Physical Modeling in Rock Mechanics and Rock Engineering

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ABSTRACT: The paper is intended to pay homage to Professor Leopold Müller who was a leading developer and user of physical models. This will be done by reviewing physical models of fundamental material behavior, geologic mechanisms, and especially jointed rock. On this basis complex models of geologic processes and of structures on and in rock masses will be discussed. As will be shown this sequence of topics also corresponds to the history of physical models. Finally, critical aspects, namely the issues of scaling and of obsoleteness because of powerful simulation models will be addressed leading to the outlook as to where physical models can and should be used.

Keywords: Physical Models, Material, Geology, Jointed Rock, Rock Structures.

1 INTRODUCTION

The paper is intended to pay homage to Professor Leopold Müller who was a leading developer and user of physical models to solve fundamental rock mechanics problems and complex engineering cases. Professor Müller not only used Physical Models extensively but also wrote a seminal paper (Müller,1980), in which he discussed other types of models and addressed the skepticism with which physical models were looked at. Today we face a similar situation in that the use of physical models is questioned in comparison to the possibilities of powerful computer-based simulations.

I intend to address this issue by looking at models of materials, of geologic processes, and of jointed rock masses to end up with complex models such as slope instabilities and tunnels in rock masses. This sequence of model topics also fits well into the history of physical models. Finally, to bring physical modeling into the context of present-day engineering and science the paper will compare the principles and use of physical models with those of simulation models.

2 MATERIAL MODELS

To start this section the discussion (dialogo secondo) by Galileo Galilei (1638) on bending (Fig. 1) is used. Figure 1 is not the picture of a physical model but of a conceptual one. It is used because it shows two aspects (among others) of Galilei's interpretations: The correct one is related to scaling: multiplying (e.g., doubling) the dimensions does not increase the loading capacity by the same factor. The incorrect one is the assumption that the tensile stresses are uniform at section A-B. If a physical model test had been run this mistake would have been discovered. This is a pertinent lead-in to the paper – PHYSICAL MODELS AND SCALING OF PHYSICAL PROCESSES are essential.

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ſin quì dichiarate, non ſarà difficile l'intender la ragione, onde auuenga, che vn Priſma, ò Cilindro ſolido di vetro, acciaio, legno, ò altra materia frangibile, che ſoſpeſo per lungo ſoſterrà grauiſſimo peſo, che gli ſia attaccato, mà in trauerſo (come poco fà diceuamo) da minor peſo aſſai potrà tal volta eſſere ſpezzato, ſecondo che la ſua lunghezza eccederà la ſua groſſezza. Imperò che figuriamoci il Priſma ſolido A B, C D fitto in vn muro dalla parte A B, e nell'altra eſtremità s'intenda la forza del Peſo E, (intendendo ſempre il muro eſſer eretto all'Orizonte, & il Priſma, ò Cilindro fitto nel muro ad angoli retti) è manifeſto che douendoſi ſpezzare ſi romperà nel luogo B, doue il taglio del muro ſerue per ſoſtegno, e la B C per la parte della Leua, doue ſi pone la forza, e la groſſezza del ſolido B A e l'altra parte della Leua, nella quale è poſta la reſiſtenza, che conſiſte nello ſtaccamento, che s' hà da fare della parte del ſolido B D, che è fuor del muro, da quella che è dentro; e per le coſe dichiarate il momento della forza poſta in C al momento della reſiſtenza che ſtà nella

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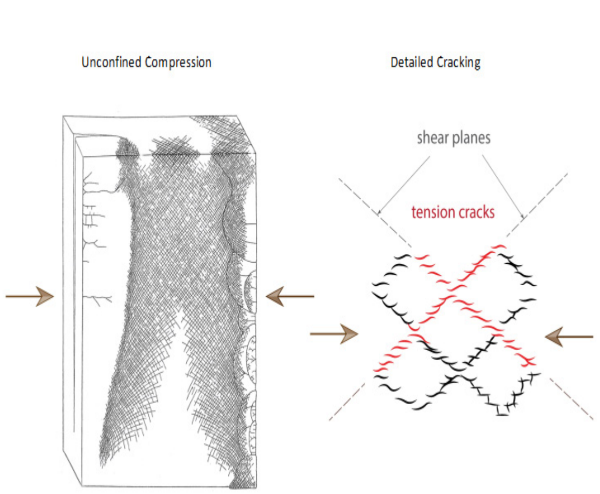

a b

Figure 1. Galilei's beam/cantilever theory (Linda Hall Library, Kansas City MO).

Figure 2. Test on blocks of paraffin at 0°F (Sheldon, 1912). a. Overall observations; b. Details.

Coulomb (1776) used detailed experiments to confirm the concept of cohesion. He measured both tensile strength and cohesion on Bordeaux limestone, the former with a direct tension test the latter with a direct shear test. Very importantly, Coulomb did multiple tests using limestone but also artificial materials (bricks and mortars) and, related to this, he mentions that one cannot generalize the results since they differ for different materials.

Mohr (1906) did not perform tests by himself but used test run by others to confirm what he proposed theoretically: The sliplines bisect the angle between the major and minor principal stresses in a tension test, and the angle between the major principal stress and the failure surface in a compression test becomes smaller with increasing material strength.

3 GEOLOGIC MODELS IN THE 19TH AND EARLY 20TH CENTURY

While Coulomb and Mohr used the physical models to prove their theories the set of model tests by Sheldon (1912), Riedel (1929), H. Cloos (1930) and E. Cloos (1955) were used to explain geologic features observed in nature. Sheldon, for example, used paraffin-resin models at 0° F to explain joint geometries she had observed in nature. The experiments showed the patterns documented in Fig. 2a, where uniaxial compression was applied along the horizontal axis resulting in new fractures inclined at 45° to the force direction implying zero frictional resistance. Most interesting is the observation shown in Fig. 2b with classic wing cracks aligned along the major principal stress direction.

4 JOINTED ROCK MODELS

4.1 *Initial work*

The next step after modeling geologic mechanism was to investigate the behavior i.e., strength and deformability of a jointed rock mass. Work by Einstein et al. (1969) presents the relatively complex triaxial testing on rock models consisting of Gypsum blocks representing a rock mass with different joint (fracture) spacings and orientations (Fig 3a). The resulting Mohr diagrams of (Fig. 3c) clearly show an effect of joint geometry leading to major follow-up research (Section 4.2.). It is important to note that around the same time others conducted analogues tests (Brown and Trollope, 1970; Hayashi, Jaeger, Rosengren, Ladanyi & Archambault, 1969).

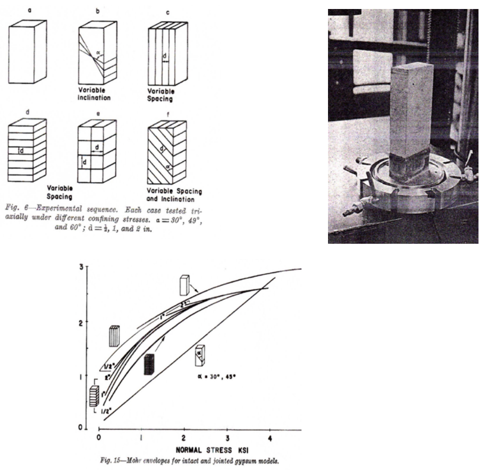

Figure 3. Triaxial Tests on Jointed Gypsum Model Rock (from Einstein et al., 1969). a. Different joint geometries/patterns; b. Triaxial test setup; c. Mohr envelopes for the joint patterns.

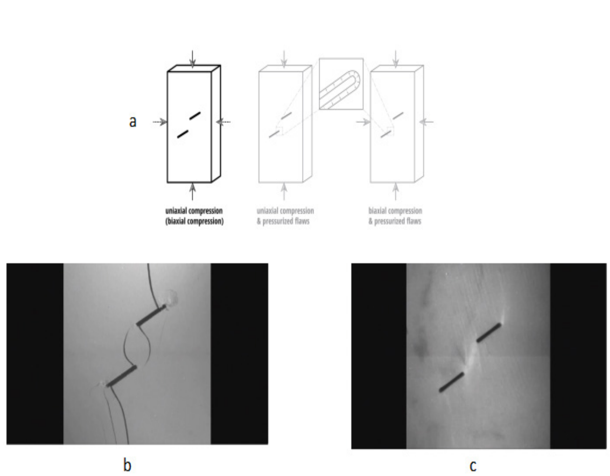

Figure 4. Physical Model Testing of the Persistence Problem. a. Prismatic model with different materials, joint geometries, and stress conditions; b. Test result with gypsum showing tensile wing cracks; c. Test result with marble showing shear failure.

To completely represent the behavior of a jointed rock mass it is necessary to model the behavior of the individual joint, specifically the effect of joint roughness on shearing resistance. Early studies using a physical model are those by Patton (1966). Interesting in this context is also what Barton (1973) and then Barton and co-workers did. Initially, joint roughness was investigated on geometrical models by having paper cuts of joints with different roughness geometries sliding over each other. Further work then used real rock and led to the expressions, similar to what Patton (1966) and Ladanyi and Archambault (1969) did, to include the resistance of the material when asperities got sheared off.

From this point on the physical modeling in rock mechanics and engineering went into two directions:

- Modeling details of the fundamental fracturing mechanisms
- Modeling of large-scale mechanisms such as slope failures and interaction with structures such as dams and tunnels

The latter will be dealt with in Section 5.

4.2 *Fracturing mechanisms in jointed rock masses*

The changing Mohr envelopes of Fig. 3c provide an idea: Analysis showed that if the physical failure surface crossed more joints this led to lower Mohr envelopes in essence indicating that these crossing points are possible fracture initiation points. Parallel theoretical investigations on the stochastic nature of joints (see e.g., Baecher et al. (1976), Call et al. (1977), Dershowitz and Einstein (1988)) eventually led to DFN's and influenced the analytical modeling of failure of jointed rock masses, which consist of a pattern of joints and intact rock. Einstein et al. (1983) showed that the eventual failure of intact rock bridges can be complex combinations of shear and tensile failure. This indicated that it was necessary to physically investigate this rock bridge failure problem also called the "persistence problem". Following numerical model work by Chan (1990) and starting with and Reyes and Einstein (1991) and Bobet and Einstein (1998) fracture interaction was studied with a combination of physical (Fig. 4a) and numerical models. The physical model tests showed different interaction mechanisms (Figs. 4b and c).

4.3 Fluid flow in jointed rock

So far, the comments on the physical models dealt with "dry" models. The following will now deal with modeling fluid-rock interaction. Flow models for individual joints mostly concentrate on the effect of irregular joint surfaces on fluid flow. Many use natural rocks with natural or artificially cut fractures/joints (see e.g., Iwano and Einstein, 1995). Using such non-transparent materials causes problems in that the flow through fractures cannot be directly observed. More informative is the approach used by Detwiler et al. (2000) with transparent fracture analogs, which was also done at MIT. Fig. 5a shows a cross-section through the MIT Hele Shaw cell with a transparent fracture analog; the transparent, rough analog is shown in Fig. 5b, and the principle of the tests in Fig. 5c. The experiment consists of applying external stresses and fluid flow in the Hele Shaw cell such that fracture surfaces get increasingly deformed and the flow (and transport) is affected (Fig. 5d). Researchers have also constructed physical models to observe flow in jointed rock masses including probably the most ambitious and complex models in this domain, namely those by Louis (1972).

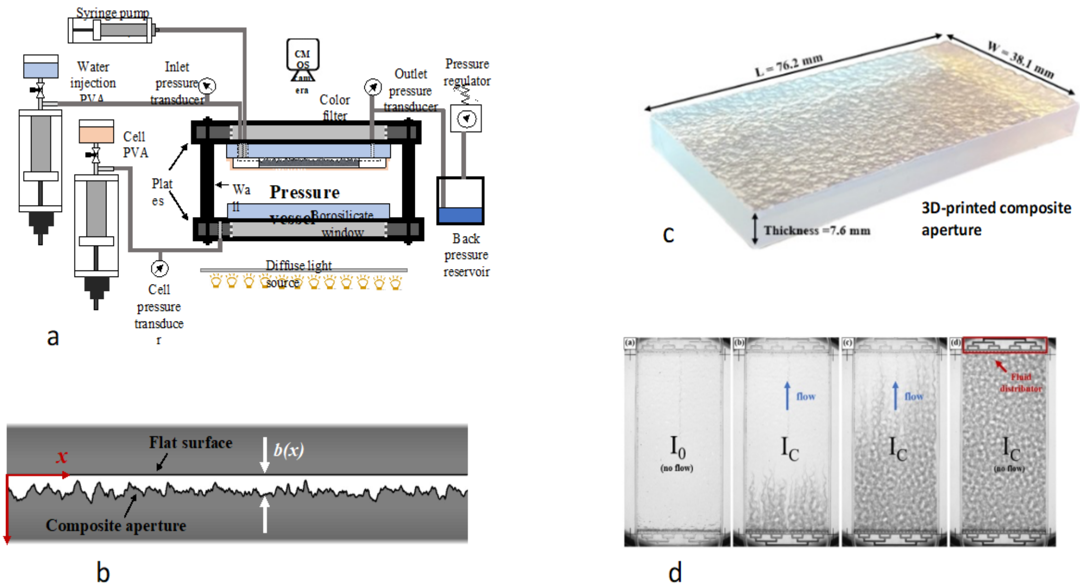

Figure 5. Flow experiments on individual fracture using a Hele Shaw cell (Villamor-Lora, 2023). a. Schematic cross-section; b. Transparent analog; c. Schematic cross-section of rough fracture; d. Deformation of contacts under increasing pressure.

5 COMPLEX MODELS

In this section physical models representing geologic mechanisms (slope failure) and the interaction of structures and geology (tunnels) will be discussed.

5.1 Physical Models of Structures in or on Rock - Slope failures

One of the best-known physical rock engineering models is the one by Müller in Karlsruhe on the rockslide at Mount Toc leading to the Vajont disaster; (Fig 6). In essence the seat shaped geometry of the jointed mass may have facilitated a collapse of the blocky mass possibly producing a sudden pore pressure increase. One needs to know, however, that other interpretations of the Mount Toc failure exist.

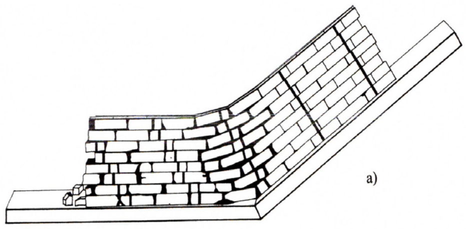

Figure 6. Physical Model of Mount Toc Slide Conducted by L. Müller (from Fumagalli, 1973).

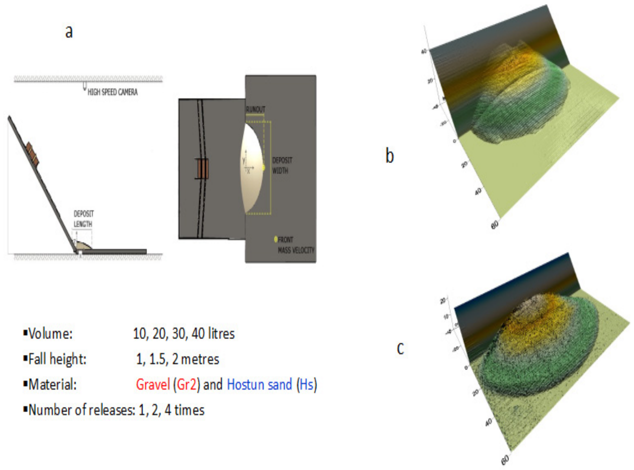

Figure 7. Rock Avalanche Experimets by Manzella and Labiouse (2009). a. Model setup; b. and c. Effect of different particle geometries on resulting morphologies of deposit.

The preceding involving some movement during slope failures logically leads to models of rock avalanches and rockfalls as were used, for instance, by Manzella and Labiouse (2009) (Fig. 7). A wide variety of parameters could be investigated with this model and then be compared to numerical models with good correspondence.

5.2 Physical models of structures on or in rock - tunnels

In contrast to most structural foundations, which load the rock mass, tunnels unload it. The correct unloading mechanism and effect of gravity can be applied in the base friction model as shown by Bray and Goodman (1981) (Fig. 8).

A step further in correct modeling of the in-situ stress field is possible with the centrifuge. Bucky (1931) used the centrifuge for mining problems. Numerous centrifuge studies with tunnel models in soil have been conducted and will not be pursued here. However, what will be mentioned is the limitation of using the centrifuge in jointed rock. While stresses and strains in continua can be properly scaled, displacements can to a limited extent only. Models of jointed rock can therefore be used conceptually (Fig.9) but not in cases where e.g., shearing resistance changing with displacement plays a role.

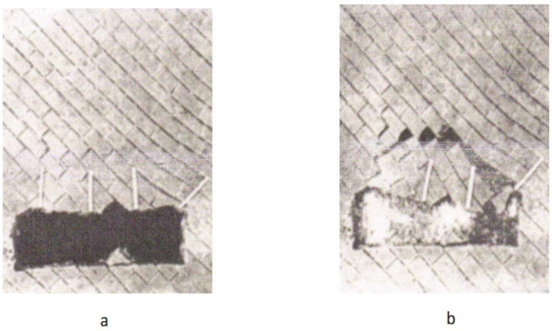

Figure 8. Base Friction Test on a Tunnel in Jointed Rock (Bray and Goodman, 1981). a. Condition before removing the bolt at the upper left; b. Condition after removing the bolt at the upper left.

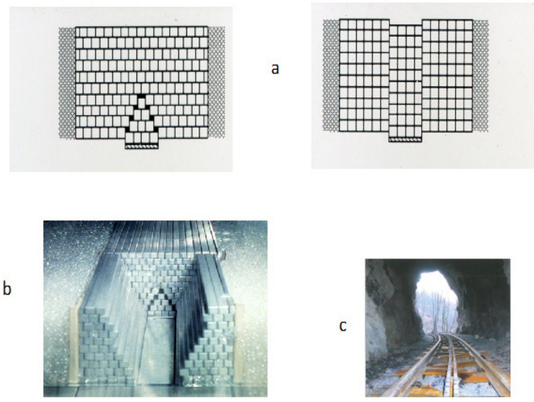

Figure 9. Trapdoor Test with the Centrifuge on Jointed Rock. a. Different rock block geometries; b. Photo of displacement after test; c. Real geometry can be conceptually modeled as shown in b.

6 PHYSICAL MODELS IN ROCK MECHANICS AND ROCK ENGINEERING - CRITICAL ASPECTS AND OUTLOOK

6.1 Critical aspects

When discussing physical models, physical properties or laboratory experiments, in general, the issue of scaling has to be addressed. As a matter of fact, this has been continuously addressed throughout this paper from Galilei to centrifuge modeling. Dimensional analysis (e.g., Lanhaar, 1951) can be used to develop a proper model. The process (as e.g., used in the jointed rock model described in Section 4.1) involves the selection of parameters that best represent the performance of the prototype, formulation of dimensionless products and variation of model material properties. Experiments are then conducted to determine which material best satisfies the requirements. Such experiments, as any laboratory experiment, are subject to experimental scatter. These comments are made to show that although mathematically clear, the combination of mathematical complexity of the scaling relations and experimental variability makes the selection of a model material and other model properties difficult.

If one compares these physical modeling problems to modeling with numerical simulations, in which many parameters, many parameter states and many external conditions can be simultaneously varied the question comes up why use physical models at all.

On the other hand, there are questionable aspects of simulation:

- To what can the simulated performance compared?
- The same simulated performance can be caused by a variety of unknown combinations of parameters, parameter states and external conditions.

Hence: Are neither the physical models nor simulations satisfactory?

6.2 Outlook -Possible role of physical models

The answers to the final question above lie in many of the examples shown previously in this paper: Coulomb stipulated cohesion and proved it by physical model testing. The flow in a rough fracture subject to combinations of external stresses and internal pressure was simulated and then physically verified in the Hele-Shaw tests. Very often is the verification process reversed: Rock avalanches or shearing and tension in the persistence problem were physically tested first and then simulated using the tested parameters.

In other words, and in conclusion:

- Physical models are essential for verifying simulation models of specific mechanisms.
- Simulation models can be used to expand the variation of parameters tested in the physical models.

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