Event tree analysis and time-integrated reliability design approach for quantifying rockfall risk reduction

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ABSTRACT: The quantification of risk in terms of loss of lives represents the essential parameter to manage rockfall risk in urbanized areas. Rockfall barriers are among the most adopted structural mitigation measures. Despite their wide use, the partial safety factors design approach is not able to guarantee a specific failure probability and, consequently, to assess the precise risk reduction. To tackle all these issues, a quantitative risk assessment method for infrastructures and a time-integrated reliability design approach for rockfall barriers developed by the Authors are combined in a unique framework to quantify risk reduction. The former computes the risk as annual probability of having at least one fatality; the latter allows defining an annual failure probability for a given product in a given site. Merging these methods, the evaluation of risk reduction in case of barrier installation or the definition of the required performances, are defined. An example of application is provided.

Keywords: Rockfall Risk, Reliability based design, Event tree Analysis, Rockfall barrier.

1 INTRODUCTION

Among natural hazard, rockfall represents one of the most dangerous phenomena, due to its unpredictability and high kinetic energy involved. Rockfall can affect public infrastructures and villages in mountain environment, as well as workers in particular contexts, e.g. open pit mines (Scavia et al. 2020). As a consequence, a quantification of the risk has become an urgent issue for public administrations and road infrastructure management bodies, to properly manage the risk, predisposing effective mitigation plans and prioritizing the interventions. Generally, the quantification of the risk is required in terms of annual probability of fatalities (Mignelli et al. 2012).

To achieve such goal, an accurate hazard analysis represents the starting point. Once the possible initiating events are identified and characterized in terms of magnitudes and associated return period, i.e. detachment probability, one or more initial realistic scenarios have to be defined, from which propagation analyses have to be performed (Moos et al. 2022). Rockfalls can indeed differ in both released and arriving block volumes, according to the occurrence of fragmentation processes.

Once defined possible released scenarios, propagation analyses have to be performed and the hazard computed (Crosta et al. 2015, Farvacque et al., 2021). Finally, selected the element at risk, the consequences, i.e. the damages, have to be quantified for each scenario.

When the obtained risk value is higher than an acceptable threshold, mitigation measures should be predisposed. Focusing on structural protective measures, flexible rockfall barriers are about the most effective for high energy events. Despite their wide adoption, their design is still under debate, even though nowadays, following the CE marking procedure (EAD 340059-00-0106, 2018) and some National Standards (UNI11211-4; ONR 24810) the current design practice is directed towards a performance-based design approach, in which energy absorption capacity, height, and deformability represent the essential characteristics. According to the results of propagation analyses, the designer selects a suitable commercial product for which it can be checked that block impact energy and passing height are smaller than barrier performances, considered as the reference values those obtained through standardized impact tests (EAD 340059-00-0106, 2018). A partial safety factor approach is generally adopted: the design values of both actions (block kinetic energy and passing height) and resistances (i.e. performances) are computed applying a partial safety factor to a characteristic value of their distribution. Due to the site variability of the problem, the distribution of actions does not follow a fixed shape (Bourrier et al. 2016). The partial safety factors proposed by the National Standards are fixed values, and thus, neglect the site specificity of rockfall phenomena. As a consequence, the use of these factors unavoidably results in designing barriers with different failure probabilities (Marchelli et al. 2020).

To overcome this issue, a time-integrated reliability based design approach has been recently proposed by the Authors (De Biagi et al., 2020, Marchelli et al. 2020, Marchelli et al. 2021a). The approach considers both the variability in magnitude of the events, and the associated detachment probability, and the intrinsic variability of the actions and their probability distributions. This approach can be embedded inside a quantitative risk assessment (QRA). In the present work a QRA proposed by the Authors for viable infrastructures is adopted. The coupling represents a valuable tool to quantify the risk reduction due to the intervention.

In the following, the mathematical framework of the quantitative risk analysis for infrastructures and reliability based design approach are illustrated, with a specific focus on their coupling. This last allows to verify the goodness of a possible intervention. An example of application is provided.

2 METHODOLOGY

2.1 Quantitative risk assessment for viable infrastructures

Rockfall is generally treated as a Poisson point process phenomenon, in which the events are independent, with an average frequency of occurrence according to their magnitude. The risk assessment must account both for the variability in magnitude, and for the discrete temporal nature of the phenomenon. Assuming the exposed area consisting of q elements at risk and p rock block volume classes that can detach, neglecting the fragmentation process, the risk R is computed as:

$$R = \sum_{l=1}^{p} \sum_{m=1}^{q} \left(P_T^l P_S^{l,m} E^m V^{l,m} W^m \right) \tag{1}$$

where P_T^l is the temporal (or detachment) probability, i.e. the frequency associated to each possible released volume (also called return period), $P_S^{l,m}$ is the spatial probability that this block reaches the m-th element at risk, and E^m, $V^{l,m}$, W^m are the exposure, i.e. the probability that a given element is at the impact location where the rock block detaches, the vulnerability and the value, respectively. As the vulnerability is function not only of the characteristics of the elements at risk but also of the intensity of the phenomenon, for each block volume, and thus for each kinetic energy at the element at risk location, the damages have to be computed. Given a volume, its detachment probability is subjected to a wide range of uncertainties and can hardly be predicted in a deterministic way (Moos et al., 2022). Statistical models are often used to approximate the block size (v) return period relationship, which has proven to be well fit by power law distributions (Hantz et al., 2003):

$$P_T(v \geq V) = aV^{-b} \tag{2}$$

where a represents the activity of the rock cliff and corresponds to the frequency of rockfall events with a volume > 1 m^3, while b, i.e. the fractal dimension, depends on the geological structure. It could be inferred that the detachment probability depends on several factors, i.e. lithology, orientation and structural configuration of the discontinuities sets on the rock face (fundamental for a failure kinematic analysis), degree of weathering, freeze-thaw cycles, other external factors, e.g. seismic actions or wildfires (Pérez-Rey et al., 2019). Nevertheless, due to the complexity and the uncertainties related to the data, the definition of P_T^l is often based on statistics of past events.

In case of infrastructures, $P_S^{l,m}$ can be referred to the system on which element at risk (P_S^l), i.e. users, are moving, i.e. the road. As people are the element at risk, the vulnerability could be considered magnitude-independent, as every block, of any volume, can cause a fatality. Thus, the correlation between release volume and return period can be neglected and P_T^l can be estimated as the mean annual frequency associated to an event. In principle, different source areas can be individuated, as well as different traffic conditions. Subdividing the road into portions equal for number of source area insisting on it and traffic conditions, it results:

$$R = \sum_{k=1}^{n} P_T^{l,k} P_S^{l,k} \left[\sum_{m=1}^{q} \left(E^{m,k} V^{l,m,k} W^{m,k} \right) \right] \tag{3}$$

it should be noted that $P_S^{l,k}$ can vary along the k-th portion, thus a homogenization process is required. To evaluate the term $\sum_{m=1}^{q}\left(E^{m,k}V^{l,m,k}W^{m,k}\right)$, for each k-th portion a method based on event tree analysis approach (ETA) has been developed by the Authors (Marchelli et al., 2021b). The ETA is a logical procedure in which both success and failure response are evaluated, starting from a single initiating event, in this case the arrival of a block on the road, and defining all the possible alternative pathway options, mutually exclusive, which can occur. The end points identify a unique outcome, whose probability is given by the conditional probability along their own pathway. The probability of more outcomes is given by the sum of the probabilities of each outcome. Figure 1 reports the proposed event tree. Once obtained $\sum_{m=1}^{q}\left(E^{m,k}V^{l,m,k}W^{m,k}\right)$ as the probability of having at least one fatality due to the occurrence of an event, this has to be inserted into Eq. (3) to consider the temporal and spatial variabilities of the events. See Marchelli et al. (2021b) for details.

In case of no sufficient data to define P_T^l and the only available data refer to events which had reached a road (as a relevant susceptible element), or a r-th section of it, at least a number of events per year N_B^r for each section can be derived. Considering thus that a section can encompass different portions, the number of events of each k-th portion can be obtained through a homogenization process based on $P_S^{l,k}$ and the length of each portion. Finally, the risk, expressed as the annual probability to have a fatality, results in:

$$R = \sum_{r=1}^{s} \sum_{k=1}^{n} \left[1 - \left(1 - \sum_{m=1}^{q} \left(E^{m,k} V^{l,m,k} W^{m,k} \right) \right)^{N_B^k} \right] \tag{4}$$

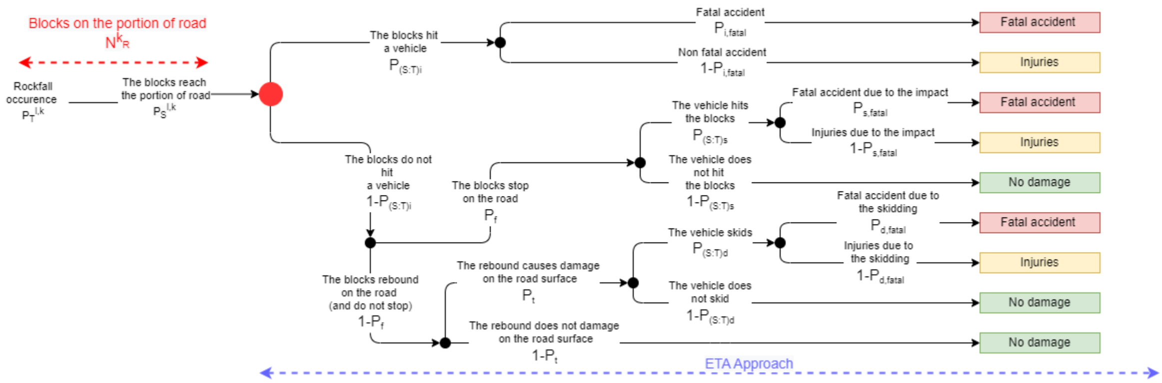

Figure 1. Proposed method for viable infrastructures.

2.2 *Reliability based design method for rockfall net fences*

The possible failure modes of rockfall barrier structures can be simplified into: (i) exceeding height when the block is not intercepted and (ii) exceeding kinetic energy, when the absorption capacity of the system is smaller than block impact energy. A failure probability can be associated to each of them, F_h and F_k, respectively. These two can be combined into a unique failure probability, named p_f. In a specific period of time τ, this can be computed as (De Biagi et al., 2020, Marchelli et al. 2020, Marchelli et al. 2021a):

$$p_f(\tau) = F_h(\tau) + F_k(\tau) = 2 - e^{-\nu\tau p_{fa,h}} - e^{-\nu\tau p_{fa,k}}, \tag{5}$$

being ν the mean expected annual frequency of a rockfall event (of any intensity) impacting on the barrier, and $p_{fa,h}$ and $p_{fa,k}$ the probability of each of the two failure modes, respectively, considering the certain occurrence of an event. As in the risk assessment procedure, all possible impacting volumes, together with their frequency should be considered. Taken $p_{fa,k}$ as an example, it results:

$$p_{fa,k} = \int_0^\infty \int_0^\infty p_{fk} \,|(V_k = \mu \text{ and } v_k = \omega) f_{V_k,v_k}(\mu,\omega) d\mu d\omega \tag{6}$$

where $p_{fk}|(V_k = \mu \text{ and } v_k = \omega)$ is the conditional failure probability if the block volume and passing height have their characteristic values V_k and v_k equal to μ and ω, respectively, i.e. random variables through which the characteristic values are defined, and $f_{V_k,v_k}(\mu,\omega)$ the probability density function of V_k and v_k, theoretically differing for each frequency. The conditional failure probability is studied through a state function that describes both the safe and the unsafe conditions, accounting for the resistances. A similar approach is adopted for $p_{fa,h}$. See the referenced papers for details.

Known the distributions of block (i) velocities, (ii) passing heights, (iii) volumes at the impact, together with their probability density functions, a value of failure probability in the period τ can be defined for the installation of a specific product in a specific site. Neglecting again the fragmentation process, the distribution of volumes at the impact depends on the detachment probability for each volume size and block reaching probability, function of the structure of the rock mass, the mechanical parameters of both block and slope and topography. When the amount of available data is reduced, a technique to determine the relationship between volume return period, based on past events information and a block volume survey at the slope toe, was developed by De Biagi et al. (2017).

If a net fence insisting on a portion of infrastructure is installed, $p_f(\tau)$ can be used for a re-assessment of the risk along the infrastructure. It should be noted that in this case, ν can be approximated to N_B^k, provided that $P_S^{l,k}$ on the portion of road is almost equal to the spatial impact probability on the net fence.

2.3 Coupling the approaches

The introduction of a rockfall barrier insisting on a portion of road varies both the spatial and temporal probabilities that a block reaches the element at risk in that portion. Provided that $p_f(\tau)$ is calculated with a time-integration for all the possible block volumes, considering τ equal to 1 year and in the hypothesis that the designed mitigation measures protect only $n_1 < n$ portions of road, Eq. (4) becomes:

$$R = \sum_{r=1}^{s} \left\{ \sum_{k=1}^{n_1} \left[1 - \left(1 - \sum_{m=1}^{q} \left(E^{m,k} V^{l,m,k} W^{m,k} \right) \right)^{p_f(1yr)} \right] + \sum_{k=n_1+1}^{n} \left[1 - \left(1 - \sum_{m=1}^{q} \left(E^{m,k} V^{l,m,k} W^{m,k} \right) \right)^{N_B^k} \right] \right\} \tag{7}$$

3 EXAMPLE

An example of application is herein provided. A local road passes along a slope onto which rockfall hazard is present. Tridimensional trajectory analyses indicate that the length of the stretch in which the natural phenomenon can create injuries to road users is 600 m long, with a reaching probability that varies along the path. A mean annual rate of 0.2 rockfall events is observed along the stretch of road. A survey of the blocks in the surroundings of the road let to identify the distribution of the sizes of the fallen values. According to the sampling method proposed by Marchelli and De Biagi (2019), the N=500 sampled volumes are distributed according to a Pareto Type I function with threshold volume $V_{th} = 0.5$ m^3 and $\alpha = 1.5$ (similar to b in Eq. 2), highlighting that the number of large blocks is limited. According to De Biagi (2017, Eqn. 5), considering an annual rate of 0.2 events per year, the volume corresponding to a return period of 100 years is 3.68 m^3 (density equal to 2700 kg/m^3). Table 1 illustrates the risk calculation along the road (Marchelli et al., 2021b) in absence of protective measures considering a travel speed of 50 km/h, a vehicle length of 4 m and a traffic of 25 vehicles/hour. The total risk, i.e. Eqn. (4), is 1.46 x 10^{-4} fatalities per year.

Table 1. Risk calculations without protective measures.

Portion	A	B	C	D	E	F
Length (m)	100	100	100	100	100	100
Reaching probability	1%	30%	10%	5%	20%	1%
Annual rate of events	0.00299	0.08955	0.02985	0.01493	0.05970	0.00299
$\sum(E^{m,k} V^{l,m,k} W^{m,k})$	7.29E-04	7.29E-04	7.29E-04	7.29E-04	7.29E-04	7.29E-04
Addends of Eqn. (4)	2.18E-06	6.53E-05	2.18E-05	1.09E-05	4.35E-05	2.18E-06

The sum of the annual rates of events is 0.2, while the total risk, i.e. Eqn. (4) is R= 1.46 x 10^{-4} fatalities per year. To mitigate the risk, protection barriers on portions B and E are planned. The probabilistic trajectory analyses provided the kinematic parameters reported in the top of Table 2. Here, the 95th percentiles of velocity and height are reported, along with the ratio between 99th and 95th percentiles to identify the shape of the rightmost part of their distributions. The annual failure probability, Eqn. (5), is computed through the approaches previously described from the values of the kinematic parameters. The sum of the annual rates of events is 0.051; as expected there are less events on the road. The total risk, i.e. Eqn. (7) is R= 3.75 x 10^{-5} fatalities per year, reduces.

Table 2. Reliability analyses on a 2000kJ / 5m barrier to protect portions B and E.

Portion	v_{95}	v_{99}/v_{95}	h_{95}	h_{99}/h_{95}	$\nu = N_B^k$	F_k (1yr)	F_h (1yr)	p_f (1yr)
	m/s		m		1/yr	1/yr	1/yr	1/yr
B	20.3	1.10	2.5	1.30	0.08955	4.20E-04	1.30E-05	**0.000433**
E	17.7	1.40	3.0	1.25	0.05970	1.10E-04	1.70E-04	**0.000280**

Table 3. Risk calculations with protective measures.

Portion	A	B	C	D	E	F
Length (m)	100	100	100	100	100	100
Reaching probability	1%	30%	10%	5%	20%	1%
Annual rate of events	0.00299	**0.000433**	0.02985	0.01493	**0.000280**	0.00299
$\sum(E^{m,k} V^{l,m,k} W^{m,k})$	7.29E-04	7.29E-04	7.29E-04	7.29E-04	7.29E-04	7.29E-04
Addends of Eqn. (4)	2.18E-06	6.53E-05	2.18E-05	1.09E-05	4.35E-05	2.18E-06

4 CONCLUSION

Quantitative rockfall risk assessment is fundamental for both public authorities and road infrastructure management bodies to properly select the mitigation measures to adopt and to prioritize the interventions. Nevertheless, to verify the goodness of the selected measures in terms of risk reduction, a procedure allowing post-intervention risk evaluation is essential.

Both quantitative risk assessment and mitigation measures design should account for a volume-frequency relationship of all the possible released and impacting block volumes. Among structural protective measures, rockfall barriers are widely adopted. These structures are generally designed with fixed partial safety factor approach, neither considering volume-frequency relationship nor designing for a specific probability of failure. A new method is proposed coupling a novel time-independent reliability-based method for net fence design and a quantitative risk assessment procedure tailored for infrastructures, both conceived by the Authors. The coupling allows quantifying risk reduction in a considered time period and, possibly, to re-design the net fences to install to obtain a target value for risk. The proposed example of application is reported highlights the capabilities of the method.

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