Shear wave propagation across jointed rocks of varying seismic impedance

Kallol Saha
Indian Institute of Technology, Ropar, Punjab, India

Resmi Sebastian
Indian Institute of Technology, Ropar, Punjab, India

ABSTRACT: Shear waves are considered as one of the damaging waves for any structure built in the rocks during earthquakes, mining, blasting etc. This paper describes the numerical simulation of a test facility that generates shear waves in rocks. The test facility comprises of a dynamic impact mechanism, friction bar, incident, and transmitted plates. When the friction bar is given dynamic impact, shear wave gets generated in the perpendicular direction in the incident plate due to friction present in the interface between friction bar and incident plate. Seismic impedance of the media plays an important role during wave propagation across the interface. The effect of change in material density across the frictional joints was monitored during laboratory testing. These tests were numerically simulated using a distinct element code. Validation of these numerical simulations have been done by monitoring the vibration amplitudes developed at various locations of the plates in the laboratory.

Keywords: Shear wave propagation, frictional jointed rocks, seismic impedance, energy coefficients, Coulomb slip joint model, continuously yielding joint model.

1. INTRODUCTION

Naturally, rock mass has multiple discontinuities, such as faults, joints, and fissures. Jaramillo (2017) addressed the need for stability analyses of subsurface structures built in rocks subjected to dynamic loads. Several analytical studies, including Pyrak-Nolte and Cook (1987) applied Displacement Discontinuity Method (DDM) to analyse the wave propagation across discontinuous media. The joints present in the rocks were proven to have a considerable impact on wave propagation. For the experimental researches on wave propagation across rock joints, wave velocities and wave attenuation are measured at various strain rates. The split Hopkinson pressure bar (SHPB) designed by Kolsky (1963) is one of the most widely used loading systems at high strain rate (10^1-10^4 sec^{-1}). Several researchers (Li and Ma 2009, Wu et al. 2013) employed the SHPB test to evaluate the dynamic response of rocks at high strains during compression wave propagation. Liu et al. (2017) designed the split shear plate (SSP) facility using the direct shear model (Wu and Zhao 2014) and SHPB theory for analyzing shear wave propagation. The study on shear wave propagation across rocks of various seismic impedances using laboratory experiments and numerical simulations is presented in this paper. Seismic impedance (defined as product of density and velocity of wave propagation in that medium) is considered as an important parameter in the propagation of waves across rock joints.

In the present study, laboratory experiments were conducted in SSP test facility (Liu et al. 2017) on synthetic soft rock samples having different materials and densities across the joints and the influence of seismic impedance ratio on shear wave propagation was analyzed. Numerical simulations were conducted using commercially available software, 3-dimensional Distinct Element Code (3DEC). Two joint models Coulomb slip (CS) joint model and the continuously yielding (CY) joint model were used for simulating the rock joint behavior. Usually, CS joint model is used to simulate joint behavior due to its simplicity. CY joint model provides more realistic perspective of joint deformation, which can be more useful in obtaining joint behavior during shear wave propagation. Analysis of joint behavior using CY model have been conducted by researchers (Cundall 1990, Gu 2013, Cui et al. 2017), however, comparison between these two joint models in a similar problem statement can provide better understanding about shear wave propagation. Results from the numerical simulations using these two joint models have been compared and have been presented here.

2. MATERIALS AND METHODOLOGY

2.1 Laboratory setup of Split Shear Plate (SSP)

Split Shear Plate (SSP) set up in the laboratory consisted of a friction bar, incident and transmitted plates, along with a supporting block (as shown in Figure 1 (a)). The friction bar and supporting block (made up of mild steel) had dimensions of 600 mm*100 mm*100 mm. Incident and transmitted plates were of the dimensions of 670 mm*300 mm*30 mm and 630 mm*300 mm*30 mm respectively. For conducting the study on influence of seismic impedance of the rocks, the properties of the plates were varied and were made using three different materials (properties are shown in Table 1). The friction bar and the supporting block exerted a uniform pressure on both the plates from the sides (see Figure 1 (b)). The compression wave in the friction bar was generated by a dynamic impact mechanism that applied a force in the +Y axis. Four supporting bars (made of mild steel) were kept at uniform spacing under the plates. The contact surface among these supporting bars and the synthetic rock plates were assumed to be frictionless. The side of the friction bar, which was in contact with the incident plate, had grooves on it to generate friction, such that when the friction bar slid in +Y direction with respect to the incident plate, shear wave generated in the incident plate.

Table 1. Engineering properties of plates made from three different materials.

Properties		Beta (β) Hemi-hydrate	OPC 43 Cement plaster	Alpha (α) Hemi-hydrate
Material Number	[-]	M1	M2	M3
Water: powder, (mass: mass)	[-]	1.00:1.80	1.00:2.50	1.00:4.00
Density	[kg/m^3]	1150	1400	1700
Uniaxial Compressive Strength	[MPa]	8	25	50
P wave velocity (V_P)	[m/s]	2250	2750	4000
S wave velocity (V_S)	[m/s]	1200	1800	2750
Seismic impedance	[kg/m^{2}s]	1380000	2800000	5100000
Shear modulus	[GPa]	1.66	4.54	12.86
Young's modulus	[GPa]	4.31	10.21	27.04

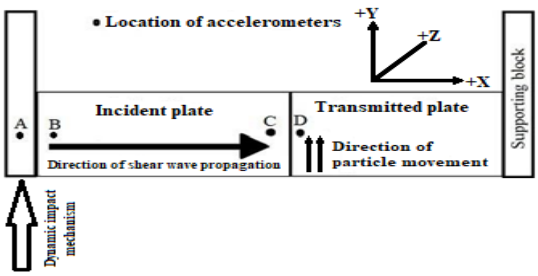

Figure 1. (a) Schematic diagram of the SSP (Liu et al. 2017).

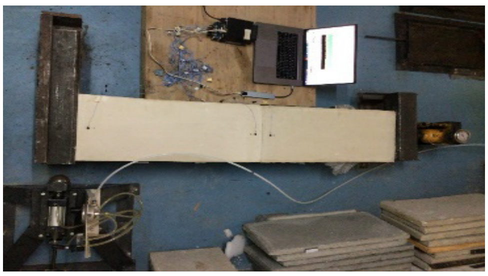

Figure 1. (b) Laboratory SSP test set up for SSP.

Four piezo-electric accelerometers were attached at four locations to record the vibrations, in the form of acceleration-time (a-t) history. These accelerometers were connected to a DAQ (Data Acquisition) system which was directly connected to a personal computer, where the data could be recorded.

2.2 Materials used in laboratory experiments

Three different materials (α hemi-hydrate, OPC 43 cement and β hemi-hydrate) were used to make synthetic rock samples. The samples were prepared using specific water to gypsum powder ratios. The mixture was kept in a mild steel mould (having dimensions of 1300 mm*300 mm*30 mm). A straight edge partitioner of 300 mm length was kept in between to keep the incident and transmitted plates separated. The plates were taken out of the mould after becoming solid and were air dried for 10 to15 days till they attained constant weight.

2.3 Laboratory experiments using Split Shear Plates (SSP) test facility

In the present study, the influence of seismic impedance on shear wave propagation was determined by comparing the transmission amplitude and energy coefficients of the propagating wave for different synthetic rock plates placed on both sides of the joint. The piezoelectric accelerometers were installed at four locations, A, B, C and D (see Figure 1 (b)). Point A was located on the upper surface of the friction bar. Points B and C were located 30 mm and 640 mm away from the beginning of the incident plate, respectively. Point D was located 30 mm from the beginning end of the transmitted plate. The a-t history obtained at these locations were used to determine the particle velocity-time (v-t) history and displacement-time (d-t) history in the given direction. Using equations given by Miller (1977,1978), the energy flux associated with the propagating waves (via both experimental and numerical simulation) were calculated as:

$$E_{I/R/T} = \rho . V_{Shear} \cdot \sum_{t}^{T+t} V_{Particle}^{2} \quad (1)$$

E_I, E_R and E_T indicate the energy flux per unit area per cycle of oscillation for the incident, reflected, and transmitted waves respectively; V_{Shear} indicates the shear wave velocity recorded at the plates; $V_{Particle}$ indicates particle velocity at specific location on the plates; ρ is density of the material. Coefficients of transmission (T_E), reflection (R_E) and absorption (A_E) were defined as,

$$T_E = \left(\frac{E_T}{E_I}\right)^{\frac{1}{2}} . \left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{2}}; \qquad R_E = \left(\frac{E_R}{E_I}\right)^{\frac{1}{2}} \qquad A_E = \left(1 - \left(\frac{\gamma_1}{\gamma_2}\right) T_E^2 - R_E^2\right)^{\frac{1}{2}} \quad (2)$$

γ_1 and γ_2 represent seismic impedance, $\gamma = (\rho G)^{\frac{1}{2}}$ of medium 1 (incident plate) and medium 2 (transmitted plate) respectively; G is the shear modulus of the material. For conducting this study, various combinations of incident and transmitted plates were tested, as provided in table 2. T_V and T_D were calculated as per equation 3 and 4 respectively:

$$T_V = \frac{\text{Maximum particle velocity measured in transmitted plate}}{\text{Maximum particle velocity measured in incident plate}} \quad (3)$$

$$T_D = \frac{\text{Maximum particle displacement measured in transmitted plate}}{\text{Maximum particle displacement measured in incident plate}} \quad (4)$$

Table 2. Details of material combinations used as incident and transmitted plate.

Case A			Case B		
Incident Plate	Transmitted plate	Seismic impedance ratio	Incident Plate	Transmitted plate	Seismic impedance ratio
M1	M3	0.295	M2	M1	1.826
M2	M3	0.539	M3	M2	1.856
M1	M2	0.547	M3	M1	3.388

2.4 *Numerical simulation using 3-dimensional Distinct Element Code (3DEC)*

Numerical modeling offers a cost-effective method to conduct research on wave propagation across rock joints when compared to theoretical and laboratory studies. In the present study, 3DEC (3-dimensional distinct element code) has been used for performing the numerical simulations. Many researchers have used 3DEC for simulating wave propagation across jointed rocks (Perino 2011, Sebastian and Sitharam 2014).

Joint constitutive models in 3DEC define the normal and shear interaction between the blocks at their contact points. For conducting the numerical analyses of the wave propagation, two joint models have been employed, namely Coulomb-slip (CS) joint model and continuously yielding (CY) joint model. The CS model works on the principle of Coulomb friction law which requires the parameters of joint stiffness (normal and shear), angle of friction and dilation, cohesion and tensile strength of the joint, for describing the joint behavior. The displacement-weakening model, CY joint model simulates the progressive damage mechanism of joint under shear and requires joint roughness parameter, joint normal stiffness exponent and joint shear stiffness exponent in addition to the parameters used in CS joint model.

Initially, the whole SSP facility was modeled in 3DEC software (Figure 2). Joints were made between friction bar and incident plate (joint 1), transmitted plate and supporting block (joint 2) and incident and transmitted plates (joint 3). High stiffness values were adopted for joints 1 and 2 to simulate the high friction existed between the bars (friction bar and supporting bar) and the plates (incident and transmitted plates). The joint normal stiffness (jkn) and joint shear stiffness (jks) of joint 1 and 2 were taken as 200 GPa/m and 50 GPa/m respectively. Joint 3 refers to the intersection between incident and transmitted plates. For both CS and CY joint models, jkn and jks of joint 3 were considered as 5 GPa/m and 1 GPa/m respectively for case of least seismic impedance ratio of 0.295 (Table 2). To match numerical simulation results with that of experiments, values of jks were obtained using trial and error method. For impedance ratios of 0.295, 0.539, 0.547, 1.826, 1.856, 3.388, the joint shear stiffness was taken as 1.00 GPa/m, 0.95 GPa/m, 0.90 GPa/m, 0.85 GPa/m, 0.80 GPa/m and 0.75 GPa/m respectively for both joint models. Joint friction angle (jfric) and insitu normal stress (in the X direction) were kept as 25° and 0.50 MPa respectively. The roughness parameter for CY joint model was taken as 0.1 mm for all the cases. The blocks were made as deformable and the average edge length of tetrahedral element was checked against the condition given by Deng et al. (2012). Particle velocities at X and Z directions were restricted for the plates, for simulating the shear wave propagation. The load was applied in the form of a v-t history obtained from the a-t history. A parameter 'velocity coefficient' of value (-)0.005 was multiplied with the velocity values loaded from the velocity–time data to simulate the same particle velocities obtained in the friction bar for laboratory experiments.

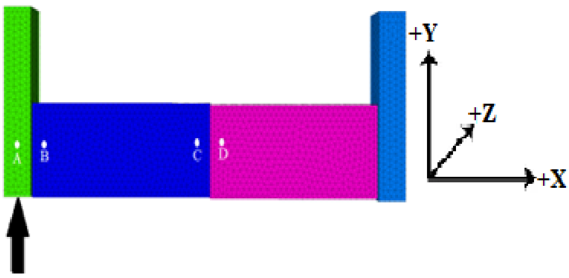

Figure 2. Numerical simulation of SSP.

3. RESULTS AND DISCUSSIONS

3.1 *Transmission coefficient*

The comparison of T_D and T_V with impedance ratio is shown in figure 3 (a) and 3 (b). With increase in impedance ratio, the T_V and T_D were found to be increasing for both laboratory tests and numerical simulations. The T_D results obtained from the numerical simulations agreed with results obtained from laboratory experiments, with variation of ±10%. The comparison of T_E obtained from both laboratory tests and numerical simulations is shown in figure 3 (c) and 3 (d) for seismic impedance ratio < 1 and > 1. T_E was found to be increasing for both laboratory tests and numerical simulations. For cases having seismic impedance ratio < 1, CS model predicted the laboratory results more precisely than the CY model. For cases of seismic impedance ratio > 1, numerical simulations results followed the same trend of T_E that was observed for the laboratory experiments. The trend observed for the two cases was different. The reason can be stated as, when a wave travels from a medium with a lower impedance to a medium with a higher impedance (case A), the transmission coefficient was lower, as the wave encounters more resistance at the interface. Conversely, when a seismic wave travels from a medium with a higher impedance to a medium with a lower impedance (case B), the transmission coefficient was found to be higher.

3.2 Reflection and absorption coefficients

Reflection and absorption coefficient (R_E and A_E) were determined using equation 2 (Figure 3(e) and 3 (f)). For laboratory experiments, with increase in impedance ratio, R_E is observed to have increasing. For both CS and CY models, R_E was almost same value of 0.67. When a seismic wave travels from a lower impedance medium to a higher impedance medium, the reflection coefficient was found to be higher. The greater impedance contrast leads to a greater mismatch in wave velocities at the interface, resulting in a stronger reflection. Similarly, when seismic wave transmits from a higher seismic impedance medium to a lower seismic impedance medium, the R_E will generally be lower, as a greater proportion of the wave energy is transmitted into the lower impedance medium, leaving less energy available for reflection back into the higher impedance medium.

For both CS and CY models, A_E varied from 0.73 to 0.745. The A_E results obtained from the numerical simulations agreed with results obtained from laboratory experiments, with variation of ±10%. As more reflections occured with increasing seismic impedance ratio, the lesser absorption occurred with increasing seismic impedance ratio.

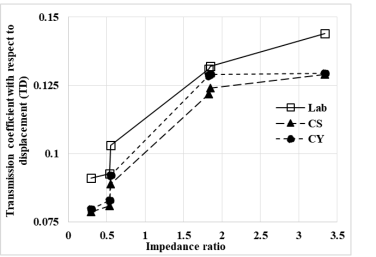

Figure 3 (a). T_D for samples having different seismic impedance ratios.

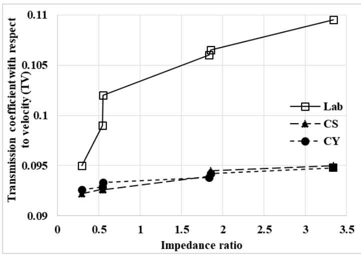

Figure 3 (b). T_V for samples having different seismic impedance ratios.

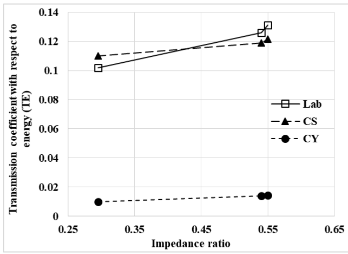

Figure 3 (c). T_E for samples having seismic impedance ratios < 1.

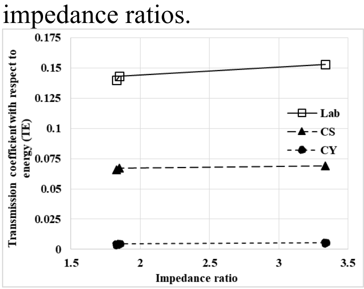

Figure 3 (d). T_E for samples having seismic impedance ratio > 1.

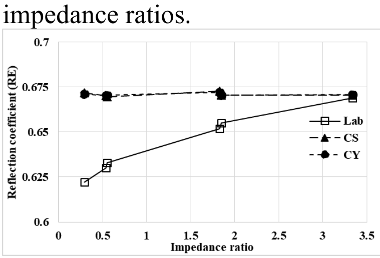

Figure 3 (e). R_E for samples having different seismic impedance ratios.

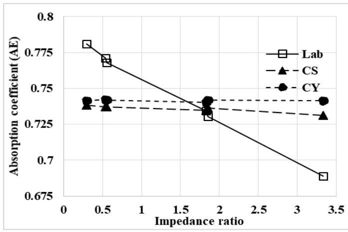

Figure 3 (f). A_E for samples having different seismic impedance ratios.

4. CONCLUSIONS

Laboratory experiments were conducted in SSP test facility to study the influence of seismic impedance on propagation of shear wave. Numerical simulations were conducted using two joint models and these results were validated with the experimental results. The conclusions obtained from the study are:

- With increase in impedance ratio, the value of transmission coefficient with respect to velocity (T_V) and displacement (T_D) increased. Coulomb slip (CS) and continuously yielding (CY) models were able to predict T_D more accurately than T_V. CS and CY model predicted almost the same T_D and T_V values for specific impedance ratios. Both the T_V, T_D and T_E were found to be increasing for both laboratory tests and numerical simulations.
- CS joint model can be reliably used for determining transmission coefficient with respect to energy (T_E) for cases where seismic wave is travelling from medium having low seismic impedance to medium having high seismic impedance.
- Reflection coefficient (R_E) was found to be increasing with increase in impedance ratio in the laboratory experiments.

The present study points to the fact that a structure constructed on rock having lower seismic impedance, beneath which rock having higher seismic impedance is present, wave energy transmission to the structure will be more causing more damage to the structure.

ACKNOWLEDGEMENTS

Science and Engineering Research Board (SERB), Department of Science and Technology (DST), India, provided funding for this research, through the research project, project No: ECR/2018/001966.

REFERENCES

Cui, Z., Sheng, Q., Leng, X. and Ma, Y., 2017. Analysis of the seismic performance of a rock joint with a modified continuously yielding model. *Rock Mechanics and Rock Engineering*, *50*, pp.2695-2707.

Cundall, P.A., 1990. Numerical simulation of fault instabilities with a continuously-yielding joint model. *Rockbursts and seismicity in mines*, pp.147-152.

Deng, X.F., Zhu, J.B., Chen, S.G. and Zhao, J., 2012. Some fundamental issues and verification of 3DEC in modeling wave propagation in jointed rock masses. *Rock mechanics and rock engineering*, *45*, pp.943-951.

Gu, R., 2013. *Distinct element model analyses of unstable failures in underground coal mines*. Colorado School of Mines.

Jaramillo, C.A., 2017. Impact of seismic design on tunnels in rock–Case histories. *Underground Space*, *2*(2), pp.106-114.

Kolsky, H., 1963. *Stress waves in solids* (Vol. 1098). Courier Corporation.

Li, J.C. and Ma, G.W., 2009. Experimental study of stress wave propagation across a filled rock joint. *International Journal of Rock Mechanics and Mining Sciences*, *46*(3), pp.471-478.

Liu, T., Li, J., Li, H., Li, X., Zheng, Y. and Liu, H., 2017. Experimental study of s-wave propagation through a filled rock joint. *Rock Mechanics and Rock Engineering*, *50*, pp.2645-2657.

Miller, R.K., 1977. An approximate method of analysis of the transmission of elastic waves through a frictional boundary.

Miller, R.K., 1978. The effects of boundary friction on the propagation of elastic waves. *Bulletin of the Seismological Society of America*, *68*(4), pp.987-998.

Perino, A., 2011. Wave propagation through discontinuous media in rock engineering. *Politecnico di Torino, Turin, Italy*.

Pyrak-Nolte, L.J. and Cook, N.G., 1987. Elastic interface waves along a fracture. *Geophysical Research Letters*, *14*(11), pp.1107-1110.

Sebastian, R. and Sitharam, T.G., 2014. Transmission of elastic waves through a frictional boundary. *International Journal of Rock Mechanics and Mining Sciences*, *66*, pp.84-90.

Wu, W. and Zhao, J., 2014. A dynamic-induced direct-shear model for dynamic triggering of frictional slip on simulated granular gouges. *Experimental Mechanics*, *54*, pp.605-613.

Wu, W., Zhu, J.B. and Zhao, J., 2013. A further study on seismic response of a set of parallel rock fractures filled with viscoelastic materials. *Geophysical Journal International*, *192*(2), pp.671-675.