Experimental and Numerical Investigations on Bi-axial Loading of Sandstone Brazilian Disks

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ABSTRACT: The behavior of sandstone Brazilian disks is investigated under loading in two perpendicular directions. The vertical force was applied at a rate of 200 N/sec while the horizontal load was adjusted to a predetermined ratio of the vertical load until the sample failure. This testing process was repeated for various ratios (including 0%, 10%, 20%, 25%, 33%, 40%, 45%, and 50%) and the failure of samples were captured using a high-speed camera at 25 kHz. While the superposition theory suggests that there will be no tension at the center of the disks in a biaxial loading mode for ratios of 33% and above, results suggest tensile crack occurrence for ratios up to 45%. The numerical results further show that there is always a tensile stress close to the Indirect Tensile Strength happening within the disks central line. It was also noted that if the material transitions into a plastic mode due to extremely high stress ratios, the induced tensile stress would be always considerably higher.

Keywords: Bi-axial Brazilian tests, Indirect tensile test, Bi-axial loading, Finite element modeling.

1 INTRODUCTION

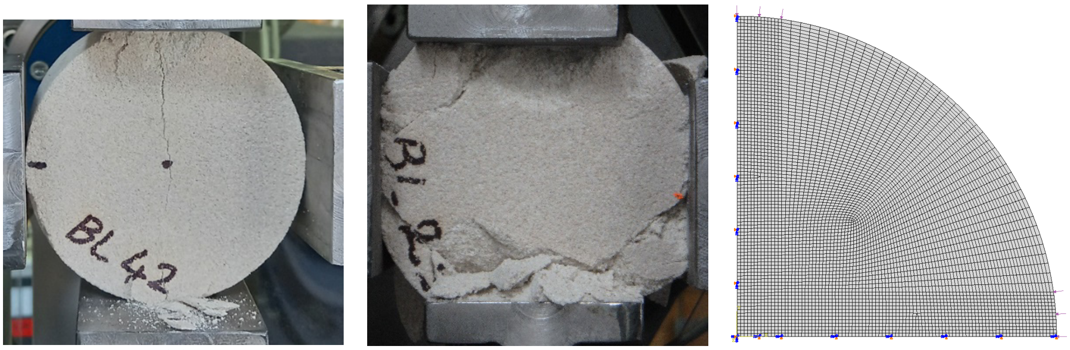
Brittle materials, such as rocks and concrete, have high compressive and shear strength but are weak in tensile and flexural behaviour. Therefore, accurate assessment of the tensile strength of these materials is crucial. The Brazilian indirect tensile strength method is preferred over direct tension test methods due to its practical convenience and ease of sample preparation. However, the accuracy of the Brazilian test has always been challenged due to several reasons. The repeatability of the test has a high range, and the tensile stress induced at the center of the disks is not pure and has a compressive principal stress three times that of the tensile stress. The contact and shape of loading platens can also lead to different results (Fairhurst, 1964; Mellor and Hawkes, 1971; Barla and Innaurato, 1973; Serati et al., 2014 and 2016). The biaxial loading of Brazilian disks with rock-like samples has also been largely under-investigated. In this study, sandstone Brazilian disk specimens under biaxial loading is investigated. Using the prescribed formula outlined in the ASTM (2008) and ISRM (2015) standard recommendations for calculating the induced tensile and compressive stresses at the centre of the disk and according to the principle of superposition, the equation (1) is obtained for the biaxial loading (Serati et al., 2012, 2018 and 2023).

$$\sigma_X\,(\mathrm{k},\mathrm{F}) = \sigma_t\,(\mathrm{F}) + \sigma_c\,(\mathrm{kF}) = \frac{-\,\mathrm{F}}{\pi \mathrm{rt}} + \frac{3\mathrm{k}\,\mathrm{F}}{\pi \mathrm{rt}} = \frac{F}{\pi \mathrm{rt}}(3\mathrm{k}-1) \qquad (1)$$

where σ_t is the tensile stress at the centre of the specimen in the normal direction to the applied force *F*, σ_c is the compressive stress at the centre of the specimen in the same direction of the applied force, and *r* and *t* are the radius and thickness of the specimen, respectively. According to equation (1), the induced tensile stress decreases with an increase in ratio of horizontal to vertical load, k, and for k> 33.3% there will be no induced tensile stress at the centre of the disk. However, the experiment showed that up to the ratio of 45%, the failure is like a tensile crack as expected in the standard Brazilian test. Therefore, in this study the numerical model is implemented to monitor the stress distribution in the disks and investigate the failure mechanism in the biaxial loading scenario and meanwhile to check whether the assumption of using the principle of superposition in this condition is valid or not.

2 NUMERICAL MODEL SET-UP

To understand the behaviour of biaxial tests done on sandstone disks, a 2D Finite Element modelling is conducted in Abaqus software. A quarter of the disk can just be modelled because of symmetry. Finer mesh is used where there is more stress gradient as can be seen in Figure 1 as well as two failed samples in the testing rig. At first, the disk and the loading platens were modelled where the contact point applies the loads to the disk, but for this study, the model with distributed pressure to the surface by the angle of 16º, as suggested by Bahaaddini et al (2019) and Serati et al(2021), is used because it is simpler, and it reached almost identical results except in the vicinity of the contact point. The friction between the platens and samples causes slightly ununiform stress distributions and resistance at the contact point, but as the contact point is relatively a small zone and the failure does not start from that zone, the assumption of ignoring the friction would not affect the result considerably.



(a) (b) (c)

Figure 1. Biaxial loading of Brazilian disk: (a) test with horizontal to vertical force ration of 33%, (b) test with horizontal to vertical force ration of 50%, and (c) Numerical model.

Gosford sandstone blocks supplied by a quarry mine in Sydney were cut into Brazilian disks with 95 mm diameter and 50 mm thickness. Table 1 provides parameters used in the simulation obtained according to ISRM recommendations. Samples were dried in an oven for 48 hours at 105 degrees and kept in isolating bags before testing.

Table 1. Parameters used for FE numerical modeling.

E (GPa)	Passion’s ratio	UCS (MPa)	Dry Density (kg/m^3)	Porosity (%)
40.0	0.40	42.26	2,183	10.88

In this study, at the Geotechnical Engineering Centre within the University of Queensland (Brisbane, Australia) a newly designed and fabricated Biaxial Testing System was used to apply the vertical and horizontal forces at a constant rate *k* ($0 \leq k < 0.5$) until failure. The actuator can apply the load in force control mode and can move up to the speed of 100 mm/sec. The advantage of this is while the actuator applies compressive lateral load, it can move in another direction to allow free expansion due to the vertical load but after the sample fails the actuators can move fast and damage the broken sample, so it might be hard to notice where the damage initiates by looking at the broken samples. The use of a Phantom v2012 ultra-high-speed camera at 100+ kHz enabled us to record the fracture initiation and damage evolution. Loading rate for the vertical actuator applies at a rate of 200 N/Sec and for the horizontal is 200×k so that at any point the sample fails the ratio of horizontal to vertical force will be equal to k. The test was done for k= 0, 10, 20, 25, 33, 40, 45, 50 % and each repeated two or three times.

3 RESULTS AND ANALYSIS

The model in Abaqus was run with the elastic properties allocated to it and results were recorded then elastic-perfectly plastic property was given to the same model to monitor how the stress distribution varies if the material reaches to a plastic state in some part of the sample. The experiment shows the crack happens at the vertical diameter of the disk for all the horizontal to vertical load ratios up to 45%. This allows us to focus on the study of the stress history along this line to understand the mechanism of failure. The average force at failure for each k-ratio was applied to the finite element model as a distributed load.

Figures 2-4 demonstrate the results and the tensile stress along the vertical radius of the disk where the maximum tensile stress is induced.

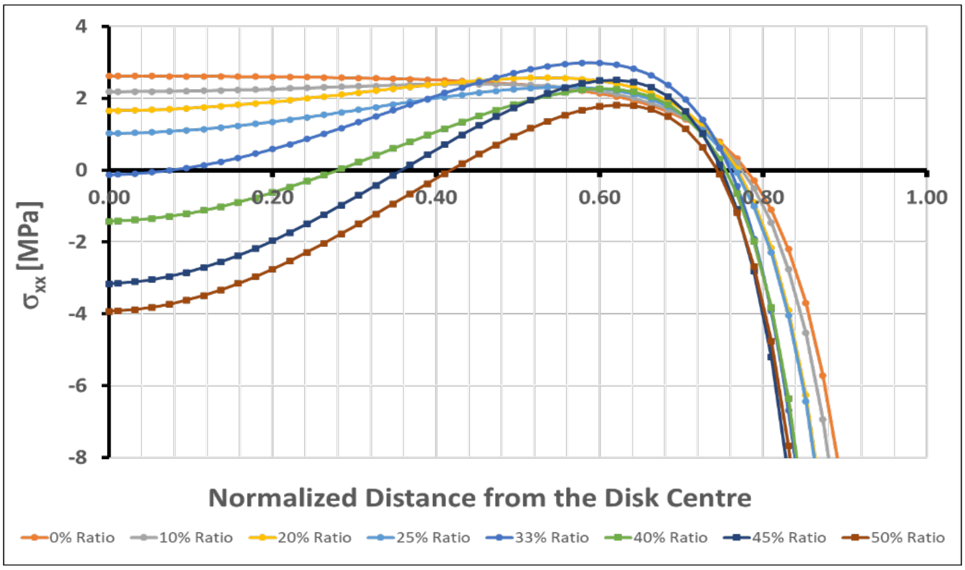
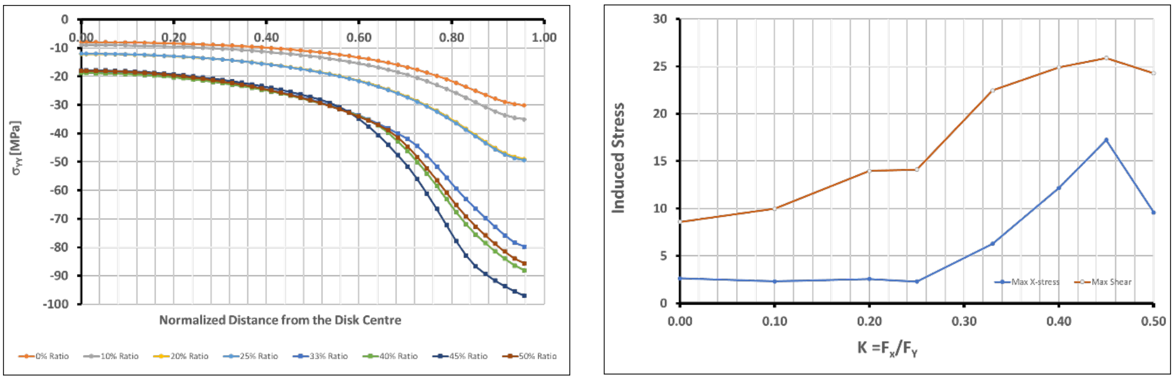


Figure 2. Induced tensile stress along the vertical radius of the disk.

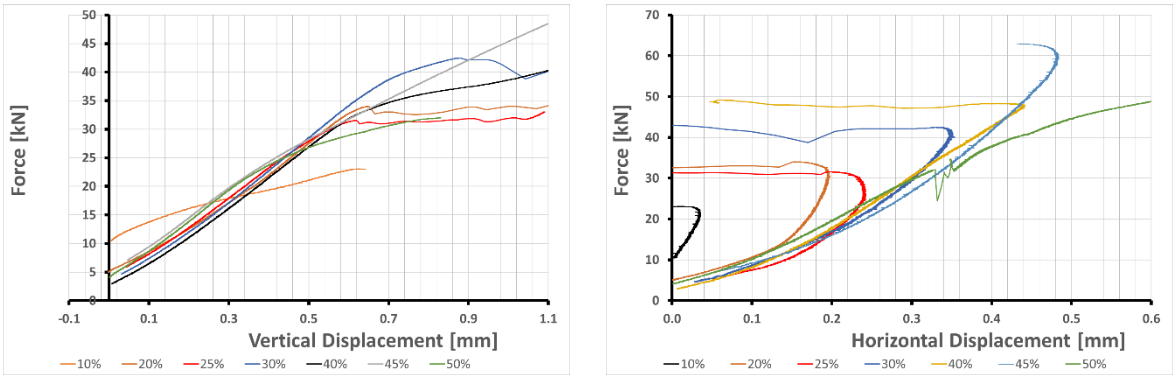
The results show that at the centre of the disks, the induced stress follows the value calculated from the formula derived from the principle of superposition. However, in the upper part of the disk, somewhere between 0.55 to 0.65 of the radius, the induced tensile stress approximately reaches the value of the Indirect Tensile Strength calculated for the Standard Brazilian Test (0% ratio). The failure mechanism in Brazilian disks is deemed to be tensile, meaning that when the tensile stress in the disk reaches a certain amount (Indirect Tensile Strength), the material fails, and the crack appears diametrically along the loading direction. The results obtained from the biaxial loading of the disks with k up to 45% show induced tensile crack across the vertical diameter of the disks similar to the

Standard Brazilian Test, therefore it is reasonable to monitor the stresses across the crack to be compared with the Standard Brazilian Test.



(a) (b)

Figure 3. (a) Induced tensile stress along the vertical radius of the disk, and (b) the maximum induced shear and tensile stress within the disk for different k.



(a) (b)

Figure 4. (a) Vertical force vs Vertical displacement of the disks. (b) Vertical force vs Horizontal displacement of the disks.

The model was run with elasto-perfectly plastic defined for the material with different yield stress. The results showed that by entering the material into the plastic phase there will be a significant change in the stress distribution especially the induced tensile stress. Figure 5 shows the tensile stress along the vertical radius of the disk with a yield stress of 47 MPa. There are no precise plastic parameters available for the material to try more sophisticated plastic models, however, the deformed shape of broken samples can suggest the possibility of local yield in the samples where the k ratio is greater than 33%. The contours of the stress in x-direction and y-direction are demonstrated in Figures 6 and 7 and the failure cracks presented in Figure 8 for different k ratios. Positive stress represents tension and negative represents compression. The induced tensile stress increases by increasing k; however, the location of the maximum tensile stress changes, and when the k is greater than 1:3 there will be no tensile stress at the centre of the disk.

4 CONCLUSIONS

The induced stress at the centre of the disk with an elastic model satisfies the calculated value using the principle of superposition. However, at a position between 55 to 65% along the vertical radius, the induced tensile stress approximately reaches the value of the Indirect Tensile Strength calculated from the Standard Brazilian Test. As the material transitions into the plastic phase, the induced tensile stress increases significantly with the increase in the horizontal to vertical force ratio (k). For the k greater than 45%, the failure deviates from the vertical diameter of the disk, indicating a different failure mechanism.

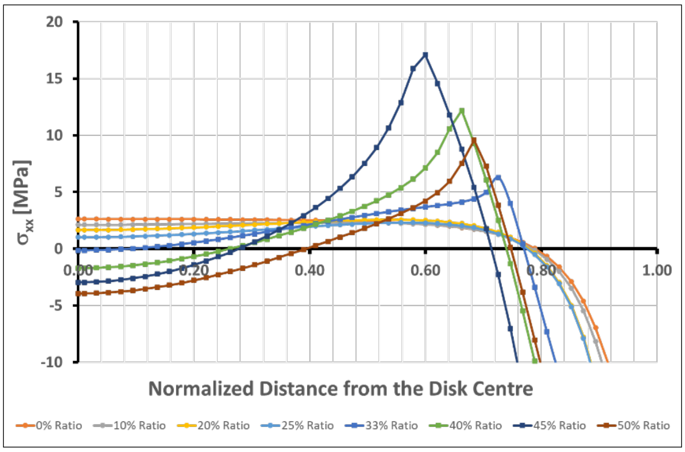


Figure 5. Induced tensile stress along the vertical radius of the disk with yield at 47MPa.

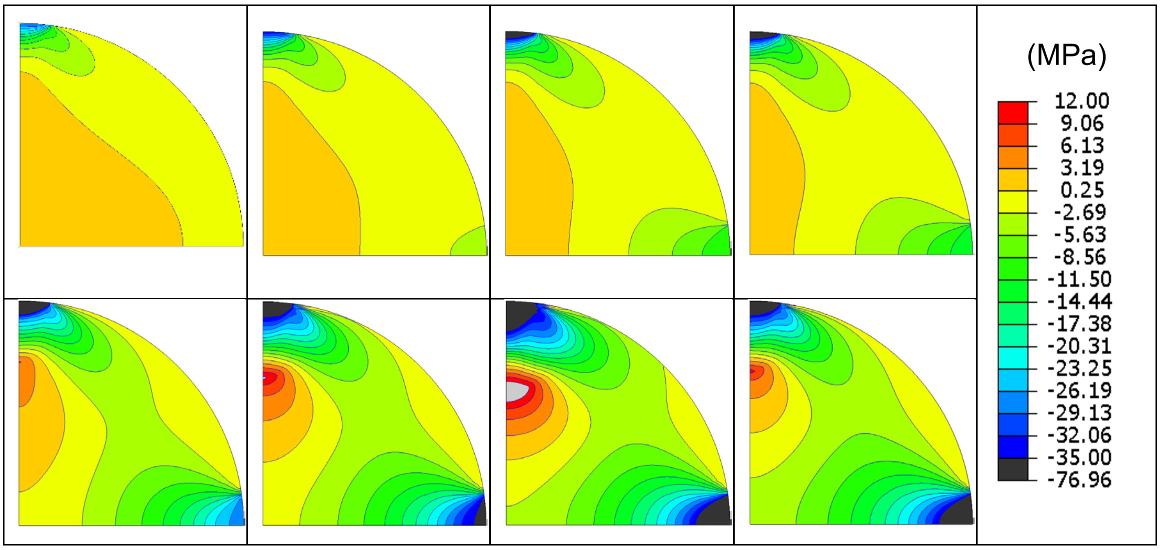


Figure 6. Stress in X-direction for k= 0, 10, 20, 25, 33, 40, 45, 50 % respectively.

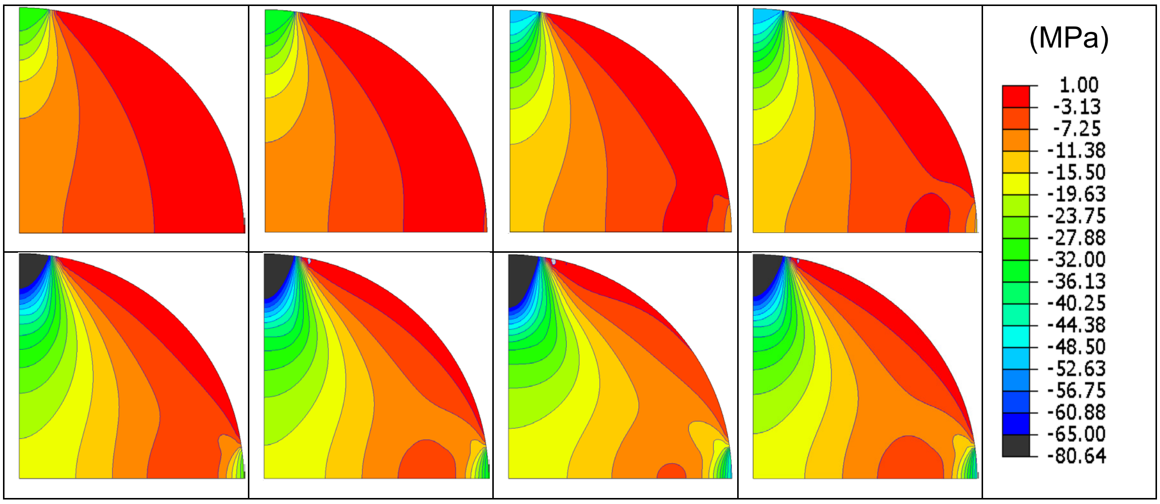


Figure 7. Stress in Y-direction for k= 0, 10, 20, 25, 33, 40, 45, 50 % respectively.

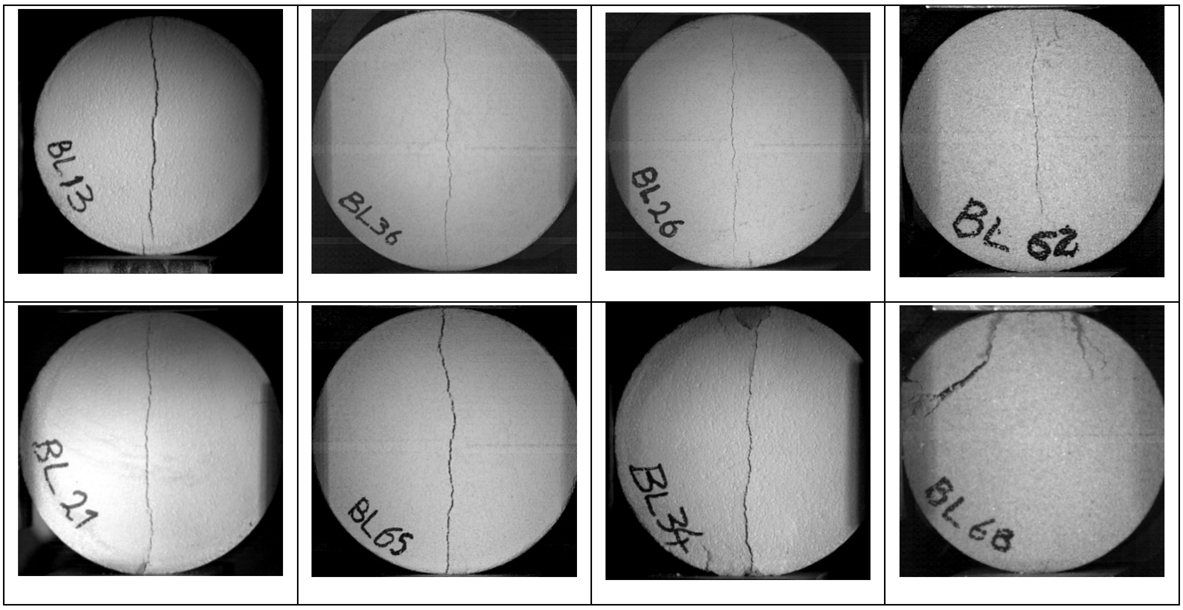


Figure 8. Failure cracks recorded by high-speed camera for k= 0, 10, 20, 25, 33, 40, 45, 50 % respectively.

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