini
IDS GeoRadar, Pisa, Italy

ABSTRACT: Time of failure (TOF) estimation of rock slope instabilities is one of the most important output of slope monitoring activities. Providing an accurate TOF estimation allows to significantly mitigate the risk associated with slope failure. Various kinematic models to describe the evolution of instabilities are available in literature, the most representative one is the hyperbolic evolution found by Saito. Different techniques have been proposed and used to fit the Saito model to the experimental data, among these, inverse velocity (IV) method and slope gradient (SLO) method are probably the most popular. However, with the recent development of near real-time monitoring systems, and with modern computing power, it is conceivable to approach the TOF estimation with powerful non-linear regression methods. In the present work, non-linear regression analysis of Saito's model is presented and tested on slope collapse datasets acquired from Ground-Based Synthetic Aperture Radar (GB-SAR) systems.

Keywords: time of failure, failure forecasting, GB-SAR, slope stability, deformation monitoring.

1 INTRODUCTION

Slope monitoring is a critical activity that aims to mitigate the risks associated with slope failure. Among its various output, time of failure (TOF) estimation is certainly one of the most important (Intrieri et al. 2019), in fact providing a reliable TOF forecast allows to significantly reduce the likelihood of both human and economic losses. With the recent development of more and more advanced monitoring technologies like Ground-Based Synthetic Aperture Radar (GB-SAR) systems (Wang et al. 2020), the forecasting methods based on deformation measures (Mercer 2006) have gained considerable importance and have proved to be remarkably effective (Praveen et al. 2022).

Various kinematic models to describe the evolution of instabilities have been proposed and tested in literature (Federico et al. 2012), certainly the most successful one is the power law evolution, originally found by Saito (1965) and later investigated by Voight (1988). Starting from this model, different methods have been developed to estimate its critical time from the experimental data, among these, the inverse velocity (IV) (Fukuzono 1985), and slope gradient (SLO) (Mufundirwa et al. 2010) are probably the most popular among technicians (Dick et al. 2015). The success of these methods is due, in addition to their effectiveness, to their computational simplicity and intuitive graphic interpretation. However, with the availability of modern computing powers it is conceivable to approach the TOF estimation with more rigorous statistical methods.

The purpose of this work is to present a TOF forecast approach based on the Bayesian estimation theory (Rossi 2018) and to verify its applicability to a real GB-SAR dataset. In section 2 the theoretical framework of non-linear regression analysis for TOF estimation is discussed. In section 3 the proposed method is tested on a slope collapse dataset, acquired by a GB-SAR system. In section 4 conclusion and further developments are presented.

2 KINEMATIC ANALYSIS METHODS

The most common TOF forecasting methods are those based on the measure and analysis of slope kinematic parameters (Mercer 2006) such as deformation η, deformation rate $\dot{\eta}$ and acceleration $\ddot{\eta}$. The basic assumption of these methods is that, during the tertiary creep stage, it is possible to define a general function $\dot{\eta}(t, \boldsymbol{\theta})$ that relates the kinematics parameters to time t, through some constant unknown parameters $\boldsymbol{\theta}$. Once this functional relationship has been defined, an estimation $\widehat{\boldsymbol{\theta}}$ of the unknown parameters can be derived empirically from the measured data. Finally, the TOF estimation $\hat{t}_f$ can be extracted by imposing a failure condition on the model. For completeness, after obtaining the TOF estimation, a further phase must also be added to the methods: the assessment of the TOF forecast reliability, including the estimation of a TOF plausibility interval.

2.1 Model definition

The functional relation between kinematic parameters and time can be defined by introducing a dimensionless parametric function $f(x, \boldsymbol{\alpha})$ that models the deformation rate time dependence as

$$\dot{\eta}(t) = v \cdot f(u \cdot (t - t_0), \boldsymbol{\alpha}) \tag{1}$$

Where v and u are two dimensional parameters having respectively dimensions of velocity and inverse of time, $\boldsymbol{\alpha}$ is a set of dimensionless parameters which may or may not be present depending on the complexity of the model, and t_0 is a reference time which can be fixed arbitrarily. Applying a simple reparameterization, $f(x, \boldsymbol{\alpha})$ can always be chosen in such a way that v represent the velocity at the reference time and $u \cdot v$ the corresponding acceleration. For sake of simplicity, in the following t_0 will be understood as the present time i.e. the time at which the forecast is performed, and will be set equal to zero.

Over the years, several parametric functions have been proposed for modelling the slope kinematics during the tertiary creep stage, although no one of them is universally accepted for describing the accelerating creep rates, most of the forecasting methods (Intrieri et al. 2019 and Federico et al. 2012), exploit the power law relation (2) coming from the seminal observations by Saito (1965) on the slopes creep behavior (generalized Saito model).

$$f(x, \alpha) = \left(1 - \frac{x}{\alpha - 1}\right)^{1-\alpha} \qquad \alpha > 1 \tag{2}$$

This class of functions is characteristic of a variety of complex systems, which exhibit self-organized criticality as precursors of large-scale disruptions (Hagstrom & Levin 2021). The main characteristic of generalized Saito model is that it has a critical time t_c in which the deformation rate diverges, and which therefore provides a natural upper bound for the TOF estimation. Combining (1) and (2) it is possible to see that the parameter u and the critical time t_c are inversely proportional.

Among all the possible values of α, the one that has proven to be the most common in slope instabilities is 2 (pure Saito model). In the present work we will focus on the pure Saito model regression, however the theoretical framework described is easily applicable to any other parametric model.

2.2 Parameters estimation

Given a set of N kinematic measurements $\{t_n, \dot{\eta}_n\}_{n=1}^{N}$ and the parametric model (1), the residuals $r_n(u, v, \boldsymbol{\alpha})$, are defined as the difference between deformation rate observations $\dot{\eta}_n$ and the kinematic model evaluated at the corresponding observation time: $vf(ut_n, \boldsymbol{\alpha})$. Residuals can be interpreted as the errors made with respect to the true underlying model. Following a Bayesian approach, it is possible to estimate the model parameters $(\hat{u}, \hat{v}, \widehat{\boldsymbol{\alpha}})$ via maximization of the likelihood function $\mathcal{L}$ that, under the normality assumption for errors, can be expressed as

$$\mathcal{L}(u, v, \boldsymbol{\alpha} \big| \{t_n, \dot{\eta}_n\}_{n=1}^{N}) = \frac{1}{\sqrt{(2\pi)^N |\boldsymbol{\Sigma}|}} \exp\left\{ -\frac{1}{2} \sum_{n,m=1}^{N} r_n(u, v, \boldsymbol{\alpha}) \Sigma_{nm}^{-1} r_m(u, v, \boldsymbol{\alpha}) \right\} \tag{3}$$

$$(\hat{u}, \hat{v}, \widehat{\boldsymbol{\alpha}}) = \arg\max_{u,v,\boldsymbol{\alpha}} \mathcal{L}(u, v, \boldsymbol{\alpha} \big| \{t_n, \dot{\eta}_n\}_{n=1}^{N})$$

where $\boldsymbol{\Sigma}$ is the measurements covariance matrix that, in most practical cases, is not known a priori. In these cases, it is therefore useful to assume a simplified model for the errors, dependent only on a standard deviation parameter σ and possibly on a set of dimensionless parameters $\boldsymbol{\kappa}$. These additional parameters can be considered as nuisance parameters, to be estimated together with the other unknown parameters.

The simplest assumption on the covariance matrix is to consider the deformation rate measures as independent and with same variance (homoschedasticity). With this simplification, the Maximum Likelihood Estimation (MLE) problem (3) is reduced to the well-known Least Square Estimation (LSE), which can be further reduced by noting that the dependence on v is quadratic and can therefore be eliminated analytically from the least squares functional.

It should be noted that the usage of LSE is justified by the assumption of normality and homoschedasticity of the errors. While these conditions seem reasonable in direct measurements $\dot{\eta}_n$ they are certainly violated after a variable transformation such as inversion $\dot{\eta}_n^{-1}$ or multiplication by time $t_n \cdot \dot{\eta}_n$. In all methods that require these transformations, like IV and SLO, the usage of LSE can lead to relevant biases in the final estimation. To reduce estimation bias, various authors (e.g. Carlà et al. 2017 and Mazzanti et al. 2015) recommend performing moving averages before implementing those methods. Temporal filtering of the measurements, while showing an excellent bias mitigation, is a data processing step that requires the arbitrary choice of an averaging window. Using a non-linear LSE directly on the measured data, allows the estimation to be performed without previous time filtering, ensuring the best possible estimation in the Bayesian interpretation.

It should also be noted that the homoschedasticity assumption is not entirely correct when velocity measurements are derived from displacement measurements. In this case two consecutive velocity measures are always anticorrelated, and the covariance is best represented by a tridiagonal matrix. A further weakness of homoschedasticity, is the assumption that errors have the same variance across all measurements. When this is not the case (heteroskedasticity), the MLE of the parameters will be biased and likelihood function should be modified to incorporate the precise form of heteroscedasticity.

All these variations on measurement errors model are clearly applicable in a non-linear regression of velocity measurements, while it is not equally clear how to apply them after a transformation of the variables, as is the case for the SLO and IV methods.

2.3 Failure condition

To determine the TOF starting from a kinematic model, it is not sufficient to estimate its unknown parameters, but it is also necessary to impose a kinematic failure condition. Usually, this condition is given by a velocity threshold v_f which identifies the occurrence of the failure:

$$v_f = v \cdot f(ut_f, \boldsymbol{\alpha}) \tag{4}$$

Inverting this relation, it is therefore possible to compute the time of failure in terms of the model parameters: $t_f(v_f, u, v, \boldsymbol{\alpha})$. For pure Saito model this correspond to

$$t_f = u^{-1}(1 - v/v_f) \tag{5}$$

which shows that, using Saito's critical time u^{-1} instead of true TOF t_f, can be considered a good approximation as long as the current velocity v is much smaller than the expected failure velocity v_f. In the following for simplicity, TOF will always be assumed equivalent to critical time.

2.4 Goodness of forecast

Once the TOF has been estimated, it is possible to investigate its plausibility range, i.e. what underlying values of t_f could plausibly have produced the observed data. The relative likelihood ratio $R(u, v|\hat{u}, \hat{v})$ helps answer this question.

Relative likelihood of any Saito's parameters values may be found by comparing the likelihoods $\mathcal{L}(u, v\big|\{t_n, \dot{\eta}_n\}_{n=1}^N)$ with the maximum one $\mathcal{L}(\hat{u}, \hat{v}\big|\{t_n, \dot{\eta}_n\}_{n=1}^N)$. Being primarily interested in the plausibility of TOF and therefore of u, the relative likelihood can be marginalized over v. Once marginalized relative likelihood $R(u|\hat{u}, \hat{v})$ is computed, it is possible to define likelihood intervals. A likelihood interval $U\gamma$ with a confidence level γ is the set of all values of u whose relative likelihood is greater than a given threshold $\lambda\gamma$, that is usually derived from the quantile function of the standard normal distribution. Finally, to assess the reliability of TOF forecast, it is possible to compare the TOF likelihood interval size with the TOF estimated value: when the ratio tends to zero, the reliability level will correspondingly tends to 1.

3 CASE STUDY

To better illustrate the method discussed above, regression analysis of pure Saito model has been applied on a slope collapse dataset, acquired by the IDS GeoRadar IBIS-FM GB-SAR system (Farina et al. 2011) The analyzed event took place in an open-pit copper gold mine at 18:48 29/11/2014 UTC time. IBIS-FM system continuously acquired deformation data throughout the evolution of the slope failure, with an acquisition time interval of 5 minutes.

To validate the non-linear regression approach, a 24 hours time interval consisting of 284 GB-SAR measurements, has been selected. The time interval was chosen to end exactly 24 hours before the actual collapse. In Figure 1 the corresponding GB-SAR displacement map is shown, with the pre-failure slope deformation clearly visible.

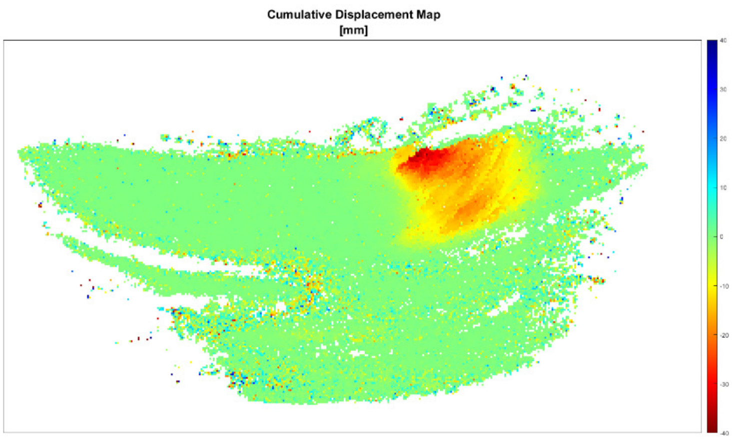

Figure 1. GB-SAR 24 hours displacement map [mm] acquired 24 hours prior the collapse.

From the reliability map (Figure 2), computed according to section 2.4, it is possible to distinguish between actually accelerating points and stable or noisy points. Figure 3 shows estimated parameter

$\hat{u}$ map, after having filtered out all the points with reliability lower than 0.5. From the spatial distribution of $\hat{u}$, it is possible to notice a substantial uniformity of the estimated TOF, which likely indicates a simultaneous collapse of the entire moving area.

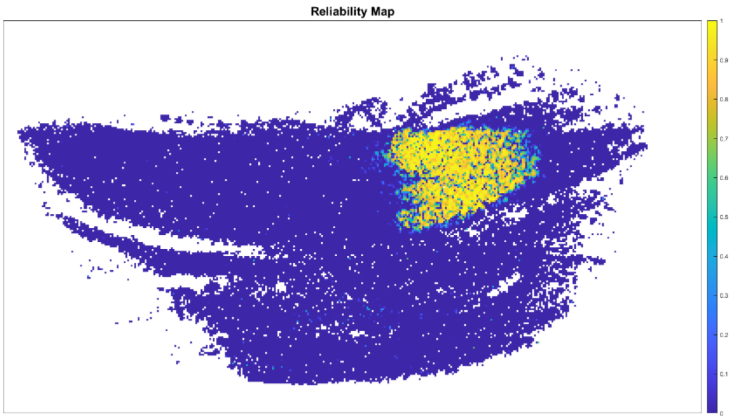

Figure 2. Reliability map obtained from pure Saito model regression.

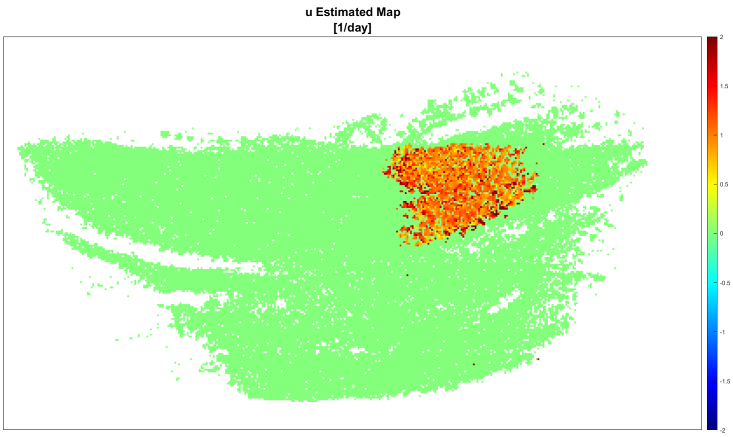

Figure 3. Estimated u parameter map [1/day] obtained from pure Saito model regression.

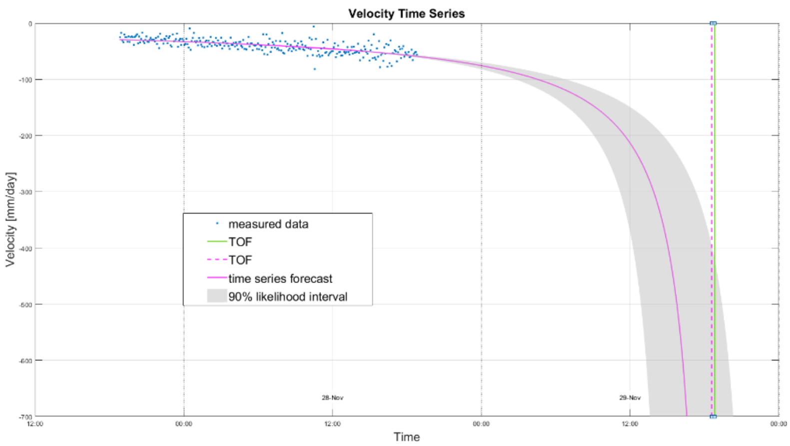

Figure 4. GB-SAR velocity time series [mm/day] and TOF forecast.

A time series has been extracted from the fastest part of the moving area. Regression analysis results are displayed in Figure 4. The difference between estimated and true TOF is only about 15 minutes. Likelihood interval at 90% of confidence provides a bandwidth on the estimated TOF of about ±3 hours, which can be a useful additional information for technicians who must decide on hazard response operations.

4 CONCLUSION

In the present work, non-linear regression analysis of kinematics models for TOF forecast has been discussed. The method presented has some advantages over other typical estimation methods, such as no time filtering needing and possibility of measurements errors modeling.

The validation on a real GB-SAR dataset showed the effectiveness of the proposed method and its potential for integration into an automatic alarm system.

Future works will focus on a systematic comparison with the well-assessed IV and SLO methods, and on the development of an automatic algorithm for the generation of real-time alarms.

REFERENCES

Carlà, T., Intrieri, E., Di Traglia, F., Nolesini, T., Gigli, G. & Casagli, N. 2017. Guidelines on the use of inverse velocity method as a tool for setting alarm thresholds and forecasting landslides and structure collapses. *Landslides* 14(2), pp. 517-534.

Dick, G.J., Eberhardt, E., Cabrejo-Liévano, A.G., Stead, D. & Rose, N.D. 2015. Development of an early-warning time-of-failure analysis methodology for open-pit mine slopes utilizing ground-based slope stability radar monitoring data. *Can. Geotech. J.* 52 (4), pp. 515–529.

Farina, P., Leoni, L., Babboni, F., Coppi, F., Mayer, L. & Ricci, P. 2011. IBIS-M, an Innovative Radar for Monitoring Slopes in OpenPit Mines. In: *Proceedings International Symposium on Rock Slope Stability in Open Pit Mining and Civil Engineering (Slope Stability 2011)*, Vancouver, Canada, September 18 – September 21, 2011, Canadian Rock Mechanics Association, Canada, CD-rom only

Federico, A.D., Popescu, M., Elia, G., Fidelibus, C., Internò, G. & Murianni, A. 2012. Prediction of time to slope failure: a general framework. *Environmental Earth Sciences* 66, pp. 245-256.

Fukuzono T. 1985. A new method for predicting the failure time of a slope. In: *Proceedings of 4th international conference and field workshop on landslides*, Tokyo, Japan, August 23 – August 31, 1985, pp. 145–150.

Hagstrom, G. I. & Levin, S. A. 2021. Phase transitions and the theory of early warning indicators for critical transitions. arXiv preprint arXiv:2110.12287.

Intrieri, E., Carlà, T. & Gigli, G. 2019. Forecasting the time of failure of landslides at slope-scale: A literature review. *Earth-science reviews* 193, pp. 333-349.

Mazzanti, P., Bozzano, F., Cipriani, I. & Prestininzi, A. 2015. New insights into the temporal prediction of landslides by a terrestrial SAR interferometry monitoring case study. *Landslides* 12(1), pp. 55-68.

Mercer, K.G. 2006. Investigation into the time dependent deformation behaviour and failure mechanisms of unsupported rock slopes based on the interpretation of observed deformation behaviour. Ph.D. thesis. University of Witwatersrand, Johannesburg, South Africa.

Mufundirwa, A., Fujii, Y. & Kodama, J. 2010. A new practical method for prediction of geomechanical failure-time. *International Journal of Rock Mechanics and Mining Sciences* 47(7), pp. 1079-1090.

Praveen, K. R., Chophel, J., Wangchuk, J., Tshewang, K. & Bansal, N. C. 2022. IBIS-FM radar as a tool for monitoring hill slopes during excavations in hydropower projects: A case study from Bhutan Himalayas. *Energy Geoscience* 3(2), pp. 211-217.

Rossi, R. J. 2018. Mathematical statistics: an introduction to likelihood based inference. John Wiley & Sons.

Saito, M. 1965. Forecasting the time of occurrence of a slope failure. In: *Proceedings of 6th international conference on soil mechanics and foundation engineering*, Montreal, Canada, September 8 – September 15, 1965, pp. 537–541.

Voight, B. 1988. Materials science law applies to time forecasts of slope failure. In: *Proceedings of 5th international symposium on landslides*, Lausanne, Switzerland, July 10 – July 15, pp. 1471–1472.

Wang, Y., Hong, W., Zhang, Y., Lin, Y., Li, Y., Bai, Z., Zhang, Q., Lv, S., Liu, H. & Song, Y. 2020. Ground-based differential interferometry SAR: A review. *IEEE Geoscience and Remote Sensing Magazine* 8(1), pp. 43-70.