Characterization of Poisson’s ratio and Elastic Modulus of granitic rocks: from micro-crack initiation to failure

Samad Narimani Ghourtlar
Dept. Engng, Geol. & Geotechn. Faculty of Civil Engng, Budapest University of Technology & Economics, Budapest, Hungary

Seyed Morteza Davarpanah
Dept. Engng, Geol. & Geotechn. Faculty of Civil Engng, Budapest University of Technology & Economics, Budapest, Hungary

László Kovács
RockStudy Ltd, Pécs, Hungary

Balázs Vásárhelyi
Dept. Engng, Geol. & Geotechn. Faculty of Civil Engng, Budapest University of Technology & Economics, Budapest, Hungary

ABSTRACT: Key geomechanical parameters utilized in rock engineering include Young's modulus and Poisson's ratio. Uniaxial compressive test results are essential in the evaluation of these values. This paper aims to study the process of changes of Poisson’s ratio and Young modulus for intact rock during loading from micro-crack initiation to failure stage. Both Young’s moduli and Poisson’s ratio were calculated using the stress-strain curves. By using parametric investigation, the crack damage stress, determined for Poisson’s ratio-axial stress graphs. Also, this research outline the findings that the variations among the three Young's moduli and Poisson’s ratio estimated for each specimen and suggest the most effective approach for doing so. It was found, that the Poisson’s rate depends on the stress value: it is linearly increasing with increasing stress till the unstable crack propagation stress. Contrary to previous ideas, our results suggest that the Poisson’s ratio is not a constant for rigid rocks.

Keywords: uniaxial compressive test, Poisson’s ratio, Young's moduli, rock mechanical parameters.

1 INTRODUCTION

In the elastic deformation of rocks and rock masses subjected to static or dynamic loads, Poisson's ratio and Young’s Modulus play an unquestionably significant role. Additionally, their impacts can be seen in a wide range of rock engineering applications, from straightforward laboratory testing on whole rocks to on-site measurements of in situ stresses or the deformability of rock masses. Rock engineering can therefore benefit from knowledge of various Poisson's ratio and Young’s Modulus features. In accordance with Bieniawski (1967), the Poisson's ratio and Young’s Module of rocks remains constant during linear elastic deformation but start to rise as a result of the emergence of new microcracks or the growth of preexisting ones.

The essential characteristic stress thresholds in the failure process are the crack initiation stress (σ_{ci}) and the crack damage stress (σ_{cd}). While crack initiation denotes the beginning of microfracturing, crack damage denotes the beginning of crack coalescence and dilatation deformation (volumetric strain). The types of rocks, the composition of the minerals, the particle sizes, and the structural types are only a few of the variables that might affect the typical stress thresholds (Malkowski and Ostrowski, 2017).

When discussing the mechanical behavior of solids, it is generally assumed that they are homogenous, continuous, and isotropic (Ulusay, 2018). However, rocks are far more complex, and their mechanical characteristics differ depending on the scale, mineral composition, or kind of matrix. Engineers, however, typically need certain values for the various rock types' rock characteristics. The only way to supply them is through intensive laboratory and field research. A uniaxial compression (UCS) is the fundamental laboratory test for rock strength. Once a rock reaches its peak strength, it acts elastically until propagating cracks cause the deformation to change to a quasi-plastic state, modifying the rock's strength characteristics (Malkowski and Ostrowski, 2017).

In theoretical investigations and numerical simulations, the Poisson's ratio is an essential variable. There are, however, little information on its evaluation, testing, and application range. Researchers will be able to conduct more accurate laboratory and in-situ studies and gain a deeper understanding of the mechanisms underlying rock (mass) deformation if they have a thorough understanding of Poisson's ratio (Dong, 2021). Due to Gercek (2007), the Poisson's ratio is an elastic constant whose relevance is typically underestimated when compared to other fundamental mechanical characteristics of rocks. Due to major heterogeneities in their geological history, mineral composition, crystallization, or depositional structure, rocks' Poisson's ratios can vary widely. Additionally, it's not obvious from the literature if the Poisson's ratios given are average or secant. Hajiabdolmajid (2002) proposed the material model which consider the effect of cohesion weakening and friction strengthening on brittle failure and crack propagation of rock.

The purpose of this study is to determine the variation of Poisson's ratio and Young’s Modulus for intact granitic rocks from crack closure until failure stage in UCS test. We study the values and variations in the Poisson's ratio and Young’s Modulus in three different scenarios, including secant, average, and tangent, in the UCS test from beginning to failure stages.

2 METHODS AND RESULTS

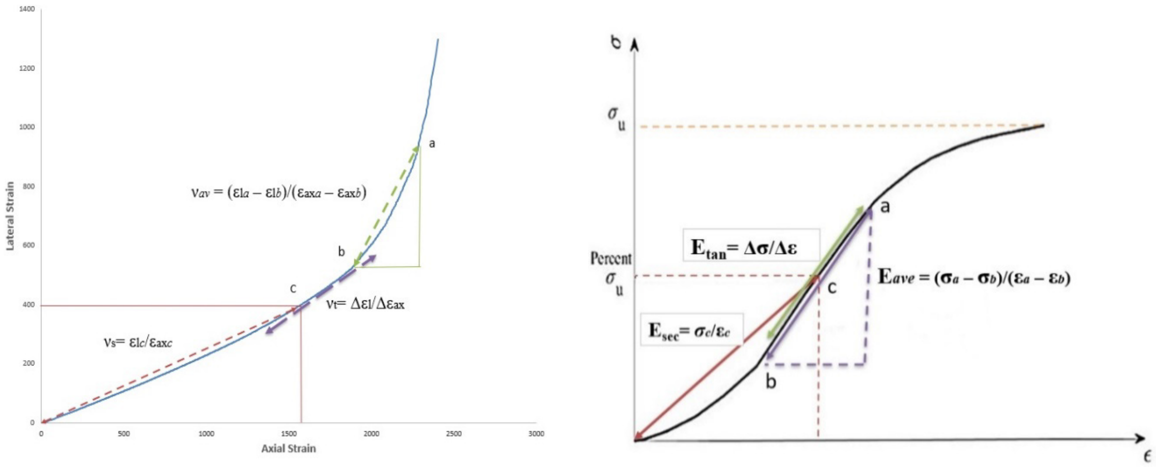
Since there is no accepted definition for σ_{cc}, the crack initiation stress, σ_{ci}, is frequently used as the lower limit of the linear segment instead. This segment of the stress-axial strain curve is roughly linear and is frequently used to describe the elastic modulus. For rock materials, σ_{ci} is roughly 30–50 % of their UCS values (Diederichs et al. 2004), and σ_{cd} is 60–80% of its UCS. In light of this viewpoint, it makes sense to determine the average elastic modulus of rocks for the great majority of rock types using the stress-strain curve that is between 40 and 60 percent of UCS between σ_{ci} and σ_{cd}.

There aren't as many researches on the behavior of the Poisson's ratio in each step of rock deformation as there are on other factors like Young's modulus and compressive strength. Yu et al. (2008) carried out tests on three different types of rocks and discovered that the tangent Poisson's ratio of rocks rises with compressive stress under compressive conditions and falls with increasing tensile stress under tensile conditions. Poisson's ratio is heavily influenced by stress, according to the findings of Davarpanah et al. (2019). Also, in Davarpanah et al. (2020a) they investigated the relationship between different mechanical parameters, such as Poisson’s ratio and Young’s modulus for Hungarian granitic rock samples. Moreover, in Davarpanah et al. (2020b) they proposed empirical relationships between tangent and secant Poisson’s ratio for different rock types.

Engineering materials' elastic moduli can be calculated using a variety of techniques. Among the techniques are the initial tangent modulus, tangent modulus of the straight-line portion of the stress-strain curve, tangent modulus at a fixed percentage of maximum strength, initial secant modulus (to maximum strength), secant modulus at a fixed percentage of maximum strength, average modulus, loading modulus, and unloading modulus. There is a wealth of literature on the application of Young's modulus to rock, but none of it discusses how Young's modulus varies from the commencement of stress to the point of failure (Santi et al., 2000). So, the other goal of this work is to identify the most effective equations for each method of measuring elastic moduli in granitic rocks in order to compute changes in elastic moduli. In a recent study, Malkowski et al. (2018) compared the tangent, secant, and average Young's moduli. Their investigation led to the recommendation that tangent Young's modulus serve as the guiding parameter at a constant range of 30–70% of the ultimate stress. Since Secant Young's modulus includes both elastic strain and pore compaction and has a range of 0 to 50 % of the ultimate stress, it should be called the modulus of deformability.

We conducted UCS on Morágy granitic rock formation in Hungary in order to investigate the variation of the ν and E by new method and formulation from the crack closure stage though failure stage. Figures 1a and 1b display the equations for calculating the tangent, average, and secant Poisson's ratios and Young’s Modulus. It should be noted that the lateral strains were regarded as positive values to make it simpler to exhibit their values in the four axial strain-lateral strain curves.

The origin of the secant Poisson's ratio was maintained at the zero-stress position, but the reference point was considered as a moving point that changes with stress in order to analyze the behavior of the secant Poisson's ratio. Secant Young’s modulus (E_s) is defined as the slope of the line from the origin to some fixed percentage of ultimate strength. The average Poisson’s ratio reflects the relative change in axial and radial strain at the upper and lower limits of some stress interval. Average Young’s modulus (E_{ave}) of the straight-line part of a curve is defined as the slope of straight-line part of the stress-strain curve for the given test. The axial strain-lateral strain curve's tangential slope is represented by the tangent Poisson's ratio. The tangent Poisson's ratio calculation is more susceptible to changes in the testing procedure and sample frequency than the secant Poisson's ratio calculation is. Tangent Young’s modulus (E_t) is defined as the slope of a line tangent to the stress-strain curve at a fixed percentage of ultimate strength (Narimani et al. 2023).



a b

Figure 1. Schematic calculation of a) secant Poisson’s ratio(υ_s), tangent Poisson’s ratio(υ_t) and average Poisson’s ratio(υ_{av}); b) secant Young’s Modulus (E_s), tangent Young’s Modulus (E_t) and average Young’s Modulus (E_{av}) (Narimani et al. 2023).

The analysis of the rock's progressive deformation process is invariably negatively impacted by the method's uncertainty and the resultant calculation uncertainty. A typical Poisson’s ratio-σ/σ_c and Elastic modulus-σ/σ_c curves for granitic specimen are shown in Figs. 2a and 2b respectively.

New method is introduced to provide the variation of υ values from crack closure to failure stage under UCS test. Based on this method, the scale of stress over peak stress(σ/σ_c) considered to vary between -80 and +80 where -80 and +80 correspond to σ/σ_c=0 and 1, respectively. To do this, for all the specimens the origin of the coordinate system moved to σ/σ_c=0.5 to make symmetrical condition and new proposed model fitted to the experimental graph based on the equation 1.

$$\upsilon = \upsilon_{0.5} + \tan(\text{degree})/B \quad (1)$$

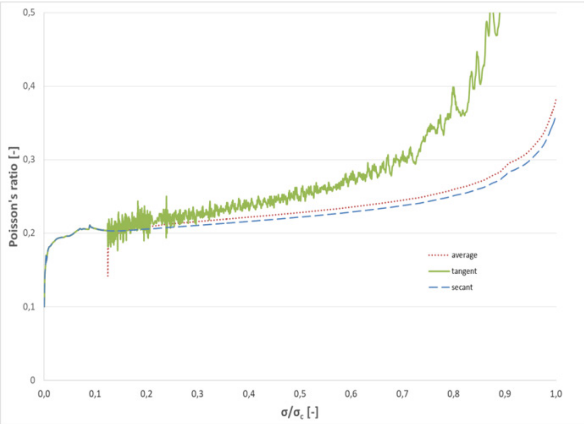
In the equation, υ_0 refers to the Poisson's ratio at σ/σ_c=0.5 which defined as constant A. In term of tan(degree), since tan90 is infinite, degree is introduced as $160\boldsymbol{\sigma}/\boldsymbol{\sigma}_c$-80. B is constant and depends on rock type. Therefore, the final equation for calculating Poisson's ratio will be as follow:

$$\upsilon = A + \tan(160\boldsymbol{\sigma}/\boldsymbol{\sigma}_c-80)/B \tag{2}$$

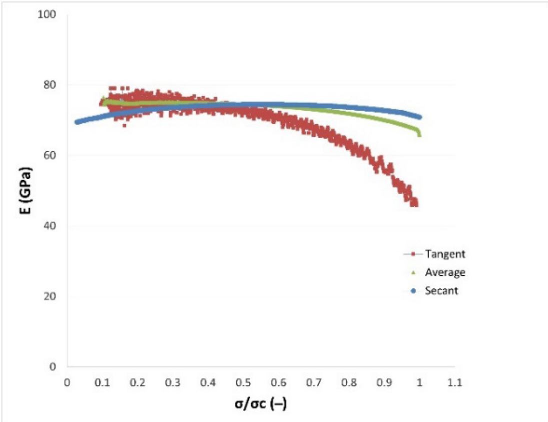
For this purpose, calculations of Poisson's ratio carried out by using the new proposed model in three different scenarios, secant, average and tangent and constants of A and B determined for each scenario. The results are summarized in Table 1.

Table 1. Equations and related constants for Poisson's ratio in different cases.

Poisson's ratio	Equation	Constant A	Constant B
Secant	$\upsilon_{sec} = A + \text{Tan}(160\sigma/\sigma_c-80)/B$	0.21 to 0.27	30 to 40
Average	$\upsilon_{ave} = A + \text{Tan}(160\sigma/\sigma_c-80)/B$	0.23 to 0.27	30 to 40
Tangent	$\upsilon_{tan} = A + \text{Tan}(160\sigma/\sigma_c-80)/B$	0.25 to 0.28	10 to 15



a



b

Figure 2. A typical secant, tangent and average a) Poisson's ratio- σ/σ_c curve; b) Young's modulus- σ/σ_c curve for granitic specimens.

Similarly, according to our analysis since all the derived curves follow the parabolic form, new quadratic equations were proposed to show the relationship between σ/σ_c and Young's modulus in different scenarios. These equations contain three different independent constants, a, b and c in each case. The obtained range for these constants is reported in Table 2.

Table 2. Equations and related constants for Young's modulus in different cases.

Young's modulus	Equation	Constant a	Constant b	Constant c
Secant	$E_{sec} = ax^2 + bx + c$	-10 to -19	9 to 19	65 to 75
Average	$E_{ave} = ax^2 + bx + c$	-14 to -24	9 to 18	61 to 76
Tangent	$E_{tan} = ax^2 + bx + c$	-22 to -67	18 to 36	59 to 77

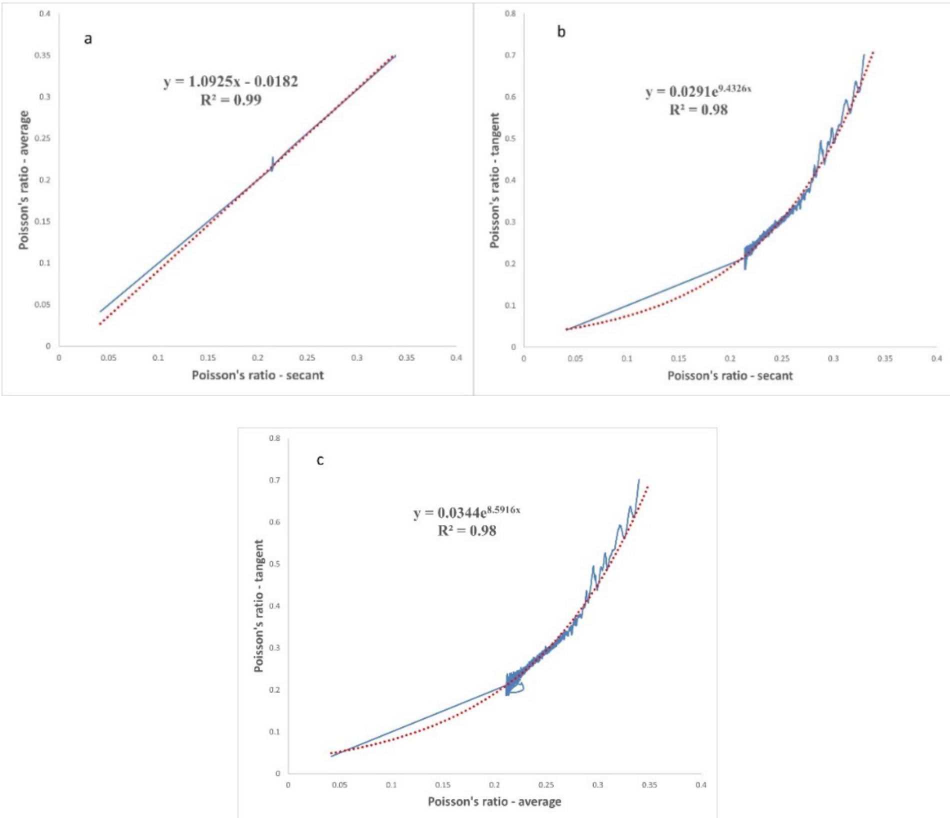


Figure 3. Relationship between (a) υ_{ave}- υ_{sec}, (b) υ_{tan}- υ_{sec}, (c) υ_{tan}- υ_{ave}.

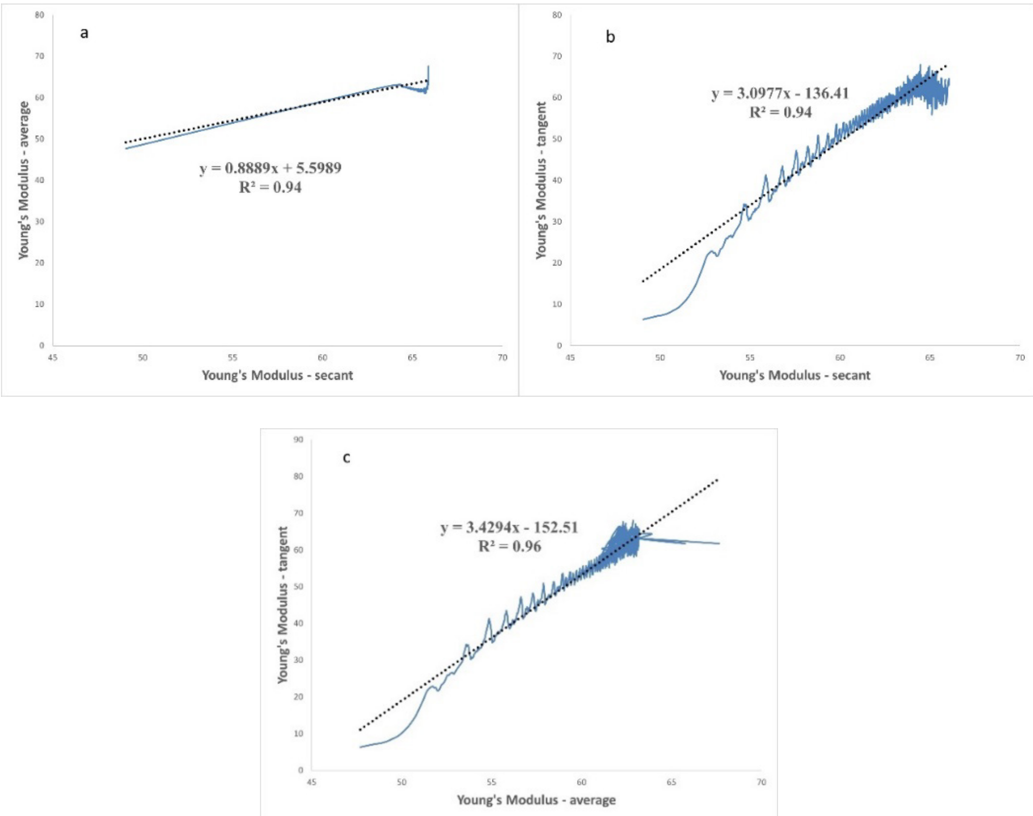


Figure 4. Relationship between (a) E_{ave}-E_{sec}, (b) E_{tan}-E_{sec}, (c) E_{tan}-E_{ave}.

Our studies (Figure 3) reveal that there have been fresh publications of linear and nonlinear correlations between deformation constants. As seen in Figure 5, the υ_{ave} and υ_{sec} values are highly linear correlated, with an R^2 value of 0.99. However, trends can be seen nonlinear between the υ_{tan} and υ_{sec} and the υ_{tan} and υ_{ave}, both of which have R^2 values of 0.98.

In other word, due to Figure 4, focusing on the E, achieved correlations for all the cases are linear which the maximum regression coefficient is related to E_{tan} and E_{ave} which R^2 value is 0.96.

3 CONCLUSION

One of the notable findings from the evaluation discussed in this study is regardless of the method, changes of υ and E from the beginning of loading through the failure stage is studied for granitic rocks for UCS test. This findings disprove the conventional idea of constant amount for υ and E of rocks. According to our findings, the representative equation for calculating the variation of υ is linear, despite the achieved equation for estimating of E follows parabolic function. Based on investigated granitic rock, constants of these proposed equations for υ and E are different. Moreover, we achieved correlations with high determination factor for E and υ in three different scenarios, secant, tangent and average. Lack of variety in rock types was one restriction on this investigation. These predictive equations can be applied to other tests to forecast υ and E with greater accuracy utilizing various rock types. Therefore, depending on the situation, the aforementioned predictive equations can be used to predict the υ and E.

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