Using borehole breakout data to constrain the *in situ* stress tensor

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ABSTRACT: Knowledge of the *in situ* stress state is critical for underground rock engineering projects, but direct measurement data are often not available. Although borehole breakouts are only indicative of the *in situ* stress state, they nevertheless represent a significant data resource. Properties such as breakout orientation, width and depth can all potentially be used to constrain the *in situ* stress state. In this paper, we present a novel global optimization and associated objective function that uses breakout data to constrain the normalized *in situ* stress tensor. This tensor is one in which the magnitudes of the three principal stresses are normalized with respect to the difference between the maximum and the minimum principal stresses, and three Tait–Bryan angles describe the principal stress orientations. The efficacy of the scheme is demonstrated by applying it to breakout data obtained from deep mining operations in the Sudbury mining district of Ontario, Canada.

Keywords: borehole breakout, in situ stress, global optimization, normalized principal stresses, principal stress orientations.

1 INTRODUCTION

In situ stress is necessary information for the design and construction of underground rock engineering projects. Borehole breakouts are stress-induced borehole diametrical elongations that occur when the induced stress exceeds the local rock strength on the borehole wall. These can be identified and are a potential data source to estimate the *in situ* stress state.

Due to the high cost and time-consuming process of direct measurement methods, necessary *in situ* stress data is often not available in most projects. Given the widespread and increasing use of Acoustic Televiewer (ATV) technology to survey boreholes, large amounts of borehole breakout data are available which are indicative of the *in situ* stress state. Borehole breakout orientation is widely used to estimate the orientations of horizontal principal stresses (Plumb & Hickman 1985; Zoback et al. 1985), but this requires the assumption that one of the principal stresses is vertical. However, this assumption is not valid in many locales, and breakout data are rarely used to estimate *in situ* stress magnitudes.

By taking advantage of a large amount of borehole breakout data to estimate the *in situ* stress tensor, we aim to constrain the normalized *in situ* stress tensor without assuming that one of the principal stresses is vertical. The normalized stress tensor used in this paper is the one in which the magnitudes of the three principal stresses are normalized with respect to the difference between the major and the minor principal stresses, and three Tait–Bryan angles describe the principal stress orientations. We use a global optimization method with breakout orientation data from many boreholes in the same locale to constrain the normalized *in situ* stress tensors at different depths.

2 BOREHOLE BREAKOUT DATA

An illustration of the trajectory and borehole breakouts of a typical borehole is presented in Figure 1. As shown in Figure 1(a), the azimuth angles of the two sides of borehole breakouts are visualized along the borehole, with the size and hue of the circles representing the length of breakouts, i.e., the bigger and darker circles represent longer breakouts. Figure 1(b) presents a circular histogram indicating breakout azimuth angles and counts. Most of the breakouts in this borehole occur with azimuth angles ranging from NW-SE to W-E counterclockwise. The wellbore trajectory (Figure 1(c)) is visualized by using the accepted industry standard minimum curvature method (Sawaryn & Thorogood 2005). The breakout orientations, the true vertical depth of each breakout, and the borehole trajectory data (borehole azimuth angle and borehole deviation angle) will be used in this work.

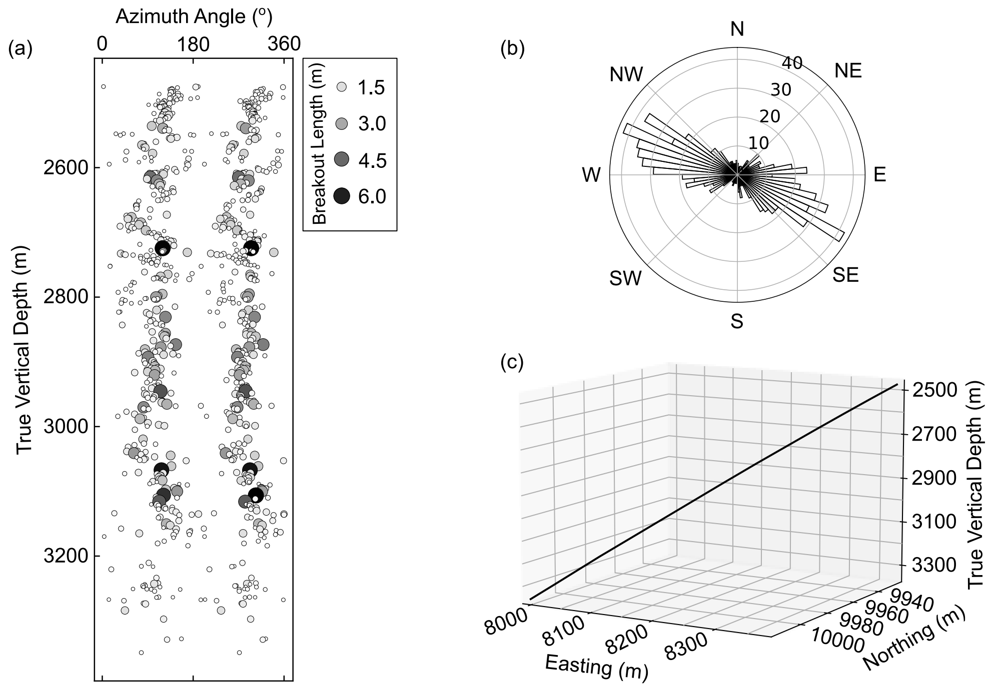


Figure 1. Illustration of borehole trajectory and breakout data of a typical borehole.

3 METHODOLOGY

3.1 Stress tensor parameterization

The stress tensor is fully determined by six components and is generally described with three normal stresses and three shear stresses. Inspired by Zajac & Stock (1997), another parameterization of the stress tensor is used in this paper. This parameterization uses the three parameters a, b, ϕ to determine the magnitudes of three principal stresses and the three Tait–Bryan angles α, β, γ to determine the orientations of principal stresses. Thus, a stress tensor **S** is given by

$$\mathbf{S} = f(a, b, \phi, \alpha, \beta, \gamma), \tag{1}$$

where

$$a = \sigma_1 - \sigma_3,\ b = \sigma_3/(\sigma_1 - \sigma_3), \text{ and } \phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3). \tag{2}$$

The inverse relationships among a, b, ϕ and σ_1, σ_2, σ_3 are

$$\sigma_1 = a(b+1),\ \sigma_2 = a(b+\phi), \text{ and } \sigma_3 = ab. \tag{3}$$

3.2 Stress state surrounding an arbitrarily deviated borehole

Borehole breakouts occur when the induced stress exceeds the local rock strength. Therefore, knowing the stress state surrounding an arbitrarily deviated borehole is necessary to investigate the properties of the breakouts. As shown in Figure 2, the stress tensor transformations among three coordinate systems (the *in situ* stress coordinate system *Xs-Ys-Zs*, the geographic coordinate system North-East-Down and the borehole coordinate system X_b-Y_b-Z_b) are required. To obtain the stress tensor in the borehole coordinate system, the stress tensor $\mathbf{S}_s$ in the *in situ* stress coordinate system is transformed into the geographic coordinate system to obtain the stress tensor $\mathbf{S}_g$. $\mathbf{S}_g$ is then transformed into the borehole coordinate system to obtain the stress tensor $\mathbf{S}_b$ (Eq. (4)). Thus,

$$\mathbf{S}_g = \mathbf{R}_s^T \mathbf{S}_s \mathbf{R}_s \text{ and } \mathbf{S}_b = \mathbf{R}_b \mathbf{S}_g \mathbf{R}_b^T, \tag{4}$$

where $\mathbf{R}_s$ and $\mathbf{R}_s^T$ are the transformation matrix and its transpose from the *in situ* stress coordinate system to the geographic coordinate system respectively, and $\mathbf{R}_b$ and $\mathbf{R}_b^T$ are the transformation matrix and its transpose from the geographic coordinate system to the borehole coordinate system respectively. The transformation matrices are available in the literature (e.g. Dahrabou et al. 2022; Peška & Zoback 1995).

3.3 Constraining the normalized in situ stress tensor

After analyzing the stresses on the borehole wall (Peška & Zoback 1995; Zajac & Stock 1997), if the ratio η is zero, the breakout orientation θ_b can be determined by the five parameters:

$$\theta_b = f(b, \phi, \alpha, \beta, \gamma). \tag{5}$$

The ratio η is given by

$$\eta = \Delta P/(\sigma_1 - \sigma_3) = \Delta P/a, \tag{6}$$

where ΔP is the difference between the borehole fluid pressure and the pore pressure in the rock mass. For the boreholes analyzed here, it is reasonable to assume that η is close to zero.

As shown in Figure 3, the borehole breakout data from 13 boreholes in the Sudbury mining district of Ontario, Canada, are used to constrain the normalized *in situ* stress tensor. The magnitudes of the three principal stresses are normalized with respect to the difference between the major and the minor principal stresses, i.e., normalized by parameter a, so the normalized principal stress magnitudes are

$$\sigma_1 = b + 1,\ \sigma_2 = b + \phi \text{ and } \sigma_3 = b, \tag{7}$$

and the principal stress orientations are determined by the Tait–Bryan angles.

Given a normalized *in situ* stress tensor, the theoretical breakout orientation can be obtained with the above methodology. The so-called Basin-Hopping global optimization method (Wales & Doye 1997) is applied to search the normalized stress tensor space and find the one that minimizes the objective function D: where

$$D = (\sigma L)^{-1} \sum \{ l_i \times |\cos(\theta_{bi} - \theta_{oi})| \} \tag{8}$$

with

$$\sigma = \sqrt{-2\ln(1 - S_o)}/2,\ S_o = 1 - (R')^{\frac{1}{4}} \text{ and } \theta'_{oi} = 2\theta_{oi}, \tag{9}$$

where l_i and L are the length of breakout i and the total length of all breakouts, θ_{bi} and θ_{oi} are the theoretical and actual orientations of breakout i, S_o and σ are the circular variance and circular standard deviation of all actual breakout orientations, and R' is the circular mean resultant length of all transformed breakout orientations θ'_{oi}.

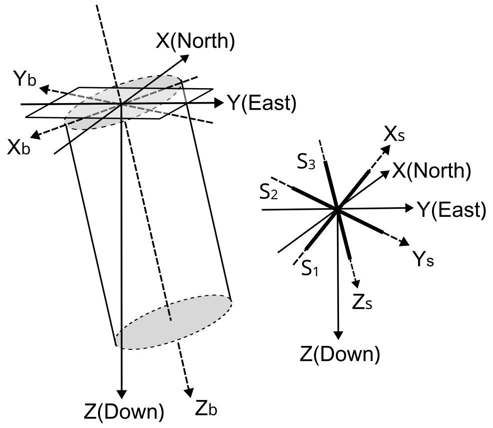


Figure 2. Coordinate systems.

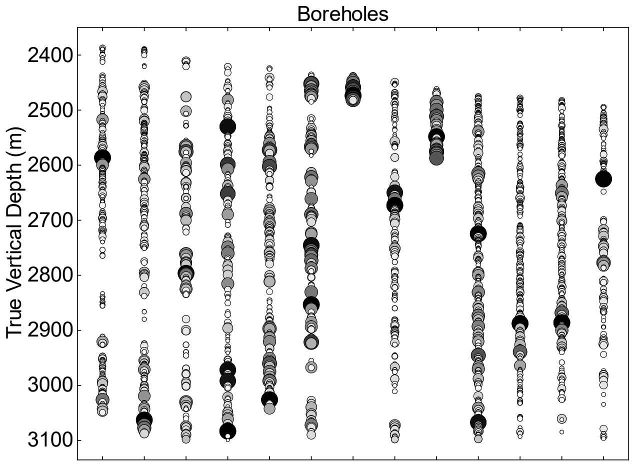


Figure 3. Borehole breakouts of different boreholes.

4 RESULTS AND DISCUSSION

The investigated depth ranges from 2400 m to 3100 m below the ground surface, and for our analysis is divided into seven depth regions each 100 m in length. To obtain robust results the global optimization procedure is conducted 100 times for each region, each time using a different initial starting point. As shown in Figure 4, the stress ratio ϕ and the orientations of *in situ* principal stresses for the 2400 m to 2700 m depth region are well constrained with low dispersion. In the plots of *b* and ϕ, the grey open circles represent all optimization results while the red filled circles are the mean values in each region. The horizontal black lines represent the 5 to 95 percentile intervals.

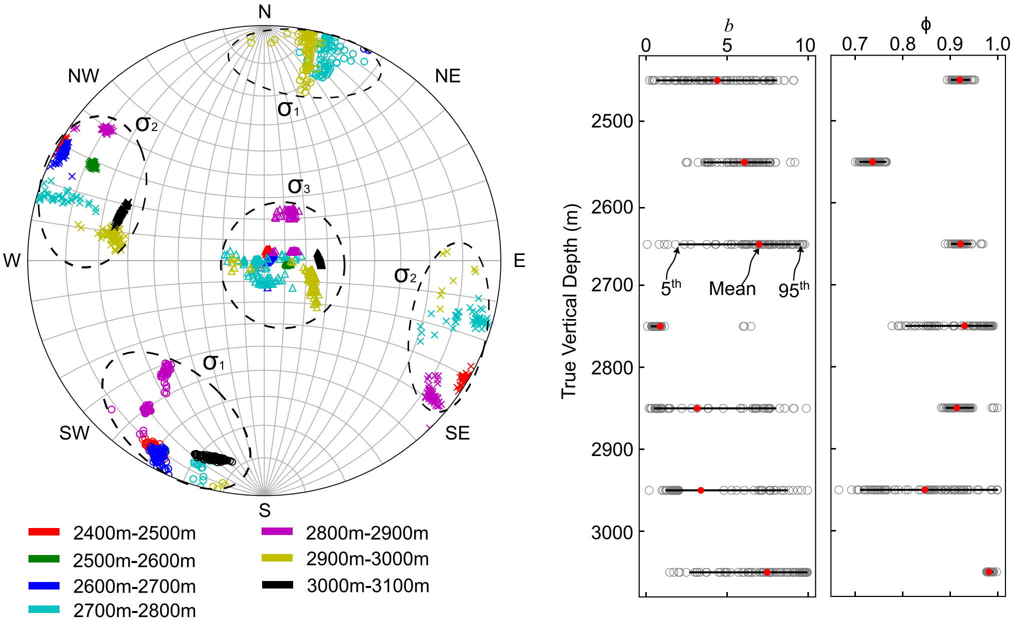


Figure 4. Normalized stress tensor at different depth intervals.

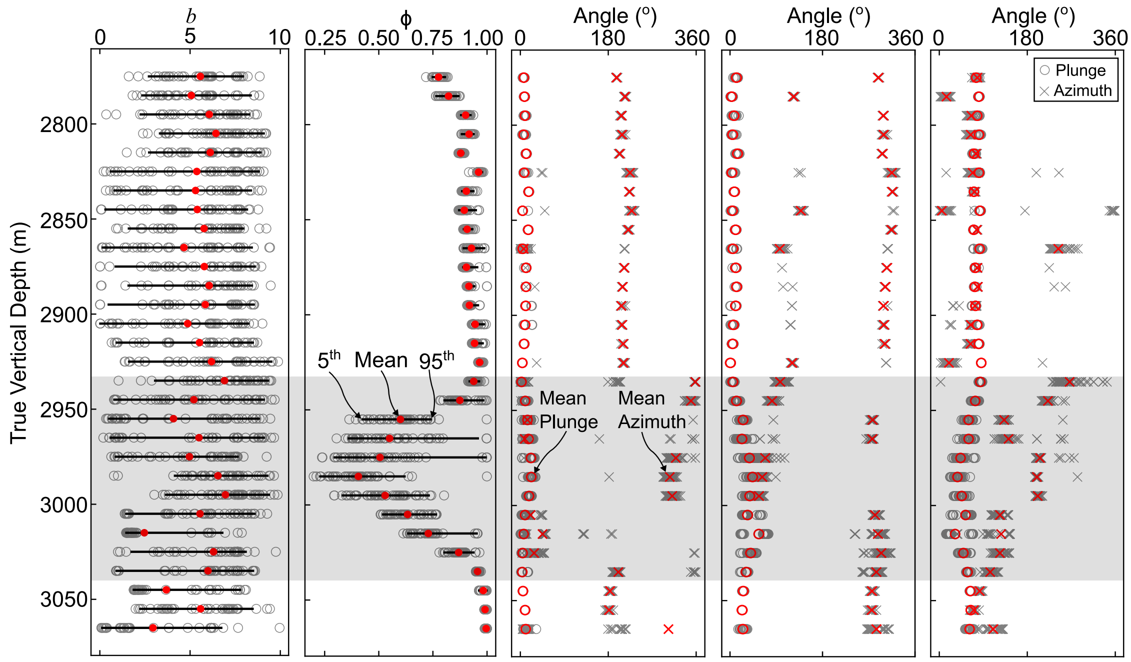
The mean value μ and the standard deviation ς of *b*, ϕ and the principal stress orientations are shown in Table 1. In terms of the azimuth and plunge angle of principal stress orientation, μ and ς represent the circular mean and circular standard deviation. These results show that *b* has high variance while ϕ has low variance, which means the breakout orientation data can be used to constrain ϕ but not *b*.

Moreover, the principal stress orientations, parameters b and ϕ all have high variance in the region from 2800 m to 3100 m.

Table 1. Mean and standard deviation of parameters.

Depth (m)	b μ	b ς	ϕ μ	ϕ ς	σ_1 Azimuth μ (°)	σ_1 Azimuth ς	σ_1 Plunge μ (°)	σ_1 Plunge ς	σ_2 Azimuth μ (°)	σ_2 Azimuth ς	σ_2 Plunge μ (°)	σ_2 Plunge ς	σ_3 Azimuth μ (°)	σ_3 Azimuth ς	σ_3 Plunge μ (°)	σ_3 Plunge ς
2400-2500	4.33	2.38	0.92	0.01	210.4	0.95	4.41	0.68	296.0	1.74	0.84	0.64	29.2	2.28	85.46	0.68
2500-2600	5.95	1.54	0.74	0.02	208.4	0.53	4.35	0.49	299.2	0.43	10.53	0.53	96.3	2.07	78.58	0.59
2600-2700	6.74	2.38	0.92	0.02	209.2	1.69	3.21	1.15	298.4	1.66	2.28	0.51	70.1	1.87	85.9	0.99
2700-2800	1.36	1.92	0.91	0.08	16.2	1.58	5.16	1.95	94.7	1.38	5.91	1.95	211.1	1.99	81.4	1.57
2800-2900	3.22	3.18	0.91	0.02	219.8	2.02	19.3	1.22	125.9	1.19	6.31	1.72	46.4	1.69	69.3	1.33
2900-3000	3.52	2.85	0.83	0.09	10.6	1.30	6.29	1.79	279.7	1.26	22.0	2.35	117.8	2.27	66.5	2.26
3000-3100	7.32	2.26	0.98	0.004	195.0	2.02	8.17	0.42	288.3	1.94	25.3	0.57	89.7	2.24	63.2	0.41

To further investigate the regions with high variance of the normalized *in situ* stress tensor (i.e. from 2800 m to 3100 m), a 50 m "scan window" is used with successive scan windows having 10 m increments in the downhole direction, and the global optimization procedure is conducted 50 times at each scan region. In Figure 5(c)-(e), grey circles and grey crosses represent the plunge and azimuth angle of the principal stress orientations for all the optimization results, while red circles and red crosses represent the mean plunge and azimuth angle of the principal stress orientations of each scan.



(a) b (b) ϕ (c) σ_1 orientation (d) σ_2 orientation (e) σ_3 orientation

Figure 5. Parameters for normalized stress tensors along the true vertical depth.

Figure 5 shows that the optimal stress tensor varies markedly in the interval 2940 m to 3040 m (i.e. the grey shaded region). This variation is seen most clearly in the value of ϕ and the orientations of the principal stresses. Our opinion is that the presence of a nearby geological structure is responsible for this perturbation.

In this work, parameter *a* is not considered, and results show that parameter *b* can not be well constrained by only using breakout orientation data. Other breakout properties such as breakout width and depth, together with mechanical properties of the intact rock can be used to constrain parameters *a* and *b* so that the magnitude of the three principal stresses can also be estimated. Investigations into this are continuing.

5 CONCLUSIONS

Borehole breakouts are stress-induced, so their properties can be used to constrain *in situ* stress state. In this paper a novel parameterization of the stress tensor together with a global optimizing procedure is applied to breakout data from 13 boreholes. The method can be used to estimate the normalized *in situ* stress tensor at different depths. The results show that the stress tensor parameters ϕ, α, β, γ can be well constrained with breakout orientation data but parameter *b* can not. The analysis indicates a marked stress perturbation in the depth interval from 2940 m to 3040 m, which may be due to the presence of a nearby geological feature. By including other breakout and intact rock properties the method can be extended to completely constrain the stress tensor.

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