A statistical damage constitutive model based on residual strength and Drucker-Prager criterion

Chongyuan Zhang, Hongtao Liu
PetroChina Tarim Oilfield Company, Korla, China

Ping Zeng, Yang Xia
China University of Petroleum, Beijing, China

Liang Wen, Bo Zhou, Bao Zhou
PetroChina Tarim Oilfield Company, Korla, China

ABSTRACT: The HTHP (high temperature and high confining-pressure) triaxial compression test has been conducted on dolomite at 8000m deep in the Tarim Basin, China. We first study the characteristics of rock deformation based on the experimental results. An improved damage constitutive model is introduced to overcome the limitations inherent in the existing models which cannot accurately characterize rock residual strength after failure. The Drucker-Prager model serves the failure criterion of the ultradeep rock. The model is used to fit the experimental data and the results show a good agreement, which indicates that the improved model can be used to predict the mechanical behavior during the whole loading process. The model provides a theoretical knowledge of the mechanical characteristics of ultra-deep rocks in Tarim oil field.

Keywords: Constitutive Model, Residual Strength, Drucker-Prager Criterion, High Confining-pressure, Statistical Damage.

1 INTRODUCTION

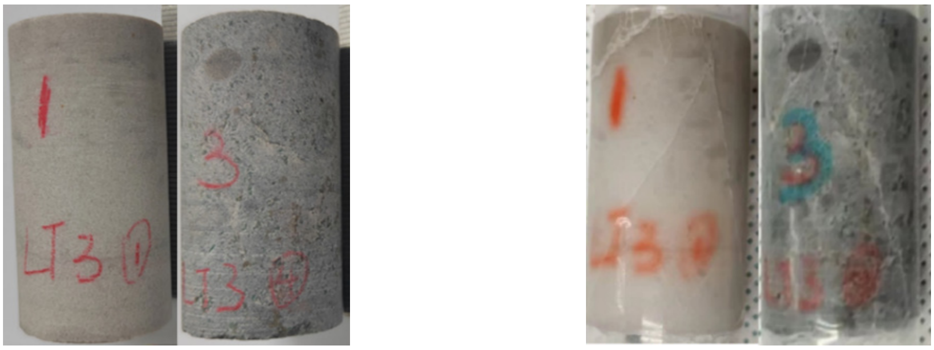
The deformation and failure of rock has always been a critical issue in the field of rock mechanics (Zhang et al.2013 and Wang et al.2018). As a heterogeneous and anisotropic material, rock contains a large number of pores and microcracks. The complexity of crack propagation evolution in rock deformation complicates the simulation of stress-strain relationship. The statistical damage function based on the Weibull distribution was first proposed by Krajcinovic & Silva (1982) and then improved by Lemaitre (1984). The correction of the model includes the following two parts. The first part is the strength criterion. Cao et al. (2005, 2008, 2012) established a constitutive model with Mohr-Coulomb criterion. The criterion is a linear failure criterion. The Drucker-Prager criterion considers the intermediate principal stress and hydrostatic pressure, which matches the failure characteristics of deep rocks (Cao et al. 1998 and Chen et al. 2021). The second part is the correction of the post-peak deformation. Cao et al. (2013) introduced a residual strength correction, but its physical significance is unclear. Cao et al. (2012) established a new damage model that can reflect the residual strength characteristics of rocks after failure. A large number of tests and constitutive models were carried out in granite (Cao et al. 2013), marble (Zhang et al. 2022), sandstone (Xu et al.2007 and Cao et al. 2012). Those rocks were all taken from shallow layers and were tested at low temperature and low confining-pressure condition. He et al. (2005) summarized the main differences between deep and shallow mining engineering, manifested in the harsh environment of "three-high-one-destabilization" (high temperature, high pressure, high karst water pressure, mining disturbances). However, there are relatively few tests carried out on limestone and dolomite at HTHP condition.

This paper studies the characteristics of stress-strain curves of dolomite under HTHP condition by means of triaxial compression tests. Based on Weibull distribution and Drucker-Prager model, a constitutive model of dolomite in ultra-deep is established. Finally, verification and model comparison are carried out according to the experimental results.

2 EXPERIMENTS

2.1 Experimental results

The experimental device is the RTR-1500 HTHP rock test system produced by GCTS Company of the United States. The dolomite samples are taken from Cambrian strata in the Tarim Basin of China. The size of dolomite samples is ϕ25mm*50mm. The pre-experiment dolomite core photo is shown in figure 1(a). The white sample is taken from 8500m depth and the dolomite content is over 95%. The gray sample is taken form 8050m depth, the dolomite content is about 50% and the calcite content is 35%-40%. The temperature gradient in this area is 1.8°C/100m and the experimental protocol is shown in Table 1.



(a) Rock samples before test. (b) Rock samples after test.

Figure 1. Rock samples.

Table 1. Experimental protocol.

sample	Length [mm]	Diameter [mm]	Weight [g]	Density [g/cm^3]	Pressure [MPa]	Temperature [°C]
1	50.49	24.57	66.81	2.79	100	160
2	49.33	24.56	66.93	2.86	80	160

The post-experiment dolomites core photo is shown in figure 1(b). The failure of dolomites under HTHP condition is mainly shear failure. The dolomite samples are accompanied by partial end face failure besides two main fractures. The experimental curve of dolomite under HTHP condition is shown in figure 2.

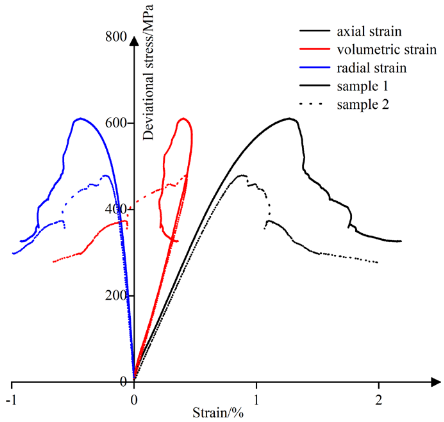


Figure 2. Experimental curve of dolomite.

2.2 Deformation characteristics of dolomite

There is no pore compaction stage as figure 2 shows as the porosity of ultra-deep dolomite rock under HTHP condition is small. With the confining pressure increases, the elastic modulus of the dolomite increases slightly, while the peak strength and residual strength increase significantly. As the confining pressure increases, the rock pores are compressed and the bearing capacity increases under low confining pressure. However, rocks cannot be compressed infinitely, so the elastic modulus of the rock may remain constant at high confining pressures (Chen et al. 2008). With the increase of confining pressure, the expansion of fractures is limited and the strength increase. That's why sample 1 is first compressed, then expanded and finally almost unchanged, while sample 2 expands after failure.

3 STATISTICS DAMAGE MODEL

3.1 Establishment of the model

According to the Krajcinovic model (1982) and Weibull distribution (1951), the damage variable *D*:

$$D = 1 - \exp\,[(-\frac{F}{F_0})^m] \tag{1}$$

where m and F_0 are Weibull distribution parameters, F is micro-unit strength.

According to Cao's assumption (Cao et al. 2012):

$$\sigma_1 = \sigma_1{}^*(1 - D) + \sigma_r D \tag{2}$$

where σ_1 is the Nominal stress, $\sigma_1{}^*$ is the equivalent stress, σ_r is the residual strength of the rock.

The undamaged part is consistent with Hooke's law:

$$\sigma_i{}^* = E\varepsilon_i{}^* + \mu(\sigma_j{}^* + \sigma_k{}^*) \tag{3}$$

where E is the elastic modulus, μ is Poisson's ratio, $\varepsilon_i{}^*$ is strain, the lateral nominal stress $\sigma_j{}^*$ and $\sigma_k{}^*$ are equal to the confining pressure σ_3.

Combining equation (1) and equation (3) with equation (2):

$$\sigma_1 = (E\varepsilon_1 + 2\mu\sigma_1)\exp\left[\left(-\frac{F}{F_0}\right)^m\right] + \sigma_r\{1 - \exp\left[\left(-\frac{F}{F_0}\right)^m\right]\} \tag{4}$$

where σ_1 is the total stress, while the test results are mostly represented by deviatoric stress, that is:

$$\begin{cases} \sigma_1 = \sigma_{1t} + \sigma_3 \\ \varepsilon_1 = \varepsilon_{1t} + \dfrac{1-2\mu}{E}\sigma_3 \\ \sigma_r = \sigma_{1r} + \sigma_3 \end{cases} \tag{5}$$

where σ_{1t} is deviatoric stress, ε_{1t} is test strain, σ_{1r} is test residual stress.

Equation (4) deforms to:

$$\sigma_{1t} = E\varepsilon_{1t}\exp\left[\left(-\frac{F}{F_0}\right)^m\right] + \sigma_{1r}\{1 - \exp\left[\left(-\frac{F}{F_0}\right)^m\right]\} \tag{6}$$

3.2 Determination of the model parameters

Rock strength is established based on the Drucker-Prager failure criterion:

$$F = f(\sigma^*) = \alpha_0 I_1 + {J_2}^{1/2} \tag{7}$$

where I_1, J_2 are the invariant of stress tensor, that is:

$$\begin{cases} I_1 = {\sigma_1}^* + \sigma_2 + \sigma_3 = E\varepsilon_{1t} + 3\sigma_3 \\ \sqrt{J_2} = [\dfrac{({\sigma_1}^* - \sigma_2)^2 + ({\sigma_1}^* - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}{6}]^{1/2} = \dfrac{E\varepsilon_{1t}}{\sqrt{3}} \\ \alpha_0 = sin\varphi/\sqrt{9 + 3sin^2\varphi} \end{cases} \tag{8}$$

where φ is rock friction angle.

Combining equation (7) and equation (8) with equation (6):

$$\sigma_{1t} = E\varepsilon_{1t}\exp\left[\left(-\frac{sin\varphi(E\varepsilon_{1t} + 3\sigma_3) + \sqrt{3 + sin^2\varphi}E\varepsilon_{1t}}{\sqrt{9 + 3sin^2\varphi}F_0}\right)^m\right] + \sigma_{1r}\{1 - \exp\left[\left(-\frac{sin\varphi(E\varepsilon_{1t} + 3\sigma_3) + \sqrt{3 + sin^2\varphi}E\varepsilon_{1t}}{\sqrt{9 + 3sin^2\varphi}F_0}\right)^m\right]\} \tag{9}$$

Relationship by stress-strain：

$$\begin{cases} \varepsilon_{1t} = \varepsilon_p, \sigma_{1t} = \sigma_p \\ \left.\dfrac{\partial\sigma_{1t}}{\partial\varepsilon_{1t}}\right|_{\varepsilon_{1t}=\varepsilon_p} = 0 \end{cases} \tag{10}$$

where ε_p is peak strain, σ_p is peak stress.

Weibull distribution parameters：

$$\begin{cases} m = \dfrac{[sin\varphi[E\varepsilon_p + 3\sigma_3] + \sqrt{3 + sin^2\varphi}E\varepsilon_p]}{(E\varepsilon_p - \sigma_{1r})[sin\varphi + \sqrt{3 + sin^2\varphi}]ln\dfrac{E\varepsilon_p - \sigma_{1r}}{\sigma_p - \sigma_{1r}}} \\ F_0 = \dfrac{F_p}{(ln\dfrac{E\varepsilon_p - \sigma_{1r}}{\sigma_p - \sigma_{1r}})^{1/m}} \end{cases} \tag{11}$$

where F_p is the peak stress of the micro-element, that is $F_p = \frac{sin\varphi[E\varepsilon_p+3\sigma_3]+\sqrt{3+sin^2\varphi}E\varepsilon_p}{\sqrt{9+3sin^2\varphi}}$

4 MODEL VERIFICATION

In order to verify the correctness of the above damage constitutive model, the date of the triaxial compression test of dolomite under HTHP condition is used to calculate damage variable according to the method of section 3.2. Further, the constitutive equation of dolomite can be obtained according to damage evolution equation and section 3.1. Combined with the constitutive model established by Cao et al. (2013) and Cao et al. (1998), the advantage of the model is compared. Comparison between dolomite test results and theoretical curves of different constitutive models is shown in figure 3.

The constitutive model sets up good agreement with the experimental data of dolomite under HTHP condition, and can reflect the deformation character of rock strain softening and residual strength, so as to verify the rationality of the damage constitutive model in this paper.

Statistical damage constitutive model has satisfied fitting in rock pro-peak deformation. For the strain softening stage and residual stage of rock post-peak deformation, the different assumptions in constitutive models induce the difference fitting. As shown in figure 3, the damage variable based on peak strength is not very suitable for dolomite with strength dropped rapidly. The constitutive model in this paper has better agreement with residual strength stage compared with the other models proposed by Cao et al (1998) and Cao et al. (2013).

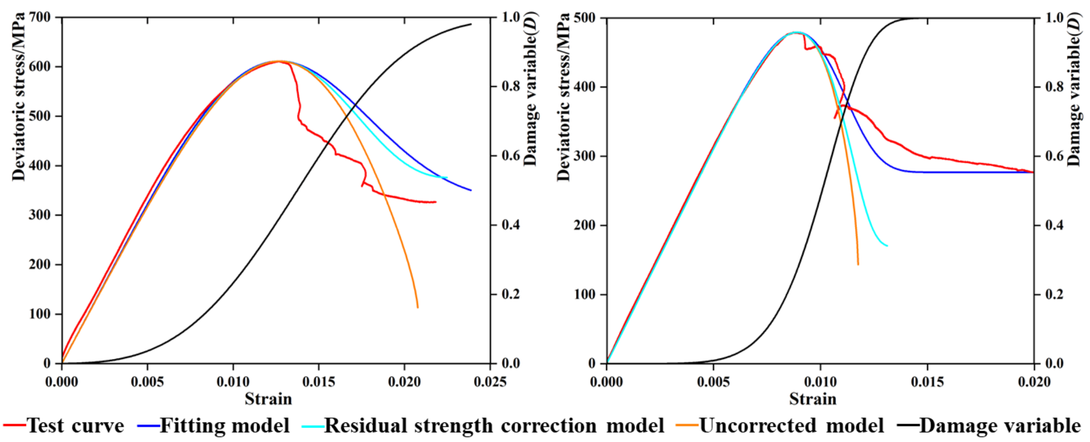


Figure 3. Comparison between test results and theoretical curves of different constitutive models.

5 CONCLUSION

In this paper, the dolomite from Tarim Basin, China of under HTHP triaxial compression were tested, and the deformation characteristics of dolomite under different confining pressure were analyzed.

Further, the statistical damage constitutive model based on Drucker-Prager criterion is established. Finally, the validity and superiority of the model are carried out. The main conclusions are as follows:

(1) In the deformation process of ultradeep dolomite under HTHP condition, the evolution of rock has not undergone the pore compression stage. Rock failure is mainly shear failure, accompanied by partial end face failure.

(2) The elastic modulus of dolomite increases slightly, and the peak strength and residual strength of rock increase significantly with the increase of confining pressure.

(3) The statistical damage constitutive model has great agreement with the pre-peak experiment data and the corrected model can reflect the post-peak deformation better. By comparing with test data, reasonable model and high degree of simulation， it can be used for engineering application.

REFERENCES

Cao, W. G., Fang, Z. L. & Tang, X. J. 1998. A study of statistical constitutive model for soft and damage rocks. *Chinese Journal of Rock Mechanics and Engineering*, 17(6), 628-633.

Cao, W. G., Zhao, M. H. & Liu, C. X. 2005. Study on rectified method of Mohr-Coulomb strength criterion for rock based on statistical damage theory. Yanshilixue Yu Gongcheng Xuebao/*Chin. J. Rock Mech. Eng.*, 24(14), 2403-2408.

Cao, W. G., Zhao, H., Zhang, L. & Zhang, Y. J. 2008. Damage statistical softening constitutive model for rock considering effect of damage threshold and its parameters determination method. *Chinese Journal of Rock Mechanics and Engineering*, 27(6), 1148-1154.

Cao, W. G., Zhao, H., Li, X. & Zhang, L. 2012. A statistical damage simulation method for rock full deformation process with consideration of the deformation characteristics of residual strength phase. *China Civil Engineering Journal*, 45(6), 139-145.

Cao, R. L., He, S. H., Wei, J. & Wang, F. 2013. Study of modified statistical damage softening constitutive model for rock considering residual strength. *Rock and Soil Mechanics*, 34(6), 1652-1660.

Chen, Y., Xiao, P., Du, X., Wang, S., Wang, Z. & Azzam, R. (2021). Study on damage statistical constitutive model of triaxial compression of acid-etched rock under coupling effect of temperature and confining pressure. *Materials*, 14(23), 7414.

He, M. C., Xie, H. P., Peng, S. P., & Jiang, Y. D. (2005). Study on rock mechanics in deep mining engineering. Yanshilixue Yu Gongcheng Xuebao/*Chin. J. Rock Mech. Eng.*, 24(16), 2803-2813.

Krajcinovic, D. & Silva, M. A. G. 1982. Statistical aspects of the continuous damage theory. *International Journal of Solids and Structures*, 18(7), 551-562.

Lemaitre, J. 1984. How to use damage mechanics. *Nuclear engineering and design*, 80(2), 233-245.

Tang, C, A. 1993. *Catastrophe in rock unstable failure*. Beijing: China Coal Industry Publishing House.

Wang, Z., Liu, D., & Zhang, X. (2018, October). Study on Strain Softening Deformation Characteristics and Damage Constitutive Model for Rocks. In *ISRM International Symposium-10th Asian Rock Mechanics Symposium*. OnePetro.

Weibull, W. (1951). A statistical distribution function of wide applicability. *Journal of applied mechanics*.

Xu, J., Li, S. C., & Liu, Y. B. (2007). Damage constitutive model of rock based on Drucker-Prager criterion. *Journal of Southwest Jiaotong University*, 42(3), 278-282.

Zhang, C., Cao, W., Zhao, H., & He, M. (2022). Statistical damage simulation method for complete stress-strain path of rocks considering confining pressure effect and strength brittle drop. *Chin. J. Rock Mech. Eng*, 44, 936-944.

Zheng, Z., Wang, W., Cao, Y. J., & Lü, J. (2013, June). A statistical damage constitutive model with damage threshold based on Mogi-Coulomb criterion. In *ISRM SINOROCK 2013*. OnePetro.