

Application of a new brittleness index to estimate the proneness to brittle failure of rock around a deep tunnel

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ABSTRACT: A major problem that may arise during the excavation of deep tunnels is the incidence of brittle failure of rock, induced by the stress release in a particularly heavy natural state of stress. The severe energy release is often associated to rapid fracturing and to projection of rock blocks inside the opening, phenomena commonly referred to as spalling or rockburst, which endanger personnel and equipment. Thus, the proper prediction of the occurrence of brittle failure is paramount in underground excavations. This paper presents the application of a new rock brittleness index based on the response of two mechanical models of rock damage, which allows the estimation of the proneness to brittle failure of rock around deep tunnels. For this purpose, the competition between ductile and brittle failure is analyzed. The calculation and usage of the index is described considering a real case study of brittle failure in a deep tunnel.

Keywords: Brittleness index, Rockburst, Spalling, Tunneling.

1 INTRODUCTION

One of the main criticalities while excavating deep tunnels in rock masses subjected to heavy natural stress states is brittle failure of the rock mass. This kind of failure usually appears as a rapid fracturing of the material, in combination with massive releases of the energy stored during the deformation due to the excavation, and the dangerous projection of rock blocks inside the opening (Cai, 2013; Diederichs, 2007; Gong et al., 2020). In less violent cases, slabbing and spalling phenomena occur, while more violent cases are usually referred to as rockbursts.

The projection of rock blocks inside the opening is usually characterized by velocities up to 6 m/s, while small fragments can reach velocities of about 50 m/s (He et al., 2022). Hence, these sudden failure phenomena can induce delays, economic losses, collapses, damage to equipments and, sometimes, casualties (Chen et al., 2021; Mazaira & Konicek, 2015).

The proper prediction of rock brittle failure is therefore crucial in the design of underground excavations. For this purpose, different empirical brittleness indexes have been proposed in the past. However, many of them proved often to provide predictions unreliable and/or inconsistent with each

other (He et al., 2022). The main reason of this limitation is that they do not consider the actual stress and energy variations induced in the rock mass by the excavation.

Trying to overcome these limitations and to help in predicting rockburst phenomena in the early stages of the design of deep tunnels, we propose a new rock brittleness index (Tunnel Brittleness Index, TBI) aimed at the estimation of the proneness to brittle failure of the rock around deep tunnels. The detailed description of the definition of TBI and the calculations involved are the subject of a research paper currently under review (Milan et al., S.d.).

In summary, TBI is calculated analytically comparing the predictions of two different analytical mechanical models of rock damage, associated to ductile and brittle failure, respectively. These models have been specially developed and are both driven by the stress release resulting from the excavation.

The purpose of this work is to show the TBI calculation procedure for a case study of brittle failure of a tunnel reported in the literature.

2 THE TUNNEL BRITTLINESS INDEX

The TBI is constructed as a non-dimensional index describing the competition between ductile and brittle failure around deep tunnels. The definition of TBI is based on the usual approach of simulating the excavation as a process of progressive stress variation around the tunnel, by means of the reduction of a fictitious distribution of tractions on the tunnel walls, within the convergence-confinement method (Alejano et al., 2009; Carranza-Torres & Fairhurst, 2000; Vlachopoulos & Diederichs, 2009).

The problem is schematized as shown in Figure 1: plane strain conditions are assumed, and fictitious internal surface forces, proportional to the original stress state, are applied to the tunnel boundary, to simulate the progressive advancement of the tunnel face and the corresponding reduction of its confinement effect. This proportionality is expressed by an unloading parameter, Λ , varying from 0 (no excavation) to 1 (complete excavation). Unlike in the case of most analytical solutions available in the literature for the convergence-confinement method, in this case an anisotropic geostatic stress state is considered. Due to this, the distribution of the fictitious internal tractions is neither uniform nor radial. Then, the tangential components at the inner boundary of the tunnel, p_t , have also been taken into account. The normal (p_n) and tangential (p_t) components of the fictitious tractions as a function of the angle ϑ are given by Eq. 1, where $\sigma_{x\infty}$ and $\sigma_{y\infty}$ are the horizontal and vertical components of the original state of stress.

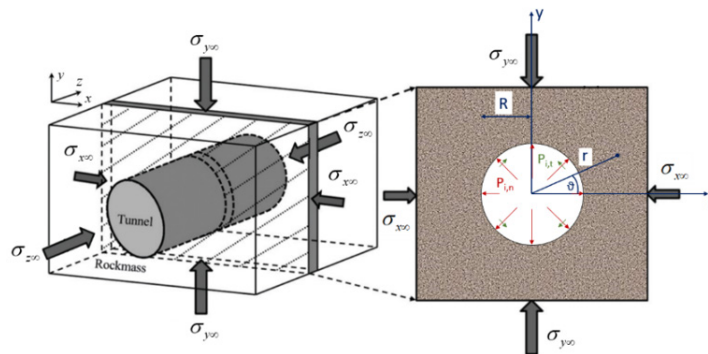


Figure 1. Schematic of the geometry and stress state of a deep tunnel (modified from Qiu et al., 2014).

$$p_n = (1 - \Lambda) \left(\frac{\sigma_{x\infty} + \sigma_{y\infty}}{2} + \frac{\sigma_{x\infty} - \sigma_{y\infty}}{2} \cos(2\vartheta) \right) \quad (1)$$

$$p_t = (1 - \Lambda) \left(\frac{\sigma_{y\infty} - \sigma_{x\infty}}{2} \sin(2\vartheta) \right)$$

Considering the analytical models associated with ductile and brittle failure, two critical values of Λ are computed, inducing one of the two failure modes each. These are called Λ_{duc} (ductile failure) and Λ_{brt} (brittle failure). Specifically:

- Λ_{duc} is calculated as the Λ value which first induces failure in one point of the rock mass at the tunnel boundary, considering an elasto-plastic model of the rock mass and the Drucker-Prager failure criterion in the form $\sqrt{J_2} = A + BI_1$, where J_2 is the second invariant of the deviatoric part of the Cauchy stress tensor, I_1 is the first invariant of the Cauchy stress tensor, and A and B are strength parameters related to the uniaxial compressive and tensile strength of the rock mass (UCS and UTS, respectively) as shown in Eq. 3

$$A = \frac{2}{\sqrt{3}} \left(\frac{\text{UCS} \cdot \text{UTS}}{\text{UCS} + \text{UTS}} \right) ; B = \frac{1}{\sqrt{3}} \left(\frac{\text{UCS} - \text{UTS}}{\text{UCS} + \text{UTS}} \right) \quad (3)$$

- Λ_{brt} is calculated as the Λ value which induces failure around the tunnel, according to a model derived from the one proposed by Bažant et al. (1993), in which regions of splitting cracks form at the sides of the cavity and create rock slabs that are assumed to buckle as slender columns.

TBI measures the proneness to ductile or brittle failure of the rock mass around the tunnel, by analyzing the competition of the two failure mechanisms and determining the prevalence of one over the other. More in detail, TBI is defined as:

$$\text{TBI} = \Lambda_{\text{duc}} - \Lambda_{\text{brt}} \quad (2)$$

TBI can assume values between -1 and 1 , and its absolute value indicates the degree of prevalence of one of the failure modes over the other, during the excavation process:

- If $\text{TBI}=0$, $\Lambda_{\text{duc}} = \Lambda_{\text{brt}}$, so failure occurs but it is not possible to predict its kind;
- If $\text{TBI}>0$, $\Lambda_{\text{duc}} > \Lambda_{\text{brt}}$, so brittle failure occurs before ductile failure;
- If $\text{TBI}<0$, $\Lambda_{\text{duc}} < \Lambda_{\text{brt}}$, so ductile failure occurs before brittle failure.

Λ_{duc} and Λ_{brt} are computed using the expressions reported in Milan et al. (S.d.) and depend on 11 input parameters:

1. The stress ratio coefficient, $k = \frac{\sigma_{x\infty}}{\sigma_{y\infty}} = \frac{\sigma_{z\infty}}{\sigma_{y\infty}}$;
2. The tunnel radius, R ;
3. The Poisson's coefficient of the rock mass, ν ;
4. The Young's modulus of the rock mass in plane strain conditions, E' ;
5. The shear modulus of the intact rock, G_i ;
6. The fracture energy of the intact rock, G_f ;
7. The maximum in situ principal stress, $\sigma_{y\infty}$ (TBI was determined assuming that the maximum in situ principal stress is vertical, but a generic orientation of the geostatic stresses can be considered by performing a rotation of the reference system);
8. The parameter A of the Drucker-Prager failure criterion of the rock mass;
9. The parameter B of the Drucker-Prager failure criterion of the rock mass;
10. An equivalent thickness of the material, λ , described as the ratio between G_i and the shear stiffness of tensile rock fractures, k_s ;
11. An empirical relative wavelength, w , which is a constant of proportionality between the half-length of spalling fractures and the tunnel radius.

In order to minimize the time needed to determine TBI, the authors developed a code for its automated calculation. This code can also manage some critical situations associated with particular Λ_{duc} and Λ_{brt} values, which are treated as follows:

- If A_{duc} is negative, greater than 1 or imaginary (hereafter referred to as "inadmissible"), while A_{brt} is a real value between 0 and 1 (hereafter referred to as "admissible"), the value $TBI=1$ is forced, corresponding to the maximum prevalence of the brittle failure mechanism over the ductile one;
- If A_{brt} is inadmissible, while A is admissible, $TBI=-1$ is forced, corresponding to the maximum prevalence of the ductile failure mechanism on the brittle one;
- If both A_{duc} and A_{brt} are inadmissible, $TBI=NaN$ (Not a Number) is forced. This means that failure is not predicted by either of the two damage models used, so the TBI is not applicable.

3 EXAMPLE OF APPLICATION OF TBI

As a part of the research described in Milan et al. (S.d.), the predictive efficacy and reliability of TBI have been tested on 21 case studies of brittle and ductile failure in deep tunnels, collected in the literature. In order to show the step-by-step procedure for calculating TBI, one of these case studies is here considered. Specifically, the spalling failure of the unsupported vertical raise of the Garpenberg mine in Sweden is examined, which is well documented by Edelbro (2006, 2009).

The tunnel under examination is a vertical raise with a diameter of 2.13 m, excavated at a depth of 830-880 m in dolomite limestone. The assumed values for the 11 input parameters are collected in Table 1. The table also reports suggestions for the definition of the design value of each parameter, in the case of an application of TBI to a tunnel still to be excavated, for forecasting purposes.

Using the information shown in Table 1 and the formulas developed by Milan et al. (S.d.), the values $A_{duc} = 1.24$ and $A_{brt} = 0.48$ are obtained. In this case, A_{duc} assumes a value with no physical meaning (inadmissible), as it is greater than 1, while A_{brt} assumes an admissible value. According to the computation algorithm described before, $TBI=1$, which corresponds to the maximum prevalence of the brittle failure mechanism over the ductile one. This result is consistent with the observed failure mode and indicates that brittle failure is likely to occur during the excavation of the tunnel, while ductile failure is unlikely. This information could be useful in the early stages of the design to choose the excavation techniques and methods, as well as the support systems.

It is important to notice that the value $A_{duc} = 1.24$ should not be interpreted as the certainty that ductile failure will not occur around the excavation. In fact, it is possible that this result was caused by an inaccurate choice of the input parameters, or that the analytical model of ductile failure is not sophisticated enough to accurately reproduce the failure phenomenon. Specifically, the model is based on simplifications, such as the assumption of a continuous, homogeneous and isotropic medium. Hence, failure phenomena associated with particular geo-structural conditions of the rock mass could occur. Nevertheless, it is possible to deduce that brittle failure is predominant and is expected more likely to occur before ductile failure.

The validation of the proposed method is ongoing with promising results, and will be presented in a forthcoming paper (Milan et al., S.d.).

4 DISCUSSIONS AND CONCLUSIONS

This paper proposes the application of a new, analytical physically-based brittleness index to assess the proneness of brittle failure of the rock mass around deep tunnels (TBI). This is based on considerations about stress and energy variations occurring in the rock mass at failure. The predictive reliability of TBI is found, at the moment, to be comparable with that of other empirical approaches in literature. However, it is necessary to enlarge the dataset of case studies for the TBI validation, in order to obtain more representative results. This is the subject of an ongoing research.

The efficacy of TBI depends, of course, on the quality of the available information on the parameters involved. In particular, the empirical parameters w and λ are hard to be properly determined. Regarding the last one, which depends on the shear stiffness of forming fractures in brittle failure mode, an experimental study is currently being carried out by the authors. The results will allow to improve the efficacy of TBI assessment, providing the tunneling community with a tool

to be adopted for provisional purposes in the design of deep tunnels, when spalling and/or rockburst phenomena are feared.

Table 1. Values of the input parameters used for the application of TBI to the case study of the vertical raise of the Garpenberg mine in Sweden. “IFL” stands for “information found in the literature”; in this case, the information comes from Edelbro (2006, 2009).

Parameter	Value	Source	Suggestions for the value determination
$\sigma_{y\infty}$	44 MPa	IFL	$\sigma_{y\infty}$ and k can be obtained from in situ tests, like the flat jack test.
k	0.55	IFL	
ν	0.17	IFL	
E'	56.6 GPa	IFL	ν can be assumed equal to the one of the intact rock, ν_i , which can be obtained from unconfined compression tests on samples retrieved on site. E' is defined as $E' = \frac{E}{1-\nu^2}$, being E the Young's Modulus of the rock mass. E can be derived from the value related to the intact rock, E_i which can be obtained from unconfined compression tests on rock samples retrieved on site. The derivation can be performed using one of the formulas proposed by different Authors, based on the rock mass quality parameters (e.g., GSI).
A	1.6 MPa	Calculated	A and B were computed using their relationship with the rock mass uniaxial compressive and tensile strengths (Eq. 3). Such values were obtained deriving the Hoek & Brown failure criterion of the rock mass, from the unconfined compressive strength of the rock $\sigma_{ci} = 105$ MPa and the material constant $m_i = 15$. σ_{ci} and m_i can be obtained from triaxial compression tests on rock samples retrieved on site.
B	0.55	Calculated	
G_i	26.5 GPa	Calculated	G_i is defined as $G_i = \frac{E_i}{2(1+\nu_i)}$.
G_f	$6.6 \cdot 10^{-6}$ MPa·m	Calculated	G_f is defined as $G_f = \frac{K_{Ic}^2(1-\nu^2)}{E_i}$. K_{Ic} (fracture toughness of the rock) can be computed as $K_{Ic} = 0.1453 \cdot UTS_i$, being UTS_i the uniaxial tensile strength of the rock (Zhang, 2002).
λ	0.64	IFL	If no information is available regarding k_s , the expression $\lambda = \frac{G_i [\text{MPa}]}{112.5 \text{ N/mm}^3}$ can be used (Bažant et al., 1993).
w	0.23	IFL	If no information regarding w is available, the value 0.25 can be assumed (Bažant et al., 1993).

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