Numerical simulation of rock's fatigue behavior using a modified indirect boundary element method

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ABSTRACT: Fatigue crack growth in brittle rocks was numerically simulated by a modified displacement discontinuity method. Mode I and Mode II stress intensity factors and their increments due to cyclic loading of brittle rock samples were numerically estimated using linear elastic fracture mechanics. Effective methodologies were introduced for modeling the fatigue crack propagation in brittle rocks under cyclic loading. The quadratic displacement discontinuity and crack tip elements were used to predict the mixed mode stress intensity factors at crack tips in the brittle material specimens. The maximum tangential stress criterion was used to predict the crack growth paths using the incrementally increasing cracks lengths in the predicted direction. Several examples were solved to evaluate the accuracy and efficiency of the proposed algorithm. It is concluded that the fatigue crack growth in brittle rocks under mixed-mode loading conditions can be effectively analyzed by the modified higher order displacement discontinuity.

Keywords: Fatigue crack growth, Boundary element method, Higher order displacement discontinuity, Crack tip elements, Maximum tangential stress criterion.

1 INTRODUCTION

The fatigue phenomenon occurs in structures that are exposed to cyclic loads. At present, three approaches of stress, strain, and fracture mechanics are utilized to investigate fatigue (Woodford, 1993). Based on fracture mechanics concepts, all materials have inherent flaws and defects. These cracks may grow due to cyclic loading and continue to grow until reaching a critical value, this approach specifically treats fatigue crack growth behavior by the principle of fracture mechanics. Several analytical methods have been presented to study the fatigue crack growth behavior (NASCRAC, 1989). Nevertheless, these methods are available only in a limited group of geometry combinations and boundary conditions. Hence, it is suitable to use numerical methods to examine the fatigue crack growth behavior and life estimation. The classical finite element method (FEM) has been extensively employed as a numerical tool for several decades to study fatigue and fracture mechanics problems (Bittencourt et al., 1996). However, this method has several limitations in meshing and, if it is employed, the mesh around the crack tip should be small enough to model the changes of the crack tip and stress gradients appropriately. Besides, at each step of crack growth, re-meshing of an existing boundary is required. The extended finite element method (XFEM) which is based on FEM has been proposed. In this method, the complex processes related to the crack growth were resolved by enriching the nodes and virtually enhancing the degrees of freedom (Surendran et al., 2019). In recent years, using meshless methods in order to develop the numerical modeling of crack growth has been at the center of interest as well. For instance, Belytschko et al., for the first time, introduced the element-free Galerkin method (EFGM) to study the growth of cracks (Belytschko et al., 1994). Free-mesh methods were later employed to develop fatigue crack growth (Kumar et al., 2014)

A 2nd order displacement discontinuity method is used to study crack propagation under cyclic loading. The capability of modeling the fatigue of the structures which have multiple cracks with different growth rates is provided. In the development of the fatigue process, the range of stress intensity factor (RSIF) was considered as the correlation parameter. The Paris law, as the equation governing the fatigue growth rate of cracks in combination with the results initiated from the HDDM-based analysis, was employed for incremental growth of cracks and life estimate of structures. Finally, using two examples, the performance of this method in comparison with the results of the experimental and other numerical methods was studied.

2 NUMERICAL MODEL

The DDM is an indirect and semi-analytical method. In this method, elements are considered in the form of a linear part having two separate surfaces. The development of higher order displacement discontinuity may be found in our previous studies (Abdollahipour & Fatehi-Marji, 2022).

In the presented algorithm, the crack growth behavior in a specified material was simulated based on the relation between crack growth rate (*da/dN*) and stress intensity factor range (ΔK) by using the Paris equation (Eq. (1)).

$$\frac{da}{dN} = C(\Delta K)^n \tag{1}$$

where a is the crack length, N is the number of loading cycles, and C and n indicate materials constants. ΔK is also the range of stress intensity factors. The effective stress intensity factor range (ΔK_{eff}) are utilized in mixed-mode loading condition, which is a combination of stress intensity factor range dependent on both opening (ΔK_I) and shear (ΔK_{II}) modes. The values of this parameter were calculated based on experiments conducted by Tanaka under fatigue loading conditions, as well as Eq. (2), which were also employed in this algorithm.

$$\Delta K_{eff} = (\Delta K_I^4 + 4 * \Delta K_{II}^2)^{(1/4)} \tag{2}$$

where $\Delta K_I = K_{Imax} - K_{Imin}$ and $\Delta K_{II} = K_{IImax} - K_{IImin}$are in one loading cycle.

Stress intensity factor range (ΔK) increases with crack length during constant amplitude loading, since the crack growth rate (da/dN) depends on ΔK, the growth rate is not constant too and increases with crack length. In other words, the crack accelerates as it grows. by considering the status of the variation of da/dN, our algorithm that is based on the fatigue higher-order displacement discontinuity method (FHDDM) utilizes integrating (Eq. (3)) to compute life time (i.e., number of cycles).

$$N = \int_{N_i}^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{f(\Delta K, R)} \tag{3}$$

2.1 Direction of fatigue crack propagation

Over the past few decades, various criteria have been presented for the onset and spread of cracks from the tip under mixed-mode loading conditions. Maximum circumferential stress criterion (σcriterion), maximum energy release rate criterion (G criterion), and minimum strain energy density criterion (S criterion) were the most applicable classic criteria, which were employed in many studies. Several modified criteria have been presented, which the F criterion (G-modified criterion) is the most important of them. Studies have been shown that similar results are obtained for brittle materials using S and σ criteria, and strain energy-based criteria have been widely used in 3D applications. In the present FHDDM algorithm, the σ criterion introduced by Erdogan and Sih has been employed to determine the direction of the crack path. Under the general conditions of mixed-mode loading, tangential and shear stresses in polar coordinates are introduced according to Eq. (4).

$$\begin{cases} \sigma_{\theta\theta} = \dfrac{K_I}{\sqrt{2\pi r}}\dfrac{1}{4}(3\cos(\theta/2) + \cos(3\theta/2)) + \dfrac{K_{II}}{\sqrt{2\pi r}}\dfrac{1}{4}(-3\sin(\theta/2) - 3\sin(3\theta/2) \\ \sigma_{r\theta} = \dfrac{K_I}{\sqrt{2\pi r}}\dfrac{1}{4}(\sin(\theta/2) + \sin(3\theta/2)) + \dfrac{K_{II}}{\sqrt{2\pi r}}\dfrac{1}{4}(\cos(\theta/2) + 3\cos(3\theta/2)) \end{cases} \tag{4}$$

Based on the Maximum circumferential stress criterion, the crack grows at the angle θ relative to its tip and in the direction that $\sigma_{\theta\theta}$ is maximum. The shear stresses will be zero when the tangential stress is maximum, so the values of the crack growth angle are obtained based on Eq. (5).

$$\theta = 2\tanh^{-1}\frac{1}{4}\left(K_I/K_{II} \pm \sqrt{(K_I/K_{II})^2 + 8}\right) \tag{5}$$

Based on the same criterion, the effective stress intensity factor is also obtained with Eq. (6).

$$K_{eff} = K_I\left(\cos\frac{\theta}{2}\right)^3 - 3K_{II}\left(\cos\frac{\theta}{2}\right)^2 \sin\frac{\theta}{2} \tag{6}$$

The σ criterion models the crack propagation path continuously when the progress of the crack length tends to zero. However, using this criterion in a discrete algorithm and mixed-mode deformation field, the extension direction regardless of the incremental length will be permanently in the same direction. The variations in the stress field with the propagation of cracks are not counted, as a consequence, the path acquired through this criterion will not be unique. This method was proposed to correct the crack propagation angle, based on which a single final crack path is obtained from different analyses of a problem with different progress step sizes. In this method, several correction levels (l) are accomplished for every single angle specified in the "s" progress step. The refinements are implemented by introducing a correction angle α; this angle is defined based on the propagation angle of the next step ($s + l$) as $\alpha^l = \theta_{s+1}/2$. where l is the representative of the number of the corrections iteration. The propagation angle of the step "s" is corrected through the correction angle α^l, so the new angle is introduced as $\theta_s^{l+1} = \theta_s^{l+} \alpha^l$. The corrections are applied for the step "s" until a correction angle smaller than the previous one is obtained ($|\alpha^{l+1}| < |\alpha^l|$).

3 FATIGUE GROWTH OF A SINGLE INCLINED CRACK IN A LIMITED PLANE

This example is established upon an experiment first conducted by Pustejovsky et al. (Pustejovsky, 1979). In these experimental investigations, crack propagation was implemented on an almost homogeneous and isotropic sample under the effect of cyclic fatigue loading at room temperature and humidity (21°C and relative humidity 70%).

A rectangular plate with a width of 2w = 76.2 mm and a height of 2l = 304.8 mm is considered, containing a single central inclined crack. The crack length and incline angle relative to the horizon surface are 2a = 13.462 mm and θ = 43, respectively (Figure 1a). Dimensions and loading and boundary conditions of rectangular-shaped plate containing one single inclined crack, b) Discretization of boundaries using ordinary and crack tip higher-order elements.a). The sample is in plane strain conditions. The cyclic tension stress with a maximum value of 172.37 (MPa) applies to the upper and, lower boundaries of the plate, and the value of the stress ratio in this cyclic loading is R = 0.1. In the practical experiment, the maximum tensile load values are designated such that the radius of the plastic zone around the crack tip is less than 10% of the other dimensions of the sample and the small-scale yield hypothesis prevails. The elastic modulus, Poisson ratio, fracture toughness, and threshold stress intensity factor are equal to 113.8 GPa, 0.34, 63.73 MPa, and 2.19 MPa, respectively. The constants of the Paris law are C=1.58*10^{-9} and $m = 3.81$; hence, the values of growth rate and stress intensity factor are in terms of mm/cycle and MPa (m)$^{0.5}$, respectively.

To perform the fatigue growth analysis, the length of each special linear segment of the crack tip is considered equal to 0.5 mm. A view of the discrete model is displayed in Figure 1a). Dimensions and loading and boundary conditions of rectangular-shaped plate containing one single inclined crack, b) Discretization of boundaries using ordinary and crack tip higher-order elements. Fig. 2 indicates the growth path of the two crack tips acquired from the analysis via the FHDDM algorithm and practical experiments. Because of the loading and geometric symmetry, both crack tips show the same path and growth rate. As can be seen, the crack growth path resulting from the displacement discontinuity method is in good agreement with the data obtained from the laboratory results. Furthermore, using the maximum circumferential stress criterion is very appropriate in determining the growth path.

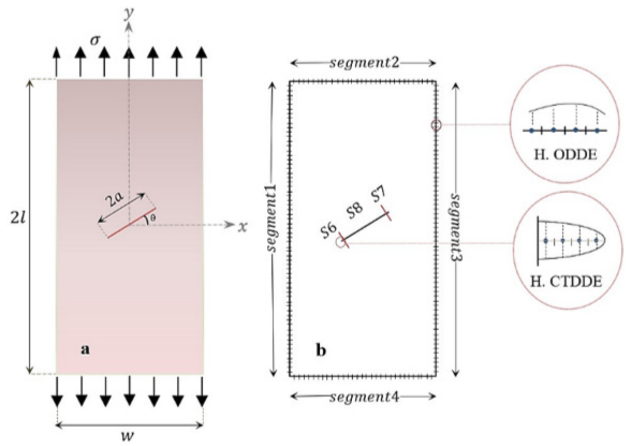

Figure 1. a) Dimensions and loading and boundary conditions of rectangular-shaped plate containing one single inclined crack, b) Discretization of boundaries using ordinary and crack tip higher-order elements.

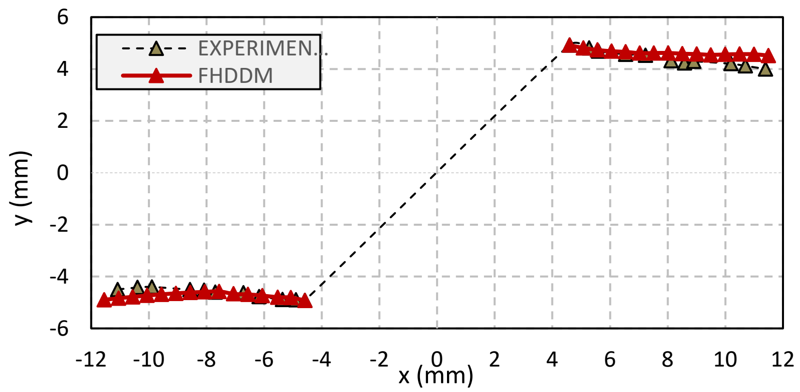

Figure 2. Fatigue growth of crack in a hole-containing plate.

4 FATIGUE CRACK GROWTH OF TWO EDGE CRACKS IN A PLATE

This example examines the growth of two cracks in a rectangular panel and studies the performance of an FHDDM-based algorithm in structures with multiple cracks. This example was investigated by two other methods of DBEM and XFEM. In the numerical model, a rectangular panel with a width of 2w = 50 mm and a height of 2l = 150 mm was considered, which contains two edge cracks with a length of 2a = 10 mm, on the opposite edge of the plate. These two cracks are located on two different surfaces with a vertical distance of 5 mm from each other (see Fig. 3a). The sample is subjected to cyclic stress with a maximum value of 100 MPa that applies to the upper edge, and the value of the stress ratio is equal to R = 0. The panel was fixed at the bottom in both directions. The elastic modulus and Poisson's ratio were 30 GPa and 0.3, respectively. The constants of the Paris fatigue crack growth law were considered as $C = 1\times10^{-12}$ (mm/cycle*(MPa (m) $^{0.5}$)^3) and n = 3.

This example is presented to assess the efficiency of the algorithm for structures with multiple cracks so that, to implement the fatigue growth analysis, the length of the initial linear segments for the crack tip is equal to 1.5 mm as the maximum size of increment and the minimum crack length increment were defined as 0.6 mm. Moreover, the maximum value of the allowable ratio between the lengths of neighboring elements is 2.5. A view of the discretized model is illustrated in Fig. 3b.

Fig. 4 indicates the growth paths of both crack tips using the FHDDM numerical method along 17 incremental extensions. The length of increments of both crack tips at the beginning of growth is almost the same, indicating that initially, both crack tips grow at nearly the same growth rate; however, the growth rate of crack No. 2 gradually becomes higher than that of crack No. 1 so that this difference is quite visible after 10 increments. in Fig. 4, the growth path resulted from the analysis by two numerical methods of DBEM and XFEM is also presented. As can be seen, there is good accordance between the results of the three numerical methods.

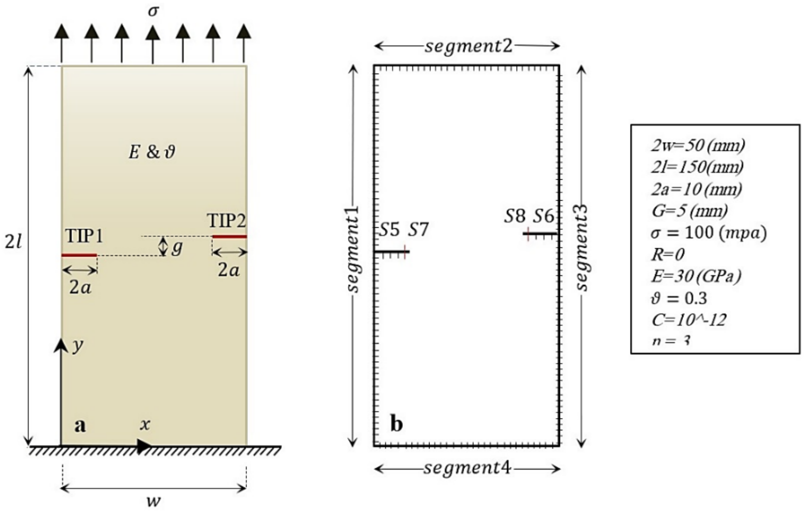

Figure 3. a) Dimensions and boundary- loading conditions for a rectangular-shaped panel consisting of two eccentric edge cracks, b) Discretization of boundaries using ordinary and crack tip higher-order elements.

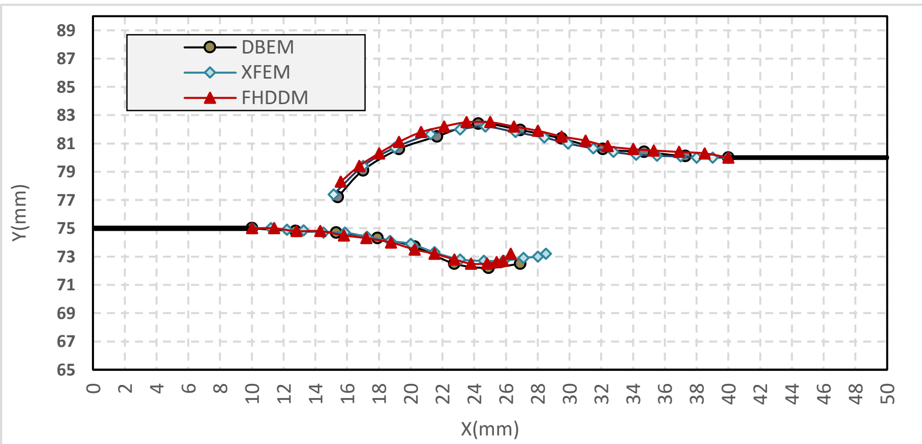

Figure 4. A comparison between the results of propagation path of the eccentric cracks located on the rectangular-shaped panel using three numerical methods of FHDDM, DBEM, and XFEM.

5 CONCLUSIONS

The displacement discontinuity method (DDM) was used to model crack growth in cyclic loading conditions. The Paris and Walker laws were employed as the governing equations of fatigue in combination with LEFM principles to model crack propagation and life estimate of brittle structures.

The method predictions are validated with the result of 2 experimental test and other numerical methods. Several major conclusions can be drawn.

- Method validation indicates an overall satisfactory result of the proposed method under the cyclic fatigue loading;
- Discretization in our method is merely performed at the boundaries, so the dimensions of the problem are decreased, and that is more cost-effective than the domain-based technique. Moreover, the need for remeshing is eliminated and the crack growth modeled by adding a new element successively;
- The possibility of fatigue crack growth prediction in mixed mode condition of loading is provided;
- The model was applied to solve multiple cracks propagation with different growth rates;
- The proposed method can handle both variable and constant amplitude loading conditions.

Finally, in this research, a good step has been taken in the development of HDDM to model fatigue growth. This numerical algorithm can be more extended to investigate the fatigue crack growth in non-homogeneous domain, especially to simulate composite structures, and this study is in progress.

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