On the crack initiation location in the Brazilian test: Griffith-based insight

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ABSTRACT: We address the controversy surrounding the use of the Brazilian test to estimate the tensile strength of rock-like materials. Due to its indirect nature, the tensile strength is inferred from the critical load by assuming that cracking initiates at the centre of the sample. We combine finite element analysis with the failure envelope of the generalised Griffith criterion to establish the crack nucleation location, and map the conditions that result in the nucleation of a centre crack. The results reveal that the regime of validity of the Brazilian test is much smaller than previously assumed, with current practices and standards being often inappropriate. An experimental protocol is developed that enables obtaining a valid estimate of the material tensile strength, and a MATLAB App is provided to facilitate the uptake of this protocol. We demonstrate the usefulness of our protocol through examples of valid and invalid tests from the literature.

Keywords: Rock mechanics, Brazilian test, Fracture, Finite element analysis, Griffith theory.

1 INTRODUCTION

The Brazilian test has been used for estimating the tensile strength of rocks and other quasi-brittle materials since it was proposed by Carneiro (1943) and Akazawa (1943). The test has been standardised since 1978, when it was included as a Suggested Method of the International Society for Rock Mechanics and Rock Engineering (ISRM) (Bieniawski & Hawkes 1978). In the Brazilian test, the tensile strength can be estimated using the Hondros (1959) solution of a disk subjected to radial loads. Standards are built upon the assumption of a zero-contact angle, simplifying Hondros (1959) solution to the case of a concentrated load. Importantly, both approaches assume that cracking initiates at the centre of the disk. The validity of the zero-contact angle and centre cracking assumptions is dependent on the test geometry and material properties, making it a subject of significant discussion in the academic literature (Fairhurst 1964). The debate is very much open and a myriad of papers have been published trying to shed light on the validity regimes of the Brazilian test using theoretical (Markides & Kourkoulis 2016), numerical (Navidtehrani et al. 2022), and experimental (Alvarez-Fernandez et al. 2020) tools. The key challenge in conducting the test is to strike a balance between ensuring that the contact angle is small enough to attain the maximum tensile stress at the centre, while also being large enough to prevent premature cracking near the loading region.

This study uses the generalised Griffith criterion (Fairhurst 1964) to investigate the location of crack initiation in the Brazilian test. Across a wide range of materials and geometries, finite element calculations are conducted to assess the viability of the test for estimating the tensile strength of rocks and other quasi-brittle materials. The use of the generalized Griffith criterion allows the determination of the crack initiation location as a function of two material properties: the tensile (σ_t) and compressive (σ_c) strengths. Maps are constructed that quantify, for a wide range of jaw radii and material properties, the admissible compression-to-tensile strength ratios above which cracking initiates at the centre of the disk, revealing a much smaller regime of validity than previously assumed. A MATLAB App is provided that easily allows determining if the test is valid *a posteriori* or making *a priori* decisions of adequate test geometries based on expected σ_t values. More details can be found in Navidtehrani et al. (2022).

2 GENERALISED GRIFFITH CRITERION FOR CRACK INITIATION

Griffith (1920) showed that when flaws are oriented at an angle relative to the principal directions of applied stress, local tensile stresses will develop near them. This leads to a tensile stress parallel to the flaw if the minimum principal stress (σ_3), has an absolute value that is lower than three times the maximum principal stress (σ_1). This analysis also implies that the compressive strength, σ_c, is eight times the tensile strength, σ_t, limiting its use to a narrow set of materials (those with $\sigma_c/\sigma_t \approx 8$). To extend and generalise Griffith's criterion, Fairhurst (1964) proposed defining a parabolic Mohr envelope that encloses both the uniaxial tensile and compressive strength circles, with the former being touched at its vertex, and the latter being tangent to the envelope. In terms of the principal stress space, the generalized Griffith criterion can be expressed as

$$\begin{cases} \sigma_1 = \sigma_t, & m(m-2)\sigma_1 + \sigma_3 \geq 0 \\ \sigma_3 = \sigma_1 - (1-m)^2\sigma_t + 2(1-m)\sqrt{\sigma_t(\sigma_t - \sigma_1)}, & m(m-2)\sigma_1 + \sigma_3 < 0 \end{cases} \quad (1)$$

where m is a material parameter defined by the compressive-to-tensile strength ratio n as follows:

$$m = \sqrt{n+1}, \quad n = -\sigma_c/\sigma_t \quad (2)$$

The generalised Griffith criterion particularises to the original Griffith criterion for $n = 8$, and otherwise extends it to arbitrary ratios of tensile to compressive strengths. It is worth noting that the adoption of the generalised Griffith criterion necessarily implies that the Brazilian test is, generally, not a suitable experiment for measuring the tensile strength of materials with $n < 8$.

3 THE APPLICATION OF GRIFFITH'S CRITERION TO THE BRAZILIAN TEST

During the Brazilian test, the material in the disk experiences a stress state that is characterised by two domains in the principal stress state. The maximum and minimum principal stresses are compressive near the jaws, whereas the maximum principal stress is tensile in other parts of the disk. To obtain a valid estimate of the material tensile strength, cracking must initiate at the centre of the disk. One can analyse the stress state in the disk using the failure envelope of the generalised Griffith criterion, and map the conditions of validity. Figure 1 shows a cloud of points representing the potential stress states in a discrete number of material points distributed within the disk, denoted as $(\sigma_1, \sigma_3)_{(x,y)}$. Two scenarios can occur. In one scenario, shown in Figure 1a, the test is invalid because the first material point reaching the failure envelope is not located at the centre of the disk; it may, for example, first occur close to the loading jaws. In the other scenario, illustrated in Figure 1b, the failure envelope is reached first by the material point located at the disk centre ($x = 0, y = 0$), and a valid estimate of the tensile strength can be obtained, which is denoted as $\sigma_t = (\sigma_1)_{(0,0)}$.

The validity of the test depends on the failure envelope, given a specific applied load, test geometry, and elastic properties of the jaws and disk. Multiple scenarios are illustrated in Figure 1c. One is for which the Brazilian test is valid, as the centre of the disk is under a stress state where $(\sigma_1)_{(0,0)} = \sigma_t$. However, if the ratio σ_c/σ_t is too low, and the stress states at multiple material points are above the failure envelope, the test is invalid, as shown by the red dotted curve in Figure 1c. Conversely, if the ratio σ_c/σ_t is large enough, only the centre point will touch the envelope, making the experiment valid, as indicated by the green dashed curve in Figure 1c. The threshold for the admissible σ_c/σ_t ratios for a Brazilian test to be valid is shown by the orange dash-dotted line in Figure 1c. Therefore, with numerical analysis, one can estimate the stress state at any point in the disk for a given load, geometry, and set of material parameters, and subsequently use the generalized Griffith criterion to determine the compressive strength associated with a failure envelope passing through that point by rearranging Equation (1b):

$$(\sigma_c)_{(x,y)} = -\sigma_\mathrm{t}\left(\frac{\left(\sigma_\mathrm{t}-\sqrt{\sigma_\mathrm{t}\left(\sigma_\mathrm{t}-(\sigma_1)_{(x,y)}\right)}+\sqrt{\sigma_\mathrm{t}\left(\sigma_\mathrm{t}-(\sigma_3)_{(x,y)}\right)}\right)^2}{\sigma_\mathrm{t}^{\,2}}-1\right) \tag{3}$$

To ensure that the failure condition is first reached at the centre of the disk, the maximum value of $(\sigma_c)_{(x,y)}$ estimated using Equation (3) must not exceed the actual compressive strength of the material. Since the compressive strength is a well-known material property that can be independently measured, combining numerical analysis with the generalized Griffith's criterion can provide a mapping of the conditions that result in failure initiation from the centre of the disk. This approach incorporates both validity conditions for the Brazilian test, namely, that cracking starting at the centre, and $(\sigma_1)_{(0,0)} = \sigma_t$, as illustrated below.

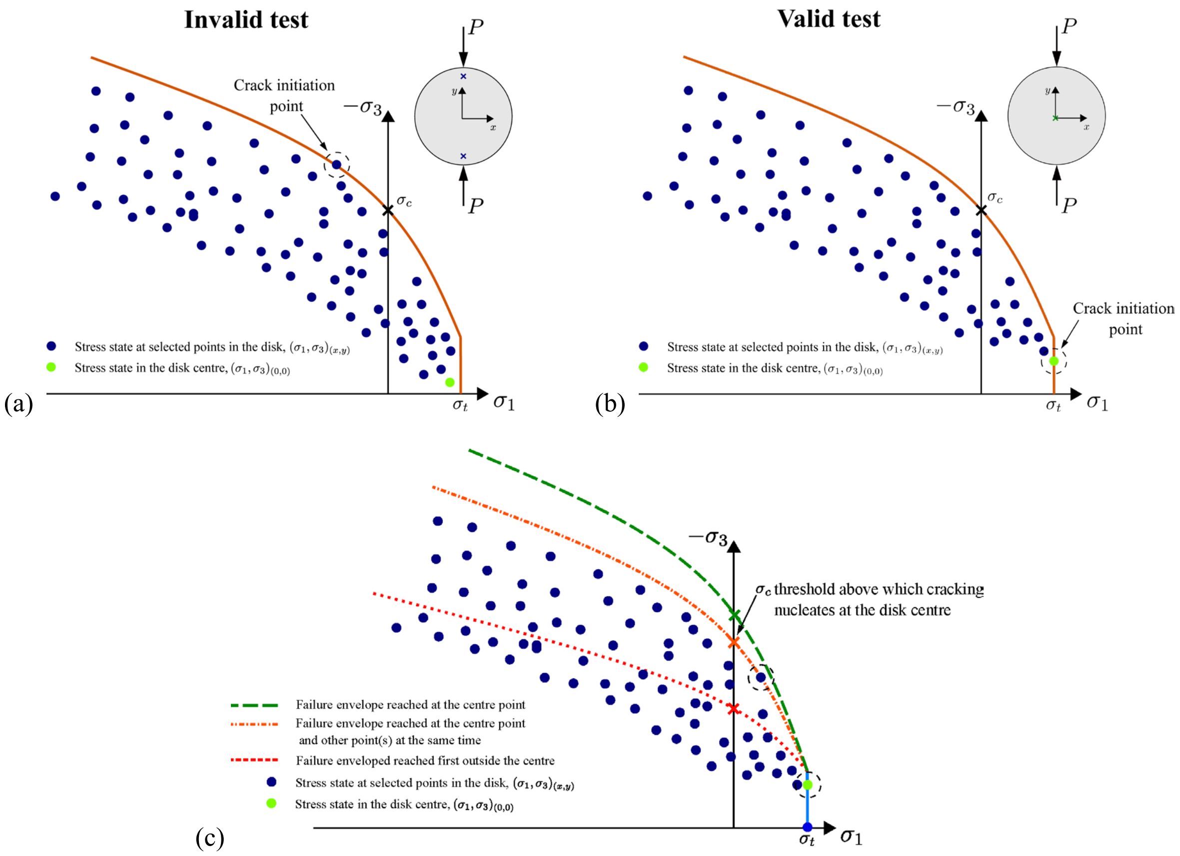

Figure 1. Stress state at a discrete number of material points within the Brazilian disk and failure envelopes based on the generalised Griffith criterion. (a) valid test, (b) invalid test, and (c) Validity of the test as a function of the failure envelope for a given stress state associated with a load P.

4 RESULTS

4.1 Preliminaries

In the Brazilian test, the location of crack initiation is dependent on several factors, including the radius of the jaws (R_j), the radius of the disk (R_d), the elastic properties of the disk (E_d, ν_d) and jaws (E_j, ν_j), and the tensile (σ_t) and compressive (σ_c) strengths of the material being tested. Assuming that cracking begins along the vertical middle axis of the disk, the position of crack initiation can be fully described by a variable Y, which is equal to 0 at the centre and equal to R_d at the edge. The solution is a function of non-dimensional sets determined by dimensional analysis, which include:

$$\frac{Y}{R_d} = F\left(\frac{R_j}{R_d}, \frac{E_j}{E_d}, \nu_j, \nu_d, \frac{\sigma_c}{E_d}, \frac{\sigma_t}{E_d}\right). \quad (4)$$

Assuming that crack nucleation occurs at the centre of the disk, $Y/R_d = 0$, as is necessary for the test to be valid, equation (4) can be rearranged as follows:

$$\frac{\sigma_c}{\sigma_t} = G\left(\frac{R_j}{R_d}, \frac{E_j}{E_d}, \nu_j, \nu_d, \frac{\sigma_c}{E_d}\right). \quad (5)$$

To determine the conditions that lead to cracking at the centre of the disk, we conduct calculations over relevant ranges of the five non-dimensional sets in equation (5). The Young's modulus of the disk is varied from 5 to 150 GPa, while Poisson's ratio is varied within the range of 0.1 to 0.4. As the jaws are typically made of steel, we assume elastic properties of $E_j = 210$ GPa and $\nu = 0.3$. Therefore, the two critical non-dimensional sets are σ_c/σ_t and σ_c/E_d. To determine the stress state within the disk, we conduct finite element simulations which account for the contact between the jaws and the sample (see Navidtehrani et al. (2022) for details).

4.2 Mapping the conditions that lead to cracking at the disk centre

Low contact angles can result in stress states that are similar to those described by the Hondros equations. However, this alone does not ensure the validity of the test, as cracking may occur outside of the centre of the disk, especially when flat or large-radius jaws are used. We determine the location of crack nucleation using a protocol that combines the generalized Griffith failure envelope and finite element analysis. Specifically, we start by assuming that cracking initiates at the disk centre, where the maximum principal stress is equal to the tensile strength. Then, we assess this assumption by comparing the compressive-to-tensile strength ratio resulting from the test with the admissible range of σ_c/σ_t ratios. If the latter is greater than the former, then cracking initiates outside of the disk centre, and the test is considered invalid. The practical steps involve conducting first a finite element analysis to estimate the principal stresses at each integration point for a range of load increments. Then, Equation (3) is used to compute the minimum admissible compressive strength (i.e., the maximum σ_c among all material points). Finally, from σ_c and the assumption $(\sigma_1)_{(0,0)} = \sigma_t$, a data point is defined that relates the material and test parameters with the threshold of admissible σ_c/σ_t values. Then, using approximately 20,000 data points, a map like the one in Figure 2 is built.

Figure 2 shows the relation between the jaw radius, the non-dimensional set E_j/E_d, and the minimum acceptable compressive-to-tensile strength ratio. We provide maps for two limit cases of disk elastic properties: $E_j/E_d = 42$ and $E_j/E_d = 1.4$, with most rock-like materials expected to fall between these two cases. Comparing Figure 2a-b, we observe that while E_j/E_d does influence the results, the jaw radius has a much greater impact. Figure 3 shows the application of maps for four common rock materials: (i) granite with $E_d = 60$ GPa, (ii) sandstone with $E_d = 20$ GPa, (iii) limestone with $E_d = 50$ GPa, and (iv) marble with $E_d = 60$ GPa. A Poisson's ratio of $\nu_d = 0.2$ is assumed in all cases. The material properties (E_d, σ_c/σ_t) are taken from the GRANTA material library. The results are shown for a range of jaw radii that very from $R_j/R_d = 1.1$ to the flat case recommended by the ASTM standard. As evident from Figure 3, the flat jaw cannot be used to deliver a valid Brazilian experiment for any of these materials. The test is found to be valid for a range of granites (Figure 3a) and sandstones (Figure 3b) if the jaw radius is chosen appropriately. These are the only rock-type materials of those considered where the ISRM jaw radius recommendation ($R_j/R_d = 1.5$) can deliver a valid test. No single type of limestone or marble is found to result in crack initiation at the disk centre when using the ISRM test configuration. In addition, as discussed by Navidtehrani et al. (2022), we evaluate the role of friction and Poisson's ratio, showing that these play a secondary role, with only Poisson's ratio influencing the results for the case of flat jaws.

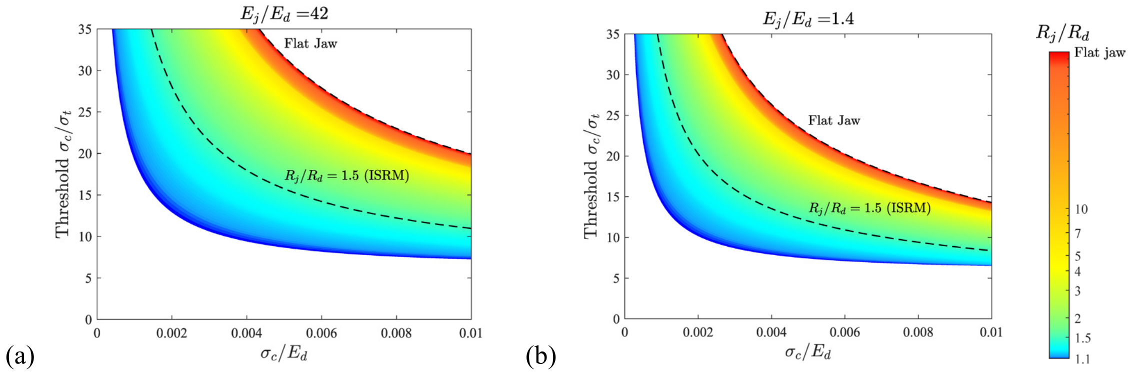

Figure 2. Influence of the jaw radius on the minimum acceptable ratio of compressive-to-tensile strength for (a) $E_j/E_d = 42$, and (b) $E_j/E_d = 1.4$. The Poisson's ratio of the disk equals $\nu_d = 0.2$.

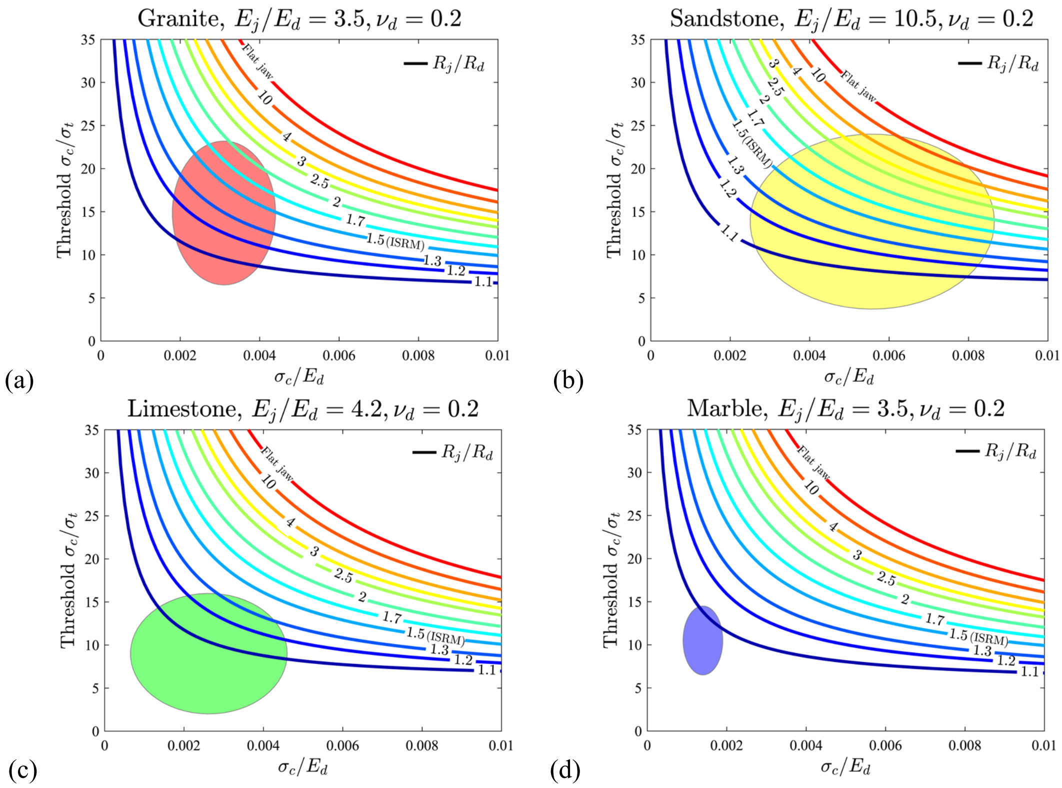

Figure 3. Maps to assess if cracking nucleates at the centre; application to: (a) granite, (b) sandstone, (c) limestone, and (d) marble. The figure shows admissible compressive-to-tensile strength ratios as a function of the jaw radius (R_j/R_d) and relevant material properties. For the test to be valid, the R_j/R_d line must be below the relevant set of material properties.

Moreover, a protocol is presented whereby the validity of the Brazilian test can be determined from the compressive strength of the material. As detailed by Navidtehrani et al. (2022), the protocol can be readily utilised by making use of the maps developed or through the use of the dedicated MATLAB App developed (BrazVal, available at https://www.imperial.ac.uk/mechanics-materials/codes/). We demonstrate the practical application of our findings through two examples of valid and invalid tests taken from existing literature. Specifically, as detailed by Navidtehrani et al. (2022), we show that the experiments of Sun & Wu (2021) on sandstone deliver a non-valid test (cracking outside the disk centre) when using the ISRM testing configuration, whereas the work by Duevel & Haimson (1997) on granite provides a valid σ_t estimate for the use of the ISRM jaw radius, but not for the ASTM one.

5 CONCLUSIONS

We have used the generalised Griffith criterion, and numerical analysis, to assess the validity of the Brazilian tensile test. By making use of the failure envelope of the generalised Griffith criterion, the location of crack initiation can be readily determined upon knowing the material compressive strength. Based on this, calculations are conducted to map the regimes of validity of the Brazilian test for most rock-type materials. The results reveal a much narrower range of conditions for which the Brazilian test is valid, with current practices and standards found to be inappropriate for a wide range of rock-like materials.

ACKNOWLEDGEMENTS

The authors acknowledge financial support from the Ministry of Science, Innovation and Universities of Spain through grants PGC2018- 099695-B-I00 and MCINN-22-TED2021-130306B-I00. E. Martínez-Pañeda additionally acknowledges financial support from the Royal Commission for the 1851 Exhibition, through their Research Fellowship programme (RF496/2018).

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