Global Unstructured Digital Image Correlation for determining strains around circular opening

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ABSTRACT: Digital Image Correlation (DIC) is a non-contact displacement measurement method that uses image correlation algorithms to measure displacement. The algorithms used are often designed for structures having uniform geometry without cavities. However, in the rock engineering problems irregular geometry and cavities are common. Therefore, an unstructured finite element based DIC (FE-DIC) algorithm is developed, which can incorporate specimens with curved geometries, and cavities. The mathematical principles behind the algorithm are discussed briefly. The validation of the developed algorithm is conducted using a deformed image created using displacements based on the Kirsch solution. The algorithm is also employed to calculate displacement field and strain visualization around a circular cavity in a concrete specimen under compressive loading conditions. The results demonstrate the potential of the unstructured FE-DIC algorithm in providing insights into the material deformation and failure behavior in the specimens.

Keywords: DIC, Unstructured Mesh, Circular Cavity, Strain Visualization, Kirsch Solution.

1 INTRODUCTION

Studying the deformation behavior of rocks under various loading conditions is crucial in designing geotechnical structures such as tunnels, slopes, underground caverns, mines, powerhouses, and dam foundations. Traditionally, laboratory experiments were conducted using contact sensors such as strain gauges and extensometers. These sensors convert mechanical movement into electrical signals and calculates the displacement and strain. However, these sensors are point-based and provide data only at a few locations, making it difficult to obtain comprehensive results. Increasing the number of sensors can make experiments cumbersome to perform. To overcome this limitation, non-contact measurement methods like digital image correlation (DIC) have been developed which provides deformation information all over the test specimen (Pan 2018).

This paper discusses an unstructured finite element based DIC algorithm that is specifically developed for specimens with irregular geometry. The paper consists of six sections. Section one presents an introduction to the digital image correlation. Section two provides a brief overview of the math behind the finite element based DIC algorithm. Section three details the numerical experiment conducted to validate the algorithm using kirsch solution, and the results obtained. Section four and five describes the lab experiment performed to investigate the deformation behavior and failure mechanism of a concrete specimen with a circular hole under compressive loading. Finally, section six concludes the paper by summarizing the findings.

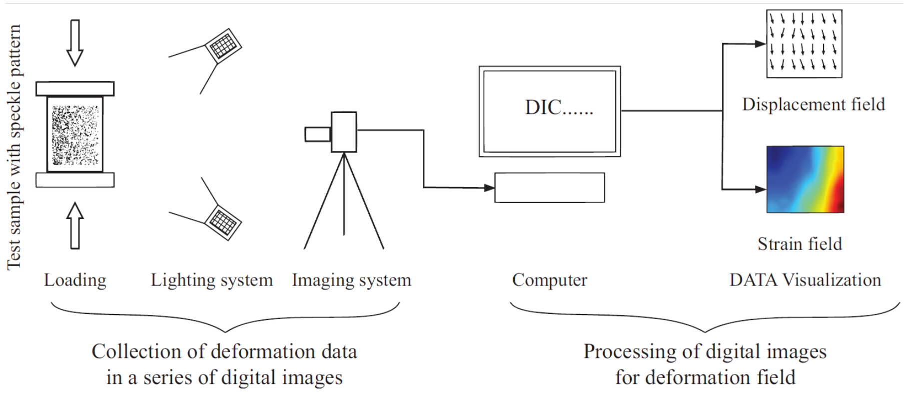

Figure 1. Digital Image Correlation (DIC) Method.

DIC is a photomechanical technique that uses correlation algorithms to measure displacements in images (Sutton et al. 2009). Data in the form of images are collected by image acquisition system (consisting of camera and illumination system) and is then analyzed using correlation algorithms to get the displacement field and strain field as shown in Figure 1 (Vemulapati and Deb 2021). Sample is tested with pattern consisting of randomly distributed circular speckles. Several correlation algorithms have been developed, which can be classified into two groups based on their mathematical approach. The first group is known subset based DIC, which employs simple image correlation techniques. The second group is finite element-based DIC (FE-DIC), which integrates the finite element framework into image correlation, enabling the use of finite element meshes for deformation analysis. The finite element-based DIC approach allows for the measurement of continuous displacement fields and it can be directly linked with numerical solutions. (Hild and Roux 2012).

The finite element based DIC approach typically uses a structured mesh to discretize the region of interest for analysis. However, this approach may encounter challenges when analyzing irregular geometries. To address this issue, a new unstructured FE-DIC algorithm has been developed, which can accomodate any 2D geometrical shape of interest without compromising the algorithm's performance.

2 FINITE ELEMENT BASED DIGITAL IMAGE CORRELATION

Let $\boldsymbol{x}$ is a cartesian coordinate vector, and $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ denote reference and deformed image pixels values at $\boldsymbol{x}$ respectively as shown in Figure 2. Considering a displacement vector $\boldsymbol{u}(\boldsymbol{x})$, based

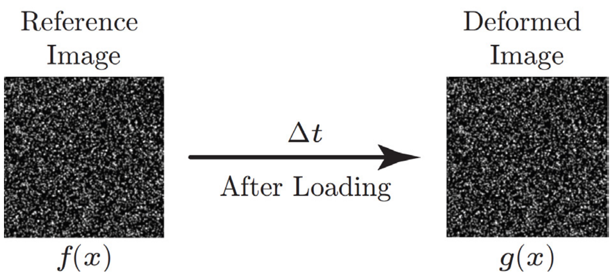

Figure 2. Deformation Process.

on the conservation of optical flow concept Equation 1 is valid. It is assumed that during this transformation lighting has not altered.

$$g(\boldsymbol{x}) = f(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) \tag{1}$$

Assuming $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are spatially differentiable, by applying first order Taylor expansion to Equation 1 results in

$$g(\boldsymbol{x}) \approx f(\boldsymbol{x}) + \boldsymbol{u}(\boldsymbol{x}).\nabla f(\boldsymbol{x}) \tag{2}$$

The total quadratic residual Ψ can be calculated by integrating the squared error in the entire domain Ω as given in Equation 3. This resembles like a least square finite element formulation.

$$\Psi = \int_{\Omega} (f(\boldsymbol{x}) + \boldsymbol{u}(\boldsymbol{x}).\nabla f(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x}))^2 dx \tag{3}$$

For an **n** noded 2D finite element, $\boldsymbol{u}(\boldsymbol{x})$ can be written in terms of nodal displacement vector $\boldsymbol{q}$, and shape functions matrix $\boldsymbol{N}(\boldsymbol{x})$, as

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{N^T q} \tag{4}$$

Replacing this relationship in Equation 3 and minimizing this with respect to $\boldsymbol{q}$, we get linear system of equations, represented in matrix from in Equation 3.

$$M_e u_e = b_e \tag{5}$$

The combination from each element is assembled into global linear system of equations. These equations are solved iteratively (Deb and Bhattacharjee 2015). Once the $\boldsymbol{u}(\boldsymbol{x})$ is known, strain tensors are estimated as:

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \tag{6}$$

3 NUMERICAL VALIDATION

In order to evaluate the performance of the unstructured DIC algorithm, a numerical experiment was conducted. For this purpose, an image dataset of two images (reference and deformed) were created with a hole in the center using the procedure given by (Pan et al 2006). The size of the image is 454 x 454 pixels, and the radius of the hole is 40 pixels as shown in the Figure. Kirsch solution displacement field is applied to the deformed image using the equations below (Deb 2010):

$$u_r = -\frac{P_o a^2}{4Gr}\left[(1+k) - (1-k)\left(4(1-v) - \frac{a^2}{r^2}\right) cos\ 2\theta\right] \tag{7}$$

$$u_\theta = -\frac{P_o a^2}{4Gr}\left[(1-k)\left(2(1-2v) + \frac{a^2}{r^2}\right) sin\ 2\theta\right] \tag{8}$$

where

a (Radius) = 40 pixels
k (Stress ratio) = 1
v (Poisson's ratio) = 0.25
P_o (Pressure) = 5×10^6 N/pixels^2
G (Shear modulus) = 5×10^9 N/pixels^2

A mesh consists of 260 nodes and 225 quadrilateral elements as shown in Figure 3 was used for processing the images with DIC algorithm, and the resulting data is presented in Figure 4. The first and second rows represent the displacements in x and y directions respectively. The first column presents the kirsch solution displacement, while the second column shows the displacement data calculated by the algorithm. The third column represents the plots of the displacement along the line-cuts along x-axis and y-axis in the first and second row, respectively. The root mean square error (RMSE) values for displacements in x and y directions are 0.0055 pixels and 0.0058 pixels respectively. Low RMSE value indicates that unstructured finite element based DIC algorithm can accurately capture the non-linear displacement fields.

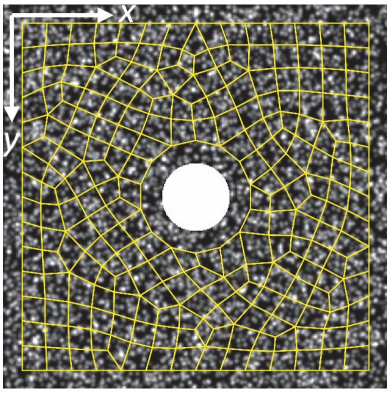

Figure 3. Unstructured mesh on digitally generated image.

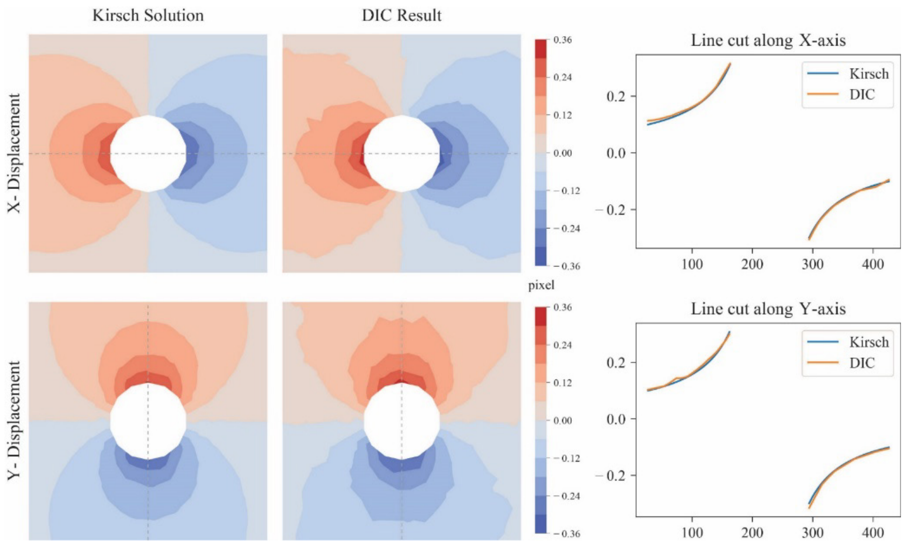

Figure 4. Numerical validation results.

4 LAB EXPERIMENT

Upon successfully validated with numerical results, a laboratory experiment was conducted to investigate the effect of a circular hole on the deformation of a 150 mm M30 grade concrete cube using the unstructured DIC algorithm. The concrete cube was prepared for the experiment and a 32 mm diameter hole was drilled at the center as shown in the Figure 5.a. A speckle sticker developed at IIT Kharagpur was pasted on one of the surface of the sample (Deb 2014). Compressive load was applied using a universal testing machine with a loading rate of 0.45 mm/min. Images of the sample were taken at regular intervals of 5 seconds using a Nikon D3400 camera as shown in Figure 5.a. Gmsh mesh-making tool (Geuzaine and Remacle 2009) was used to generate a unstructured mesh with 997 nodes and 917 quad elements, shown in Figure 5.b. From the load-deformation data, stress-strain plot is made and shown in Figure 5.c. Three locations on the stress-strain graph are annotated with A, B, and C. The DIC results for these three locations are discussed in the next section.

5 RESULTS AND DISCUSSION

Figure 6 presents displacement vectors and strain (ϵ_{xx} , ϵ_{yy}) when sample was at locations A, B, and C on the stress-strain graph. The first row of the figure presents the image of the sample taken during

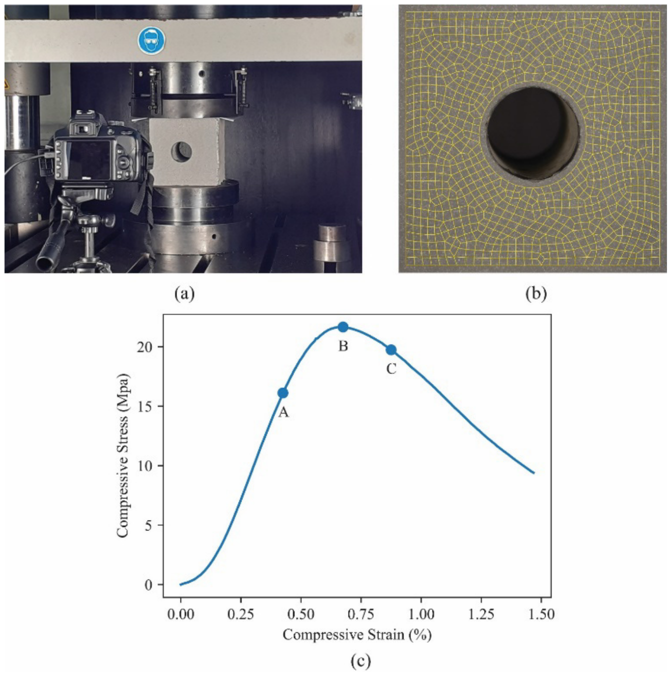

Figure 5. a) Experimental setup b) Unstructured mesh superimposed on the sample surface c) Stress-strain plot of the experiment.

the experiment, providing a visual reference of the sample. The second row shows the displacement quiver diagram of the sample, which depicts the direction and magnitude of the surface displacement vectors. The magnitude of the displacement increased by 173 % and 336 % as the strain jumps from 0.42 % to 0.67 % and 0.87 % respectively. This information can help to understand the surface motion of the sample during the loading. The third and fourth rows of the figure present the lateral and longitudinal strain of the sample. These strain maps provide information on the locations of the high strain on the surface of the sample, which can help to identify areas of potential failure. Overall, the DIC results presented provide rich insights into the material's failure behavior and deformation characteristics. By analyzing this data, engineers and researchers can gain a better understanding the effect of the circular opening on the mechanical behavior of the material, which can inform the design of new materials or improve the performance of existing ones.

6 CONCLUSION

An unstructured finite element based DIC algorithm is developed for finding full field displacement and strain fields. This algorithm can find deformation behavior of complex geometries like circular hole or any irregular shape object under load. Compared to a contact based strain measurement like strain gauge, a DIC experiment yields full field strain in a non-contact manner that can be used to predict the crack well in advance before it appears visually. The obtained displacement fields in the experiment are well in accordance with the fundamental theories of rock mechanics which backs the results of DIC experiment. Hence, DIC has broad use in structural health monitoring of civil, mining, aerospace, and infrastructure designing. The goal of future research is to deploy DIC in the field to track field structure deformation.

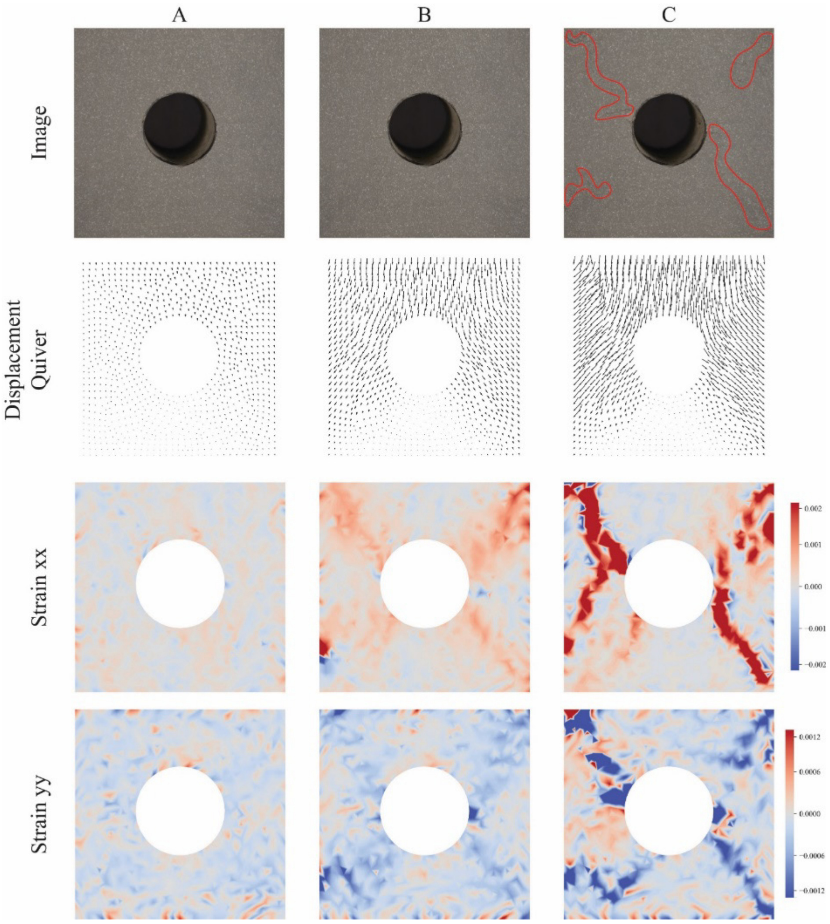

Figure 6. FE-DIC results of the experiment.

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