A reliability-based design approach for geotechnical domain modelling in pit slope design

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ABSTRACT: The inherent variability of rock mass properties unavoidably leads to imprecisions in open pit geotechnical models, which can increase the likelihood of failures in pit slopes. To overcome this, a reliability-based design approach is advocated to allow for adequate consideration of the design's uncertainties. In this study, the First Order Reliability Method (FORM) was used to determine the reliability of a pit slope design through a probability of failure. Data from the Bozshakol copper mine located in north Kazakhstan was used in this study. The input data for the FORM analysis were the rock mass properties and the slope design parameters corresponding to selected sectors of the pit. The reliability indices evaluated for each geotechnical domain show good agreement with the slope displacement observations. The results of this study illustrate the importance of a reliability-based design and its capability to be used as basis for improvement of pit slope design.

Keywords: Reliability analysis, reliability index, probability of failure, open pit slope, geotechnical domain modelling, factor of safety.

1 INTRODUCTION

A major risk to the reliability of the geotechnical model, and hence to the slope design performance, arise from data uncertainties associated with the inherent variability of rock mass properties (Valerio et al. 2013). In practice, these uncertainties are usually handled by implementing a conservative design (e.g., the worst-case scenario) through a deterministic approach in which a single value for the safety factor of the slope is used to represent the overall stability of the slope while the variability of input design parameters and other uncertainties are not explicitly accounted for. However, experience shows that the conservative designs are not always free from failure (Duzgun et al. 2003; R Jimenez-Rodriguez et al. 2006; Phoon 2008).

To overcome these shortcomings of geotechnical modelling, it is necessary to implement a reliability-based design (RBD) approach that allows for adequate consideration of the uncertainties associated with the design. The RBD methods for slope design have been now adopted by many practitioners and researchers (Carter & Barnett 2022; Gaida et al. 2021; Hu et al. 2022; R. Jimenez-

Rodriguez & Sitar 2007; Read & Stacey 2009; Wang et al. 2013; Zuo et al. 2021). In general, the main purpose of the RBD methods is to determine the likelihood of achieving a certain performance target against failure. This would allow to determine the probabilities of failure of the slope appropriate decision making. Driven by these motivations, in this paper, the reliability indices and probability of failure of an open pit slope for each geotechnical domain are assessed using the First Order Reliability Method (FORM). Data from the Bozshakol Mine located in Kazakhstan are used in the case study.

2 CASE STUDY DESCRIPTION

The Bozshakol Copper Mine, owned by KAZ Minerals, is an open pit operation located in Pavlodar Region, eastern Kazakhstan. The mine exploits a copper-porphyry deposit to produce 7Mtpa of 20-28% copper concentrate (Kazminerals 2023). The pit extends to northeast direction and is divided into 8 sectors. Currently, the depth expansion of the pit is at about 190 m, 700 m in width and more than 2km in length. However, in the next decade, it is expected that the pit will reach the depth of 570 m and the length of 4.5 km. Currently, the pit is exploited in the northeast direction toward sector 8. The pit design parameters are as follows: slope angle varies between 55 and 65°, bench height 10-30m, bench width 5-9.5m and the inter-ramp slope 40-52°, depending on the rock domains.

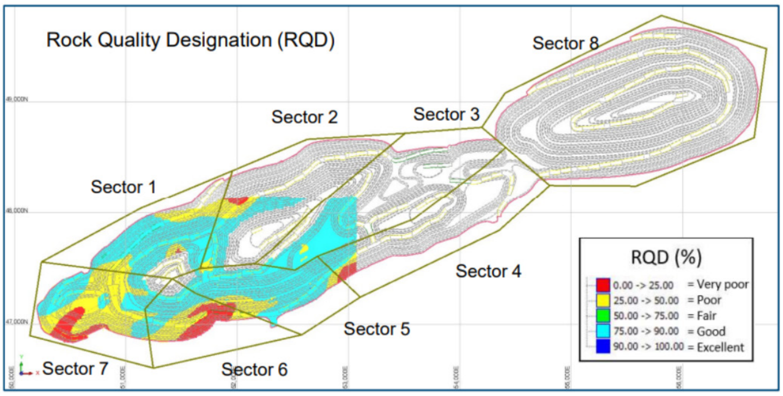

Figure 1. RQD distribution within the pit boundaries.

Table 1. Sample of the geotechnical data.

Sample		1	2	3	4	5
Rock domain		Granodiorite	Andesite	Saprolite	Andesite	Breccia
RDQ	[%]	75-90	25-50	60	50-75	80-85
GSI	[-]	70-75	30-35	35	60-65	70-75
Joint orientation	[°]	303/36	007/40	130/30	204/30	140/40
Joint cohesion	[MPa]	4.13	0.24	0.15	1.2	0.08
Joint friction angle	[°]	49	29	26	35	25
UCS	[MPa]	150	65	50	80	52

The rock mass is characterized by numerous discontinuities contributing to the formation of the folded-block structure, magmatic complexes, and ore-metasomatic systems. The presence of faults influences the block structure of the deposit with extended cracks. The extension of these faults along the deposit´s fault zone in the longitudinal east-northeast direction contains confined ore-bearing intrusive minerals such as granitoid dikes. The featuring fault zones along the northeast strikes of the deposit produce several displacements. This fracturing has primarily been identified within the pit geometry and consists of 3 major systems with the dip directions/dip angles as follows: 160°–190°/60°–80°; 220°–260°/20°–70°; 310°–340°/30°–50°, respectively. The first system corresponds to crushing zones along the main direction, which can cause slope instability issues. The deposit was divided into several geotechnical domains, including, weathered (clay), fractured (saprolite), indigenous (andesite, gabbro), intrusive (breccia, granodiorite, diorite), and sedimentary. Figure 1 shows the RQD distribution per sectors of the pit. Table 1 presents a sample of the geotechnical data.

3 RELIABILITY ANALYSIS AND PROBABILITY OF FAILURES

3.1 Overview of the First Order Reliability Method (FORM)

The First Order Reliability Method (FORM) is a probabilistic approach to evaluate the reliability of a system by searching for a "most probable point" on the limit state of a design. For example, when determining the factor of safety of a rock slope, the boundary dividing the safe domain (R) and failure domain (S) is the limit state surface (boundary) expressed as:

$$G(\mathbf{x}) = R - S \tag{1}$$

where **x** denotes a vector of variables controlling the slope stability and is considered as random variables. The $G(\mathbf{x})$ function can be used to represent the overall performance of the design with $G(\mathbf{x}) > 0$ corresponding to safe design while $G(\mathbf{x}) < 0$ considered as unsafe design i.e. failed slope. However, in most situations, $G(\mathbf{x})$ is not known explicitly and, therefore, its approximate determination can be implemented implicitly through a numerical procedure, such as, the finite element method. The failure probability of the slope P_F, can be calculated via a multidimensional probability integral defined as (Ang & Tang 1975):

$$P_F \equiv P\big(G(\mathbf{x}) \leq 0\big) = \int_{G(\mathbf{x})\leq 0} p_X(\mathbf{x})d\mathbf{x} \tag{2}$$

In Eq. (2), $G(\mathbf{x})$ is defined on a space of x random variables and $p_X(\mathbf{x})$ is the joint probability density function (PDF). Since $p_X(\mathbf{x})$ is unknown and G(**x**) is non-linear and complex, approximate solutions such as the algorithm proposed by Low & Tang (2007) are used in this study. The probability of failure is governed by a reliability index β and defined as:

$$\beta = \min_{x \in F} \sqrt{\left(\frac{x-\mu}{\sigma_k}\right)^T (\mathbf{R})^{-1} \left(\frac{x_k-\mu_k}{\sigma_k}\right)^T} \tag{3}$$

where: **R** is the correlation matrix, μ_k and σ_k are the mean and standard deviation of random variable x_k, respectively. The design point i.e., the most probable failure point is expressed by the x_k values, which are determined by minimizing the square root of the quadratic form represented in Eq.(3). Low & Tang (2007) developed an alternate and convenient FORM algorithm in excel spreadsheet by varying basic random variable **x**. Eq. (3) is rewritten as:

$$\beta = \min_{x \in F} \sqrt{\mathbf{n}^T \mathbf{R}^{-1} \mathbf{n}} \tag{4}$$

where: **n** is a column vector of n_k. Once the reliability index β is obtained, the probability of failure PF can be evaluated as:

$$P_F \approx 1 - \Phi(\beta) \tag{5}$$

where: Φ is the cumulative density probability (CDF) of a standard normal distribution.

3.2 Results

In this study, three parameters were selected to calculate the reliability index associated with the slope stability: the RDQ, RMR, discontinuity orientations, intact rock strength and the slope geometry; depending on the geotechnical domains and the pit sector.

The reliability index β for the slope stability considering each geotechnical domains (rock types), was determined using the EXCEL spreadsheet tool developed by Low & Tang (2007), where the performance function was defined as $G(x) = FS - 1$, FS being the Factor of Safety. These calculations enabled hand-on reliability analysis and were successfully employed in previous studies (Adoko et al. 2022; Goh & Zhang 2012).

An example of the reliability index computation corresponding to sector 6 is shown in Figure 2. The input parameters used were RDQ/Jn denoted as P1, the joint strength given by the angle of internal friction ϕ (P2) and the height to width ratio of the pit bench (P3). The geotechnical data indicated that these parameters were normally distributed as specified in cells A2:A4 (Figure 2). Cells C2:C4 correspond to the mean values while cells D2:D4 correspond to the standard deviations. The correlation matrix **R** (Eq. 3) is represented by cells K2:M4 showing that the variables are non-correlated. The n_x vector in cells N2:N4 is specified according to Eq. 4. The design points (x^* values), were initially assigned the mean values then the SOLVER search algorithm was invoked. Subsequently, iterative numerical derivatives and directional search for the design point x* were automatically carried out and the reliability index was computed. The calculations were performed for all domains and for each sector. The results are summarized in Table 2 and Figure 3.

	A	B	C	D	G	H	I	J	K	L	M	N	O	P	Q
1	Distribution		Para1	Para2	x^*	μ^N	σ^N		Correlation matrix			n_x	g(x)	β	Pf
2	Normal	P1	7.78	14	0.0263	7.78	14		1	0	0	-0.554	7E-08	0.571	28%
3	Normal	P2	26	4	25.442	26	4		0	1	0	-0.139	=YZ - M		
4	Normal	P3	0.67	0.5	0.6704	0.67	0.5		0	0	1	0.0008	(x* values)		
5															
6												(x* - μ^N)/σ^N			
7															

Figure 2. Example of reliability index β calculated for rock domain 3 in pit sector 6.

Table 2. Range of the computed reliability indexes and probability of failure.

Domain	#1	#2	#3	#4
Lithology/rock type	Clay	Saprolite	Andesite	Breccia
Reliability index range	2.051-0.844	1.88-0.02	1.468-0.101	1.647-0.335
Probability of failure range	2-20%	3-50%	7-45%	5-37%

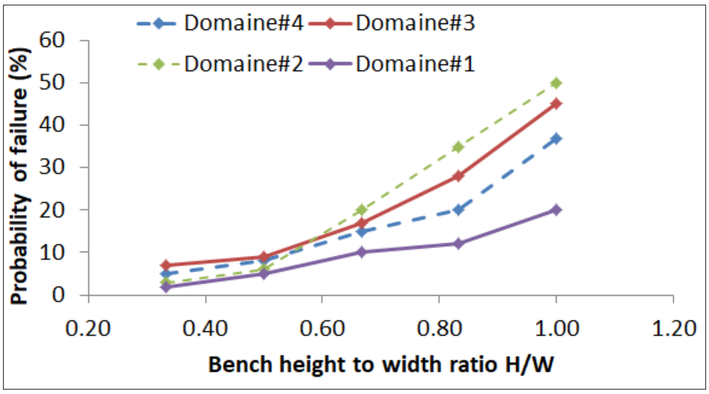

Figure 3. Probability of failure vs H/W.

3.3 *Discussions*

Table 2 indicates that the reliability index values vary depending on the rock domains. Domain #1 was shown as the most reliable while domains #2, #3 and #4 were the least reliable. Figure 3 shows the probability of failure vs the slope geometry. It can be seen that the probability of failure increases with an increase of the H/W ratio, which agrees well with the principles of a pit slope design. In addition, it was observed that the probability of failure was very sensitive to the coefficient of variation of RDQ/Jn (P1) and, to a lesser extent, to the strength of the rock domain (P2). It should be noted that the highest value of reliability index is 2.051, which suggests a poor performance in term of expected reliability level for geotechnical structure (Adoko et al. 2022). This seems to concord with the field observations and the monitoring of the daily slope displacement rates.

In May 2022, the south wall of sector 6 experienced some instabilities as shown in Figure 4. Unexpected displacement patterns were recorded on a bench located between levels -140 and -120. The displacement velocity in the active phase of failure went up to 7.9 mm/h. Based on the historical slope displacement data , the pit slopes present low geotechnical risks, as evidenced by the daily displacements (0-5mm/day), which contradicts the instabilities observed in the field (Kazminerals 2022). In addition to the blasting activities, the reliability of geotechnical domains used for the design impacted to slope stability. As the pit expands, more geotechnical data will be available to adjust the design.

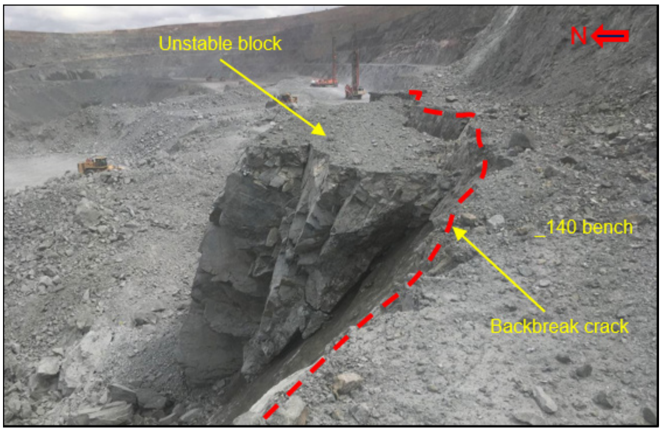

Figure 4. Example of slope instabilities (Sector 6).

4 CONCLUSIONS

This study was aimed at evaluating the probability of failure of pit slopes in Bozshakol mine. This was achieved through the use of a built-in spreadsheet optimization routine, which implemented the First Order Reliability algorithm. The selected input parameters were RQD, the joint number (Jn), joint strength, and the pit bench width and height.

The calculated reliability indices and the probabilities of failure vary from 2.05 to 0.02 and from 2 % to almost 50 %, respectively. The reliability analyses indicated that the probability of failure was significantly influenced by RQD/Jn and, to a lesser extent, by the ratio of bench height to the bench width, depending on the rock domains. This means that, for the same design, the stability is governed by the variability of the rock parameters. The results concord with the actual slope performance. Hence, it is suggested that displacement monitoring and the reliability index be used together as complementary to manage pit slope stability issues.

The results of the present study serve as the basis to fine tune the displacement thresholds used in the geotechnical risk assessment implemented in a particular mine site. Further studies will introduce additional parameters influencing slope stability, including joint orientations, rock strength, and ground vibration to better reflect real site conditions.

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REFERENCES

Adoko, Amoussou Coffi, Yakubov, Khamit, & Kaunda, Rennie. (2022). Reliability Analysis of Rock Supports in Underground Mine Drifts: A Case Study. *Geotechnical and Geological Engineering, 40*(4), 2101-2116. doi: 10.1007/s10706-021-02014-4

Ang, Alfredo H S, & Tang, Wilson H. (1975). *Probability concepts in engineering planning and design-Basic principles* (August 4, 1975 ed. Vol. 1). New York Wiley.

Carter, Trevor G., & Barnett, Wayne P. (2022). Improving Reliability of Structural Domaining for Engineering Projects. *Rock Mechanics and Rock Engineering, 55*(5), 2523-2549. doi: 10.1007/s00603-021-02544-6

Duzgun, HSB, Yucemen, MS, & Karpuz, CELAL. (2003). A methodology for reliability-based design of rock slopes. *Rock Mechanics and Rock Engineering, 36*(2), 95-120.

Gaida, M., Cambio, D., Robotham, M.E., & Pere, V. (2021). *Development and application of a reliability-based approach to slope design acceptance criteria at Bingham Canyon Mine*. Paper presented at the SSIM 2021: Second International Slope Stability in Mining, Online and Perth, Australia. https://papers.acg.uwa.edu.au/p/2135_02_Robotham/

Goh, A. T. C., & Zhang, W. (2012). Reliability assessment of stability of underground rock caverns. *International Journal of Rock Mechanics and Mining Sciences, 55*, 157-163. doi: https://doi.org/10.1016/j.ijrmms.2012.07.012

Hu, J. Z., Zhang, J., Huang, H. W., & Zheng, J. G. (2022). Assessing expected benefit of site investigation program for reliability-based design of slope. *Engineering Geology, 306*, 106749. doi: https://doi.org/10.1016/j.enggeo.2022.106749

Jimenez-Rodriguez, R, Sitar, Nicholas, & Chacón, José. (2006). System reliability approach to rock slope stability. *International Journal of Rock Mechanics and Mining Sciences, 43*(6), 847-859.

Jimenez-Rodriguez, R., & Sitar, N. (2007). Rock Wedge Stability Analysis Using System Reliability Methods. *Rock Mechanics and Rock Engineering, 40*(4), 419-427. doi: 10.1007/s00603-005-0088-x

Kazminerals. (2022). Monthly review: Geotechnical-July: Mining Department, Bozshakol Mine.

Kazminerals. (2023). The official website of the Kazminerals company. from https://www.kazminerals.com

Low, B. K., & Tang, Wilson H. (2007). Efficient Spreadsheet Algorithm for First-Order Reliability Method. *Journal of Engineering Mechanics, 133*(12), 1378-1387. doi: doi:10.1061/(ASCE)0733-9399(2007)133:12(1378)

Phoon, Kok-Kwang. (2008). Reliability-based design in geotechnical engineering: computations and applications: CRC Press.

Read, John, & Stacey, Peter. (2009). *Guidelines for Open Pit Slope Design* CSIRO Publishing.

Valerio, M., Clayton, C., D.'Ambra, S., & Yan, C. (2013). *An application of a reliability based method to evaluate open pit slope stability*. Paper presented at the Slope Stability 2013: Proceedings of the 2013 International Symposium on Slope Stability in Open Pit Mining and Civil Engineering, Brisbane. https://papers.acg.uwa.edu.au/p/1308_29_Clayton/

Wang, Lei, Hwang, Jin Hung, Juang, C. Hsein, & Atamturktur, Sez. (2013). Reliability-based design of rock slopes — A new perspective on design robustness. *Engineering Geology, 154*, 56-63. doi: https://doi.org/10.1016/j.enggeo.2012.12.004

Zuo, Shi, Hu, Changwen, Zhao, Lianheng, Jiao, Kangfu, Lei, Zhibin, Huang, Dongliang, & Zhu, Zhiheng. (2021). Reliability back analysis of a 3D wedge slope based on the nonlinear Barton-Bandis failure criterion. *Engineering Failure Analysis, 128*, 105601. doi: https://doi.org/10.1016/j.engfailanal.2021.105601