Polymorphic uncertainty modelling of rock properties coupled with combined probabilistic and non-probabilistic framework for rock tunnel stability analysis

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ABSTRACT: A polymorphic uncertainty model is proposed considering the combined effect of aleatory and epistemic uncertainties of rock properties on the stability analysis of rock tunnels. The model incorporates fuzzy logic to represent epistemic uncertainties in the Geological Strength Index (GSI), transformational uncertainty of empirical models, systematic uncertainties due to discrepancy between field and laboratory conditions, and stochastic methods to represent aleatory uncertain properties. Further, detailed guidelines are proposed for the characterization and fuzzification of epistemic uncertain properties. An extended Convergence-Confinement Method (CCM) is proposed and illustrated by performing the stability analysis of a railway tunnel in Jammu and Kashmir, India under the framework of combined probabilistic and non-probabilistic methods. Further, the results obtained from the developed methodology were systematically compared with those of traditional reliability-based results and it was concluded that the proposed methodology is in order with the available input parameters having different uncertainty types.

Keywords: Fuzzy approach; polymorphic uncertainty; probabilistic methods; non-probabilistic methods.

1 INTRODUCTION

Rock mechanics has always found it difficult and demanding to model uncertainties relating to rock characteristics and model parameters. In this discipline, the application of probabilistic analysis paired with reliability techniques such as First/Second-Order Reliability Method Point Estimate Methods (PEMs), Monte-Carlo Simulation (MCS), etc. (Hoek 1999 and Tiwari & Latha 2017) are the most renowned.

There are primarily two sorts of sources of uncertainty for intact rock and rock mass attributes. Aleatory uncertainty (caused by innate variability) and epistemic uncertainty (resulting from a lack of knowledge) (Bedi 2014). Systematic uncertainties resulting from variations in the laboratory and in-situ conditions and transformational uncertainties associated with the empirical relations are subcategories of epistemic uncertainty (Spross 2016 and Tiwari & Latha 2019). Because there is a lack of accurate or sufficient rock data, probabilistic approaches cannot accurately represent the uncertainties resulting from many sources. Numerous researchers have employed non-stochastic techniques to take into account epistemic uncertainties, such as fuzzy set theory and intervals (Alefeld & Mayer 2000; Park et al. 2012). However, accurate uncertainty modelling of structural response parameters of rock structure necessitates the complete uncertainty (epistemic + aleatory) quantification in intact rock and rock mass properties.

To the best of the author's knowledge, very few efforts have been undertaken to give a full and unified uncertainty framework for calculating combined/polymorphic (epistemic + aleatory) rock parameter uncertainty and using the results to analyse the stability of rock structures.

In this spirit, this paper proposes a polymorphic uncertainty model coupled with a combined probabilistic and non-probabilistic framework for accurate uncertainty modelling and safety analysis of rock structures. The applicability of the method is demonstrated for a tunnel case study along a proposed railway line in Jammu and Kashmir State of India.

2 DETAILS OF POLYMORPHIC UNCERTAINTY QUANTIFICATION MODELS

2.1 Polymorphic uncertainty quantification in intact rock properties

Total uncertainty in intact rock properties is the accumulation of systematic errors (epistemic uncertainty) arising from differences between laboratory and in situ conditions due to factors like water saturation and sample size etc. and inherent randomness (aleatory uncertainty) (Ang and Tang 1984). Considering these elements, modified intact rock properties (X^T) can be calculated as

$$X^T = \prod_{i=1}^{n} N_i^X \hat{X} \tag{1}$$

where, N_i^X are the correction factors accounting for systematic uncertainties. $\hat{X}$ is the estimator of X accounting only for the inherent variability. Inherent variability in laboratory measured properties can be modelled using probabilistic parameters (mean, standard deviation,PDF). The ratio of the values of a rock property under in-situ and standard laboratory testing circumstances is used to estimate correction factors (N_i^X) compensating for systematic errors. Section 4 presents a demonstration of measurement and modelling of systematic correction variables related to the effect of water saturation and the size of the rock sample for the current issue.

2.2 Polymorphic uncertainty quantification in rock mass properties

Total uncertainty in rock mass property is the cumulative uncertainty due polymorphic uncertainty in intact rock properties and transformational in GSI-based empirical models used for predicting the rock mass property. Transformational correction factors (C_m) can be approximated as:

$$C_m = \frac{True\ value\ of the\ property\ (X)_T}{Predicted\ value\ of\ the\ property\ (X)_P} \tag{2}$$

For the proposed tunnel analysis, transformational uncertainties were needed for the following relations:

2.2.1 Associated with the Hoek-Brown criterion, C_{m1}

$$(\sigma_1 - \sigma_3)_T = (\sigma_1 - \sigma_3)_P \times C_{m1} = \left[\sigma_{ci}{}^T (m_b \frac{\sigma_3}{\sigma_{ci}{}^T} + s_b{}^T)^{\ 0.5}\right] \times C_{m1} \tag{4}$$

where, $(\sigma_1 - \sigma_3)$ and σ_3 are deviatoric and axial stresses obtained from triaxial tests; $\sigma_{ci}{}^T$ is the modified uniaxial compressive strength of intact rock (Equation-1); m_b and s_b are Hoek-Brown strength parameters of rock mass.

2.2.2 Associated with the GSI based deformation modulus relation, C_{m2}

$$(E_m)_T = (E_m)_P \times C_{m2} = \left[E_i^{\ T} \left(0.02 \frac{1}{1+e^{\frac{60-GSI}{11}}} \right) \right] \times C_{m2} \tag{5}$$

where, $E_i^{\ T}$ is modified elastic modulus of intact rock (Equation-1); $(E_m)_T$ is modified deformation modulus of rock mass.

2.3 Polymorphic uncertainty quantification in rock-tunnel response parameters

CCM is a popular analytical method for tunnel stability analysis. Hoek-Brown (1980) have derived expressions for the response parameters of the rock tunnel i.e., plastic zone radius $'r_p'$ and the radial deformation,$'\ d_i'$, considering the rock mass as Hoek-Brown material. In this study, we derived expressions of response parameters (similar to Hoek-Brown (1980)) using the modified Hoek-Brown strength model (Equation 4) considering the transformational correction factor C_{m1}.

Assuming the tunnel to be a plane-strain axisymmetric problem, and rock mass to be an elastic-perfectly plastic material, expressions for response parameters of the tunnel of radius $'r_i'$,were derived. Following is a possible calculation order for tunnel response parameters:

$$M = \frac{1}{2}\left\{ \left(\frac{m_b C_{m1}}{4} \right)^2 + \frac{m_b p_0}{\sigma_{ci}^{\ T}} + s_b \right\}^{0.5} - \frac{m_b C_{m1}}{8} \tag{6}$$

$$G = \left\{ \frac{-m_b}{m_b + 4\left\{ \frac{m_b}{\sigma_{ci}^{\ T}}(p_0 - M C_{m1} \sigma_{ci}^{\ T}) + s_b \right\}^{0.5}} \right\} \tag{7}$$

Case 1: If support pressure (p_i) > critical stress $(p_i^{cr} = p_0 - \sigma_{ci} C_{m1} M)$

$$\frac{d_i}{r_i} = \frac{(1+\nu)}{(E_m)_T}(p_0 - p_i) \tag{8}$$

Case 2: If $p_i < p_i^{cr}$

$$r_p = r_i exp\left[\frac{2}{C_{m1}}\left(\frac{p_0 - M\sigma_{ci}^{\ T} C_{m1}}{m_b \sigma_{ci}^{\ T}} + \frac{s_b}{(m_b)^2} \right)^{0.5} - \frac{2}{C_{m1}}\left(\frac{p_i}{m_b \sigma_{ci}^{\ T}} + \frac{s_b}{(m_b)^2} \right)^{0.5} \right] \tag{9}$$

$$\text{For } \frac{r_p}{r_i} < \sqrt{3}, \quad R = 2\ ln\left(\frac{r_p}{r_i}\right) G; \qquad \text{For } \frac{r_p}{r_i} > \sqrt{3}, \quad R = 1.1\ G \tag{10}$$

$$A = \left\{ 2 \times \left(\frac{(1+\nu)}{(E_m)_T} \sigma_{ci}^{\ T} C_{m1} M \right) - \left(\frac{2\left(\frac{(1+\nu)}{(E_m)_T}\sigma_{ci}^{\ T} C_{m1} M\right)\left(\frac{r_p}{r_i}\right)^2}{\left\{\left(\frac{r_p}{r_i}\right)^2 - 1\right\}\left(1+\frac{1}{R}\right)} \right) \right\} \left(\frac{r_p}{r_i}\right)^2 \tag{12}$$

$$\frac{d_i}{r_i} = 1 - \left[\frac{1 - \left(\frac{2\left(\frac{(1+\nu)}{(E_m)_T}\sigma_{ci}^{\ T} C_{m1} M\right)\left(\frac{r_p}{r_i}\right)^2}{\left\{\left(\frac{r_p}{r_i}\right)^2 - 1\right\}\left(1+\frac{1}{R}\right)} \right)}{1+A} \right]^{0.5} \tag{13}$$

where, p_0 is hydrostatic in-situ stress; ν is Poission's ratio of rock mass; $m_i^{\ T}$ is modified Hoek-Brown strength parameter of intact rock that can be obtained from the Equation 1.

3 COMBINED PROBABILISTIC AND NON-PROBABILISTIC MODELS

3.1 Fuzzy model and p-box representation of fuzzy model

Two stages can be used to calculate the fuzzy number's membership function $(\mu_X(x))$ (Moller & Beer 2004): a) Plot the data as a histogram. b) Find the left and right branches of $(\mu_X(x))$ by fitting the data linearly in such a way that the sum of the squared discrepancies between the actual number of sample elements in each bin range and the functional value $(\mu_X(x))$ in the center of the bin is smallest. This is done while keeping the bin range with the highest frequency in common. Additionally, normalise the resulting membership function by setting $\mu_X(x) = 1$ to the intersection point of the left and right branches of ($\mu_X(x)$). A non-parametric p-box with the same level of information can also be used to represent fuzzy numbers [p, q, r] (Bedi 2014). The methods listed below can be used to obtain the pbox's:

a)- Generate n random probability values $f_i : (f_1, f_2, f_3, \dots, f_n)$ where, $f_i \in [0, 1]$; $i = 1,2, \dots, n$

b)- For lower bound cdf ($\underline{P}_X$), estimate $x_i = U^{-1}(f_i)$ where, U^{-1} denotes inverse uniform distribution and $x_i \in [p, q]$.

c)- Similarly, for upper bound cdf ($\overline{P}_X$), estimate $x_i = U^{-1}(f_i)$ where, $x_i \in [q, r]$.

3.2 *Double loop uncertainty propagation model*

The double loop uncertainty propagation aims at determining the output boundary distributions $\underline{P}_Y$, $\overline{P}_Y$, i.e., the output p-box, propagated from the inputs p-box variables. Steps for performing double loop uncertainty propagation are given as follows (Zhang et al. 2010):

a) Generate m×n standard uniform random probability values f_{ij} ; $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$; $f_{ij} \in [0, 1]$ where m and n are the number of input variables and sample size generated by MCs.

b) Estimate interval realizations of all input variables X_{ij} corresponding to probability values f_{ij} where, $X_{ij} = [\underline{P}_{X_i}^{-1}(f_{ij}), \overline{P}_{X_i}^{-1}(f_{ij})]$; $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$

c) Estimate the lower and upper bounds of output response parameters by solving the following constraint optimization problem:

$$\underline{R}_j = minimize\ g(X_{1j}, X_{2j}, X_{3j}, \dots, X_{mj})\ ;\ s.t.\ X_{ij} = [\underline{P}_{X_i}^{-1}(f_{ij}), \overline{P}_{X_i}^{-1}(f_{ij})];$$

$$\overline{R}_j = maximize\ g(X_{1j}, X_{2j}, X_{3j}, \dots, X_{mj})\ ;\ s.t.\ X_{ij} = [\underline{P}_{X_i}^{-1}(f_{ij}), \overline{P}_{X_i}^{-1}(f_{ij})];$$

Where, $R_j = g(X_1, X_2, X_3, \dots, X_m)$ is given response function.

4 APPLICATION CASE STUDY

The suggested methodology was used to analyse a hypothetical railway tunnel in Jammu and Kashmir, India, in this section. The main reason to select this tunnel was the availability of detailed investigation data for a nearby rock slope with similar rock mass conditions. It should be noted that the major aim of the study was to demonstrate the analysis procedure and advantages of the proposed methodology over traditional methodology. With a radius (r_i) of 3 metres and an in-situ hydrostatic pressure (p_0) of 3.5 MPa, the tunnel was considered to be circular. The surrounding rock mass had a Poisson's ratio (ν) of 0.23. Two performance functions (PFs) were employed to evaluate the stability of the tunnel (Lü & Low 2011); a) $M_1 = 2 \times r_i - r_p$, b) $M_2 = 0.02 \times r_i - d_i$; where, r_p is the plastic zone radius, r_i is the radius of the tunnel, d_i is the tunnel deformation.

The statistics and best-fit PDFs for m_i, E_i and, σ_{ci} are shown in Table 1. Figure 1h depicts the histogram of the GSI, which was subjectively calculated by experts at the tunnel site.

Table 1. Statistical parameters and PDFs of intact rock properties.

Property		Mean	Std	PDF
$\boldsymbol{m_i}$		12.8	4.08	Weibull
$\boldsymbol{E_i}$	[GPa]	65.7	18.03	Weibull
$\boldsymbol{\sigma_{ci}}$	[MPa]	114.5	50.13	Weibull

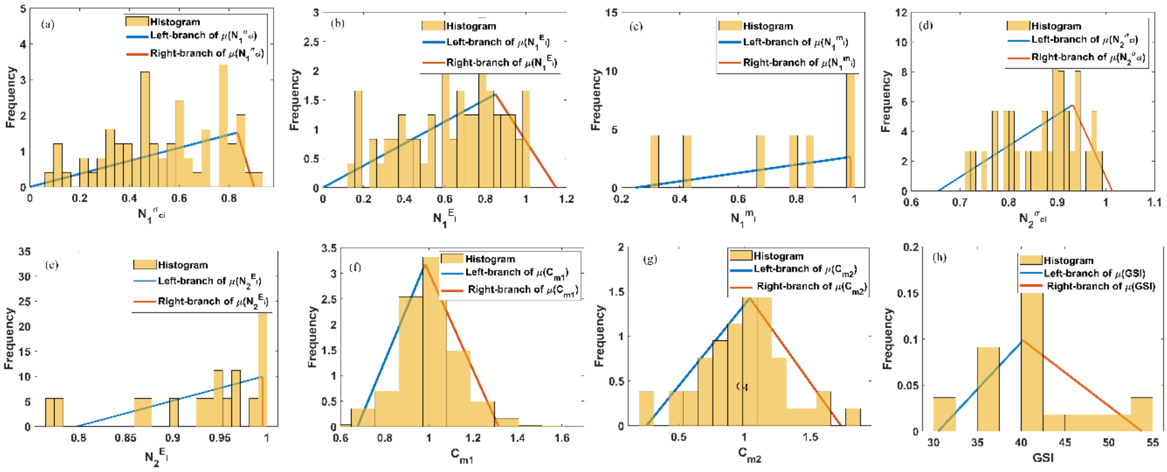

Figure 1. Best fitted linear left and right branches of membership functions on histograms of a) $N_1^{\sigma ci}$ b) $N_1^{E_i}$ c) $N_1^{m_i}$ d) $N_2^{\sigma ci}$ e) $N_2^{E_i}$ f) C_{m1} g) C_{m2} h) GSI.

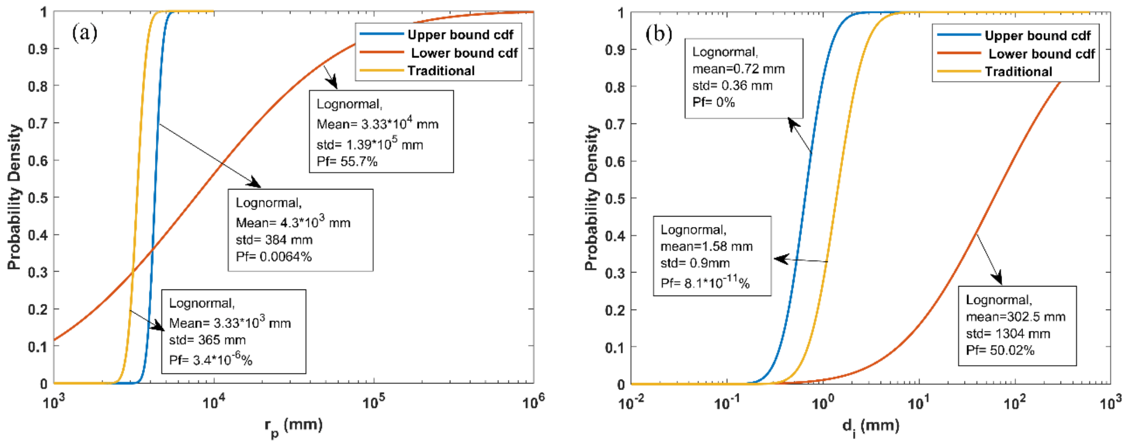

Figure 2. P-boxes of response parameters a) radial deformation i.e., d_i b) plastic zone radius i.e., r_p.

After a thorough review of the literature, triaxial test data for the rock was acquired for the quantification of C_{m1} (Zhang et al. 2018). The papers Hoek et al. (2019), and Kayabasi et al. (2003) provided information on the deformation modulus for C_{m2}. Considering that the entire rock sample size (dia > 54mm) was relatively large in the field and the laboratory testing samples were dry and small (dia <=54mm), yet there was a chance of saturation owing to rainfall or a rise in the water table, systematic correction factors corresponding to water saturation (N_1) and the size of the rock (N_2) sample were estimated based on the data obtained from the literatures (Wong et al. 2016; Jamshidi 2014; Komurlu 2018). Since these correction factors may differ for various rocks, it is preferable to characterise it probabilistically by gathering data for a wide range of rocks as opposed to giving a set deterministic number. The collected data was modelled using non-probabilistic approaches in consideration of this as well as the lack of precise and enough information. Figure 1 provides the histograms and corresponding fuzzy membership branches of transformational (C_{m1} and C_{m2}) and sytematic ($N_1^{\sigma ci}$, $N_2^{\sigma ci}$ (for UCS), $N_1^{E_i}$, $N_2^{E_i}$ (for elastic modulus) and $N_1^{m_i}$(for Hoek-Brown constant)) correction factors. The resultant fuzzy membership functions were further normalised and turned to non-parametric p-boxes. Tunnel stability analysis was performed, and p-boxes of tunnel response parameters, including plastic zone radius (r_p) and deformation (d_i) were determined, utilising double loop uncertainty propagation methods and equations described in section-2.3. Furthermore, a conventional stability analysis of the rock tunnel was performed, discarding systematic errors and transformation model uncertainties (taking, $C_{m1} = 1, C_{m2} = 1$) and accounting for the GSI as a probabilistic variable (mean = 41.13; Std = 6.45; PDF- lognormal). Figure 2 displays the research's findings after employing both traditional and proposed methods.

5 CONCLUSION

An efficient polymorphic uncertainty model coupled with a combined probabilistic and non-probabilistic framework has been provided for rock tunnel stability analysis. According to the current case study, the stability analysis of the tunnel yielded safe (Pf <1%) and unsafe (tunnel failure) (Pf > 1%) results, respectively. It was shown that the traditional approaches' underestimating of epistemic uncertainty resulted in a 100 % underestimation of the chance of tunnel failure concerning $\boldsymbol{r_p}$ and $\boldsymbol{d_i}$.

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