Simulating on the evolution of natural pores and induced microcracks in rock samples: an elastoplastic damage Gurson type model

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ABSTRACT: A mechanically acceptable model to explain the evolution of discontinuities inside rock mass and their effect on the entire rock mechanical performance is still required. In this study, an elastoplastic (hardening and softening) damage Gurson-type model is proposed. The behaviour of discontinuities is separated into natural voids and induced microcracks. The behaviour of natural void is further explained by Gurson's model, considering both the healing mechanism in the initial compaction stage and the damage effect in the crack development stage. The induced microcracks are considered as internal damage, manifested as a damage variable, which directly affects the yielding envelope of porous media (coupled with Gurson's model) and rock grains (Drucker-Prager law in the effective stress domain). The proposed model shows great consistency with laboratory observations.

Keywords: microcracking, rock mechanics, constitutive model, Gurson's model.

1 INTRODUCTION

Many pioneering studies pose the importance of microcracking effect inside rock samples (You et al. 2021; Li et al. 2023), where the performance of microcracking can significantly affect the entire rock mechanical behaviour. From nano-size microcracks to geological faults, stress is redistributed around the tips and the existence of microcracks dominates the marco scale mechanical properties of rock material. The influence of microcracking behaviour is obvious, even from a laboratory perspective (Li et al. 2023). A few studies notice an initial concave stage in conventional triaxial tests of intact rock samples, resulting in the effect of microcracking closure (Li et al. 2022). On the other hand, the non-linear stress-strain relationship beyond the yielding surface indicates the development of microcracks, manifested by the deterioration of rock samples. In addition, the notched rock sample indicates the propagation mechanism of pre-existing discontinuities, such as pre-slip and triggering, to conclude the discontinuity effect in rock mechanics.

We also noticed that some previous studies empirically investigate the effect of microcracks of rock samples, such as using a double yield model to simulate goaf compaction in longwall coal mining (Shabanimashcool and Li 2012). The effect of goaf compaction (microcrack healing) is related to the plastic strain generated in the goaf zone. Another approach is to use a damage variable to represent the deterioration of Young's modulus, which is introduced as a typical microcracking effect (Shen et al. 2022). The damage variable is considered an independent variable and affects the evolution of rock mechanical response. Statistical analysis is another effective way to consider microcracking effects, owing to the randomness of pre-existing cracks, as a typical inhomogeneous character of rock samples (Cai et al. 2018). However, although previous research achieves convincing research in fitting rock stress-strain relationships and explains geotechnical issues in engineering problems, the constitutive relationship they applied does not obey the thermodynamic framework. Those models may easily violate under complex in-situ geotechnical conditions.

In this study, we attempt to construct a new elastoplastic (hardening and softening) damage Gurson-type model following the framework of thermodynamics. The Gurson model has been embedded into the damage theory as well as the plastic theory to achieve the best fitting of laboratory results. The proposed model is capable of reproducing the laboratory results.

2 METHOD

2.1 *Macroscopic model: Modification of Gurson's model*

The microcracking behaviour can be separated into initial (pre-existing) cracks and induced cracks. The initial cracks are naturally situated in rock samples including the pore structures to dominate the initial compaction stage of rock samples. On the other hand, the induced cracks are more likely to relate to the damage effect, as the deterioration of Young's modulus. In this study, those two effects are separated based on a previous study (Shen et al. 2022).

$$\kappa_{mac} = \frac{4(1-f)\kappa_{mes}\mu_{mes}}{4\mu_{mes} + 3f\kappa_{mes}} \tag{1}$$

$$\mu_{mac} = \frac{(1-f)\mu_{mes}}{1 + 6f\dfrac{\kappa_{mes} + 2\mu_{mes}}{9\kappa_{mes} + 8\mu_{mes}}} \tag{2}$$

$$\kappa_{mes} = \frac{4(1-d)\kappa_{mic}\mu_{mic}}{4\mu_{mic} + 3d\kappa_{mic}} \tag{3}$$

$$\mu_{mes} = \frac{(1-d)\mu_{mic}}{1 + 6d\dfrac{\kappa_{mic} + 2\mu_{mic}}{9\kappa_{mic} + 8\mu_{mic}}} \tag{4}$$

where, κ_{mic} and μ_{mic};κ_{mes} and μ_{mes} are the bulk and shear modulus on the microscopic and mesoscopic scale. Equations (1) to (4) enable to calculate the porosity from the modulous of rock samples. The original Gurson's model only considers one pore-related variable (e.g., porosity), whereas two crack-based variables (initial cracks and induced cracks) are engaged in this study (Tvergaard and Needleman 1984). Hence, we need to modify Gurson's model to consider the initial cracks and induced damage. The modified Gurson's model is written as:

$$\Phi = \left(\frac{\sigma_e}{\Sigma_M}\right)^2 + 2q_1 f \cosh\left(\frac{q_2}{2}\frac{\sigma_{kk}}{\Sigma_M}\right) - [1 + (q_1 f + q_3 d)^2 - 2q_3 d] = 0 \tag{5}$$

where, f is the porosity of rock sample (initial pore); σ_e is the macroscopic stress in the RVE of Gurson's model (see Figure 1) and its expression will be introduced later. σ_{kk} is the hydrostatic stress in macroscopic stress space. q_1, q_2 and q_3 are fitting parameters in Gurson's type envelope. d is the damage variable, represented by the induced microcracks in rock samples. Σ_M is the yielding stress based on the failure envelope of solid grains. We assume that a Drucker-Prager envelope can be a reasonable approximation and the transformation from mesoscopic stress to macroscopic stress follows $(1-f)\Sigma_m = \sigma_m$ (Zhou and Zhu 2010). Thus, the yielding surface of rock grains can be simplified as:

$$\alpha_{mes}\Sigma_m + \Sigma_M - k_{mes} = 0 \tag{6}$$

where, α_{mes} and k_{mes} are fitting parameters for mesoscopic rock grains.

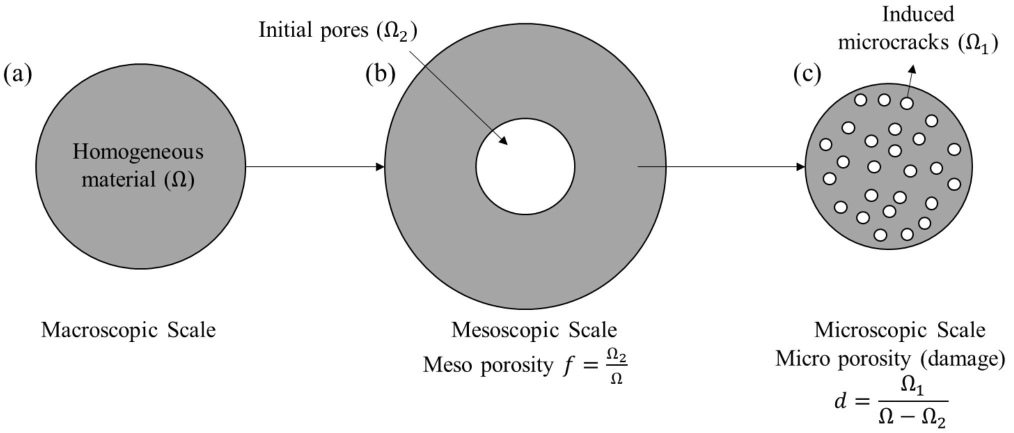

Figure 1. The RVE of (a) homogenous rock material, (b) the mesoscopic scale model with initial pore structures and (c) the microscopic scale model with induced microcracks.

2.2 Microscopic model: Drucker-Prager model

According to Section 2.1, the selection of Gurson's model requires setting the yielding surface of rock grains according to the Drucker-Prager law.

$$\alpha_{mic}\tilde{\sigma}_m + \sqrt{\tilde{J}_2} - k_{mic} = 0 \tag{7}$$

where, α_{mic} and k_{mic} are fitting parameters for the microscopic scale Drucker-Prager model and $\tilde{\sigma}_m$ is the effective hydrostatic stress in the microscopic scale and $\tilde{J}_2$ is the effective deviatoric stress in the microscopic scale. The transformation between mesoscopic stress to microscopic stress starts from damage mechanics in a previous study (Shao et al. 2006), such that $\tilde{\sigma}_{ij} = (1-\chi d)\sigma_{ij}$. σ_{ij} is the microscopic stress and χ is a fitting parameter determining the damage effect from the mesoscopic to the microscopic scale. From Sections 2.1 and 2.2, in the proposed model, there are two yielding envelopes controlling the plastic flow of rock samples according to Equations (5) and (7). Hence, the plastic flow applied in this study is a 'double-yield' yielding surface. The plastic flow will require considering the expansion of both yielding surfaces.

2.3 Plastic flow and hardening

For the modified Gurson's model, we assume the plastic strain is coming from pre-existing pore structures. We hence assume that the change in pore structure does not affect the shape of entire pore structures, namely that the plastic strain generated from initial pore structures only generates hydrostatic plastic strain rather than deviatoric plastic strain. Also, the hardening variable for Equation (5) is macroscopic porosity (f). Since only hydrostatic plastic strain is considered:

$$df = A_N d\varepsilon_{kk}^{p_G} \tag{8}$$

where, A_N is a parameter related to the hydrostatic plastic strain to be defined in Equation (8). $\varepsilon_{kk}^{p_G}$ is the hydrostatic plastic strain engaged in Gurson's model.

$$A_N = \frac{f_n}{S_n\sqrt{2\pi}} e^{\left[-\frac{1}{2}\left(\frac{\varepsilon_{kk}^{p_G} - \varepsilon_n}{\varepsilon_n}\right)^2\right]} \tag{9}$$

where, f_n, S_n and ε_n are three fitting parameters, controlling the relationship between the increment of volumetric plastic strain and the change in the initial pore structures. On the other hand, regarding the microscopic model, the hardening variable is selected as deviatoric plastic strain (Υ_p). The hardening law of the microscopic model is based on a well-applied model and the relationship between k_{mic} and Υ_p can be drawn:

$$k_{mic}(\Upsilon_p) = k_0 + (k_{max} - k_0)\sqrt{1 - \left(\frac{\Upsilon_p - \Upsilon_p^t}{\Upsilon_p^t}\right)^2} \tag{10}$$

where, k_0 is the k_{mic} at the initial yielding surface, at which the non-linear plastic behaviour happens. k_{max} is the maximum hardening variable, dominating the maximum expansion of the yielding envelope. Υ_p^t is the maximum hardening parameter, to regulate the level of plastic strain that the tested rock sample can approach.

3 RESULTS

The laboratory test results come from a previous study where triaxial tests on limestone samples were conducted (Meng et al. 2021). In this study, we mainly focus on the axial and lateral stress-strain relationships of the 5MPa confinement case. The axial stress-strain relationship and the percentage of initial pore structures and induced cracks are presented in Figure 2. The axial stress-strain relationship is modified since we remove the loading-unloading cycles inside the test to achieve a better illustration. The stress-strain relationship clearly shows the influence of microcracking behaviour on the mechanical response of limestone samples. Hence, the percentage of initial pore structures and induced microcracks are determined based on the previous study (Shen et al. 2022) and the computational results are presented in Figure 2. Interestingly, at the initial crack-closure stage, the initial pore decreases, indicating an effect of crack closure. The initial pore then decreases to zero when the axial load is high. However, the development of initial pores is observed at the post-peak stage, and analogously, the induced crack also experiences an increasing trend at the post-peak stage.

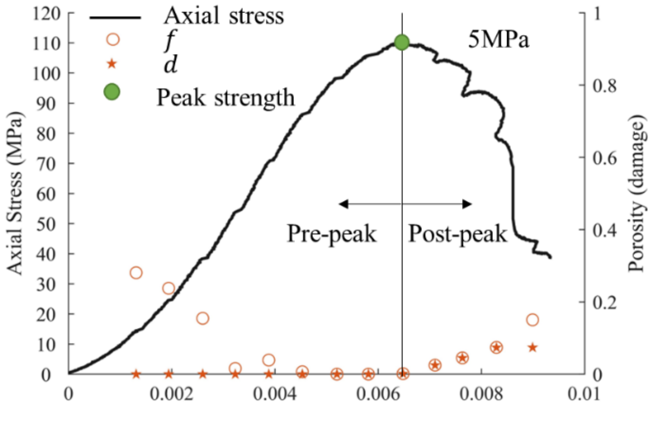

Figure 2. Evolution of stress-strain relationship of limestone rock sample with 5MPa confinement. The solid line indicates the axial stress-strain relationship of limestone samples; orange circles represent the evolution of pre-existing pore structures inside the rock sample and the orange stars represent induced cracks, especially at the post-peak stage.

Even though we observe a closure (decreasing) trend of initial pore structures in Figure 2, similar to the healing mechanics (at least mechanically equivalent). In other words, we do not simulate the initial pore closure stage but fully focus on the post-peak stage, where the induced damage parallelly develops with the initial pore structure. The evolution of the initial pore structure (f) is considered a hardening variable for Gurson's type model mentioned in Equation (5), related to the external load applied from the platen and hydraulic oil in the triaxial test. The evolution of induced damage (d), on the other hand, needs to be specified, especially its evolution law. We review the evolution of induced damage from a traditional damage mechanics point of view and the damage loading model (Shao et al. 2006):

$$d = c_d^1 (Y_d - Y_{d(max)}^0) \tag{11}$$

The fitting parameters for the proposed model are illustrated in Table 1.

Table 1. Fitting parameters in the proposed model.

Parameters					
κ_{mac}	10GPa	μ_{mac}	16.69GPa	q_1	1.5
q_2	0.1	q_3	1	α_{mes}	-0.2114
k_{mes}	35	α_{mic}	0.212	k_{mic}	27
f_n	0.05	S_n	-0.038	ε_n	-0.1161
Y_p^t	0.002	k_{max}	35		

The result of the proposed model is presented in Figure 3. We notice that the axial plastic strain is overestimated and the lateral plastic strain is underestimated, owing to the divergent data set in the fitting of plastic potential. The plastic strain on the axial stress-strain relationship is overestimated whereas the lateral stress-strain relationship is underestimated. In addition, the effect stress in the post peak zone is still plausible since that the parameters f and d in the post-peak stage can not well describe the drop of stress in macroscopic sacle. More studies are still required to better fit the model into the laboratory observations of rock samples.

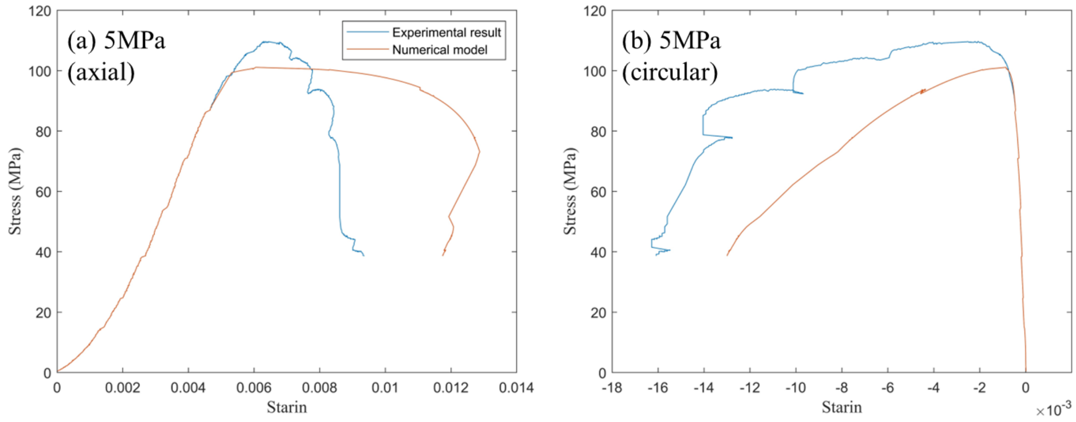

Figure 3. Comparison of computational model results and laboratory observation of limestone sample under 5MPa confinements for (a) axial stress-strain relationship and (b) lateral stress-strain relationship.

4 CONCLUSION

In this study, we proposed a new elastoplastic (hardening and softening) damage Gurson-type model based on our observations from a triaxial lab test. The initial concave stress-strain relationship and the following-up damage development indicate the microcracking behaviour is ignorable when performing rock constitutive modelling. Hence, we consider both initial pore structures and induced cracks in the proposed model. The initial pore structure indicates a clear pore closure behaviour at the initial loading stage, regulated by the Gurson-type model. The induced microcracks on the other hand are considered a damage variable, where a traditional Drucker-Prager yielding surface is applied to simulate the laboratory observations. The proposed model, however, still needs more future attempts to improve the stress-strain relationship.

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