# Experimental study and damage-mechanism-based modelling of creep behavior of fractured granite

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ABSTRACT: Experimental investigations on creep behavior of fractured granite have been performed under multi-step stresses. The significant stress effect on the creep strain rate is presented. A coupling damage of the initial macroscopic damage of fractured granite and the microscopic damage induced by external load is deduced. A damage-mechanism-based creep model for fractured granite has been proposed by introducing the damage evolution in the constitutive relations. The model parameters are determined by fitting the experimental results of the time-dependent deformation of fractured granite. It is indicated that the creep constitutive model proposed can provide a precise description of the full creep process in fractured granite under different stresses. The Burgers model is indicated to be a special case of the damage-mechanism-based creep model.

*Keywords: Fractured granite, creep model, damage, fractional derivative.*

## 1 INTRODUCTION

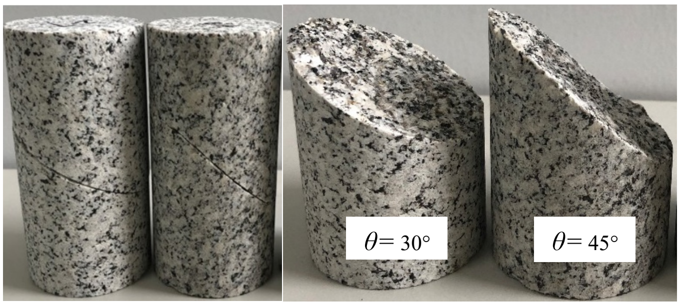
Deep geological disposal is considered to be a feasible and safe option to deal with the high-level radioactive waste (HLW). The deep geological repository (DGR) is consequently considered to be built in many countries. Meanwhile, a number of underground research laboratories (URLs) for demonstrating the long-term safety of DGRs are under construction around the world. In order to ensure the long-term stability and safety of DGRs and URLs, a good understanding and the reliable modeling method for the time-dependent behavior of host rock is essential.

During several decades many efforts have been directed toward the study on creep constitutive model. The component model has been widely studied and applied because of its intuitive concept and clear physical meaning (e.g., reviews by Tomanovic 2006, Sterpi & Gioda 2009, Fahimifar et al. 2015, Aditya et al. 2018). Zhou et al. (2011, 2012) proposed a fractional derivative model by replacing the Newtonian dashpot with the variable-viscosity Abel dashpot in Nishihara model. Based on the Burger's body, Fahimifar et al. (2010) presented a new formulation to determine the wall displacement and convergence of tunnel. Zhang et al. (2019) established a nonlinear creep damage model by introducing the internal variables into the creep model. Cao et al. (2020) gave a unified creep model to describe the typical creep behavior stages for rock under multiple stress levels based on the time-hardening theory and the damage theory. However, most of these improved models are complex with many components and complicated combination forms, or in the form of implicit functions and the explicit expression cannot be derived, which may bring many difficulties to the engineering application. In this paper, the experimental data of fractured granite samples in creep test is first analyzed. On this basis, a new creep model for fractured granite is formulated through introduction of damage variable in the constitutive relations. The model parameters are determined by fitting the experimental curves of fractured granite samples.

## 2 EXPERIMENTAL STUDY ON FRACTURED GRANITE

### *2.1 Preparation of specimens with fractures and testing procedures*

The granite studied in this paper is taken from Beishan area, which is the most potential area for China's HLW repository (Wang 2006). The intact standard samples were first prepared about 100 mm in height and 50 mm in diameter. With the help of a special rock splitting device, the granite samples with fractures were produced after bring the intact samples to separation, as shown in Figure 1. The inclination angles ($\theta$) of the fractures in this study are 30° and 45°.




(a) overall view (b) pre-made fracture surface

Figure 1. Granite samples with fractures (fracture angles: 30° and 45°).

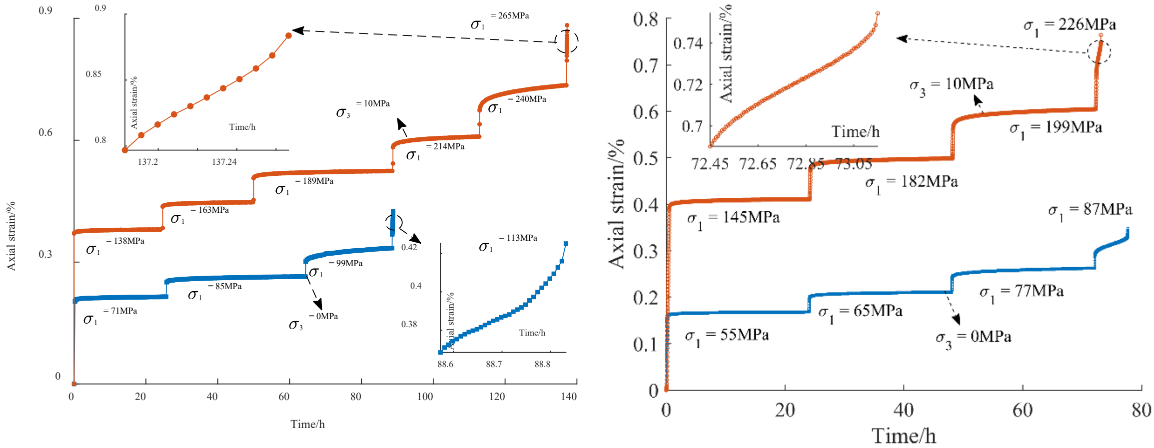
The creep tests of fractured granite were conduct in a multi-step process at confining pressure of 0MPa and 10MPa. The axial stress was applied stepwise at a certain percentage of the peak strength ($\sigma_p$) of the fractured granite, that is about 50%$\sigma_p$, 60%$\sigma_p$, 70%$\sigma_p$, 80%$\sigma_p$, 90%$\sigma_p$ and 100%$\sigma_p$ for each step. The axial stress was keeping constant for about 24h during each step. All tests were performed at room temperature.

### *2.2 Experimental results*

The creep strain curves of fractured granite samples at different confining pressures are presented in Figure 2. The inset is a zoomed-in view of the axial strain curve during last step of creep test. It can be noticed that, when the applied stress is relatively small, the transient creep stage with a decreasing strain rate and steady creep stage with a constant strain rate both can be observed. In the last step of creep test, the creep deformation is entering a rapid increasing phase with the increase of the axial stress and the accumulation of the microcracks inside specimen. In addition to the transient creep stage and steady creep stage, there is an accelerated creep stage with a rapidly increasing strain rate until sample failure. Above all, the typical three stage creep curve can be observed for fractured granite samples.

Figure 3(a) shows the steady-creep strain rate at different axial stresses. With the increase of axial stress, the creep strain rate increases significantly. The last stress level results in creep strain rate improvements of roughly two orders of magnitude. Failure 3(b) gives the photos of the half section of the fractured granite sample after creep test. It can be noticed that, under uniaxial compression, the broken degree of the specimen with 30° inclination fracture is higher than that of specimen with

45° inclination fracture. The broken degree of fractured specimen at confinement of 10MPa is lower than that without confinement.




(a) fracture angle $\theta$: 30° (b) fracture angle $\theta$: 45°

Figure 2. Creep strain curves of fractured granite samples at different confining pressures.

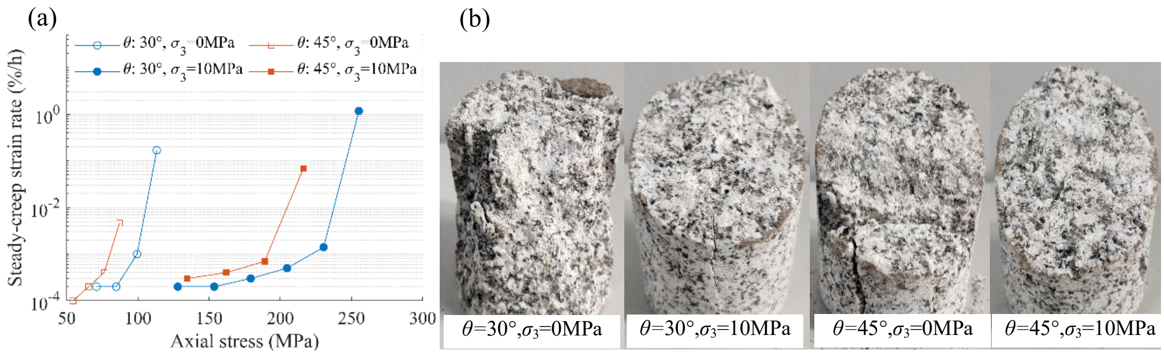



Figure 3. (a) Steady-creep strain rate variation with axial stress. (b) Half section of the failure specimen.

## 3 DAMAGE-MECHANISM-BASED MODEL OF FRACTURED GRANITE

### 3.1 *Establishment and solution of the creep constitutive model*

The damage-mechanism-based model of fractured granite is composed of a Hooke element, a fractional derivative viscoelastic element and a damaged elastic-visco-plastic body (Figure 4), with strains $\varepsilon_e$, $\varepsilon_{ve}$ and $\varepsilon_{evp}$, respectively. The total strain is given by

$$\varepsilon = \varepsilon_e + \varepsilon_{ve} + \varepsilon_{evp} \tag{1}$$

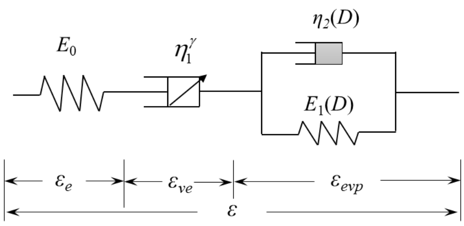



Figure 4. Schematic view of the damage-mechanism-based model.

The constitutive equation of the Hooke element is given by

$$\varepsilon_e = \sigma / E_0 \tag{2}$$

where $\sigma$ is the constant stress, $E_0$ is the elastic modulus.

The constitutive equation of the fractional derivative viscoelastic element is given by (Zhou et al.2012)

$$\varepsilon_{ve} = \frac{\sigma t^\gamma}{\eta_1^\gamma \Gamma(1+\gamma)} \tag{3}$$

where $\eta_1^\gamma$ is the fractional viscosity coefficient, $\gamma$ is the fractional derivative order with values between 0 and 1, $t$ is the creep time.

As shown in Figure 4, the damaged elastic-visco-plastic body is formed by connecting a damaged Newtonian dashpot in parallel with a damaged spring, in which the elastic modulus $E_1(D)$ and viscosity coefficient $\eta_2(D)$ are no longer constant, but are related to the degradation of rock properties induced by accumulation of damage $D$:

$$\begin{cases} E_1(D) = (1-D)E_1 \\ \eta_2(D) = (1-D)\eta_2 \end{cases} \tag{4}$$

In Eq. (4), the total damage $D$ of fractured rock simple during creep test is a coupling damage of the initial macroscopic damage induced by the pre-made fracture and the microscopic damage of rock material induced by external load. The damage of rock material in the creep process is usually assumed to be a negative exponential function over time. Based on the Lemaitre equivalent strain principle, the total damage of fractured rock can be written as

$$D = 1 - \frac{1-D_1}{(1-D_1)\exp(\alpha t) + D_1} \tag{5}$$

where $\alpha$ is the parameter reflecting the damage rate; $D_1$ is the initial macroscopic damage, which can be defined as the deterioration of elastic modulus. According to the results of uniaxial compression test conducted on fractured granite, the initial damages are 0.06 and 0.23 for granite samples with 30 or 45-degree inclination fractures respectively.

Based on the combination mode of the damaged elastic-visco-plastic body, the constitutive relation can be deduced:

$$\sigma = E_1(D)\varepsilon_{evp} + \eta_2(D)\dot{\varepsilon_{evp}} \tag{6}$$

By substituting Eq. (4), Eq. (5) into Eq. (6), the strain formula of the damage elastic-visco-plastic body can be obtained:

$$\varepsilon_{evp} = \frac{\sigma}{E_1+\alpha\eta_2} e^{\alpha t} - \left[\frac{\sigma}{E_1+\alpha\eta_2} + \frac{\sigma}{E_1}\frac{D_1}{1-D_1}\right] e^{-\frac{E_1}{\eta_2}t} + \frac{\sigma}{E_1}\frac{D_1}{1-D_1} \tag{7}$$

Combining the three strains $\varepsilon_e$, $\varepsilon_{ve}$ and $\varepsilon_{evp}$, the constitutive relation of the damage-mechanism-based model of fractured granite is given by

$$\varepsilon(t) = \frac{\sigma}{E_0} + \frac{\sigma t^\gamma}{\eta_1^\gamma \Gamma(1+\gamma)} + \frac{\sigma}{E_1+\alpha\eta_2} e^{\alpha t} - \left[\frac{\sigma}{E_1+\alpha\eta_2} + \frac{\sigma}{E_1}\frac{D_1}{1-D_1}\right] e^{-\frac{E_1}{\eta_2}t} + \frac{\sigma}{E_1}\frac{D_1}{1-D_1} \tag{8}$$

where $\sigma$ is the constant stress; $E_0$, $E_1$, $\eta_1^\gamma$ and $\eta_2$ are the model parameters related to the rock material; $\gamma$ is the fractional derivative; $\alpha$ is the damage factor; $D_1$ is the initial damage of fractured rock.

Under three-dimensional stress condition, the total strain ($\varepsilon_{ij}$) of the damage-mechanism-based model can be expressed in the form of tensor:

$$\varepsilon_{ij} = \frac{1}{2G_0} S_{ij} + \frac{1}{3K_0} \delta_{ij} \sigma_m + \frac{S_{ij}}{2\eta_1^{\gamma}} \frac{t^{\gamma}}{\Gamma(1+\gamma)} + \frac{S_{ij}}{2G_1+2\alpha\eta_2} e^{\alpha t} - \left[ \frac{S_{ij}}{2G_1+2\alpha\eta_2} + \frac{S_{ij}}{2G_1} \frac{D_1}{1-D_1} \right] e^{-\frac{G_1}{\eta_2} t} + \frac{S_{ij}}{2G_1} \frac{D_1}{1-D_1} \quad (9)$$

where $S_{ij}$, $\sigma_m$ and $\delta_{ij}$ are the deviatoric stress tensor, spherical strain tensor and Kronecker function respectively; $G_0$ and $G_1$ are the shear moduli; $K_0$ the is bulk modulus.

In conventional triaxial compression test, the confining pressure $\sigma_2$ is equal to $\sigma_3$, the axial strain of the damage-mechanism-based model is obtained as:

$$\varepsilon(t) = \frac{\sigma_1-\sigma_3}{3G_0} + \frac{\sigma_1+2\sigma_3}{9K_0} + \frac{\sigma_1-\sigma_3}{3G_1+3\alpha\eta_2} e^{\alpha t} + \frac{\sigma_1-\sigma_3}{3G_1} \frac{D_1}{1-D_1} - \left[ \frac{\sigma_1-\sigma_3}{3G_1+3\alpha\eta_2} + \frac{\sigma_1-\sigma_3}{3G_1} \frac{D_1}{1-D_1} \right] e^{-\frac{G_1}{\eta_2} t} + \frac{\sigma_1-\sigma_3}{3\eta_1^{\gamma}} \frac{t^{\gamma}}{\Gamma(1+\gamma)} \quad (10)$$

The relationship between shear moduli $G_0$, bulk modulus $K_0$ and elastic modulus $E_0$ is

$$G_0 = \frac{E_0}{2(1+\mu)}, K_0 = \frac{E_0}{3(1-2\mu)} \quad (11)$$

Substituting $D_1$=0, $\alpha$=0 and $\gamma$=1 into Eq. (8) and Eq. (10), that means taking no account of damage influence and without using the theory of fractional calculus, the proposed model can be written as

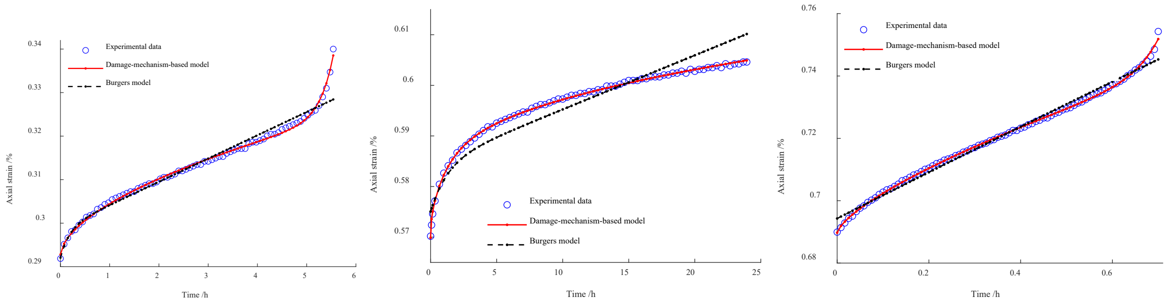
$$\varepsilon(t) = \frac{\sigma}{E_0} + \frac{\sigma}{\eta_1} t + \frac{\sigma}{E_1} (1 - e^{-\frac{E_1}{\eta_2} t}) \quad (12)$$

$$\varepsilon(t) = \frac{\sigma_1-\sigma_3}{3G_0} + \frac{\sigma_1+2\sigma_3}{9K_0} + \frac{\sigma_1-\sigma_3}{3\eta_1} t + \frac{\sigma_1-\sigma_3}{3G_1} (1 - e^{-\frac{G_1}{\eta_2} t}) \quad (13)$$

Eq. (12) and Eq. (13) are just the expression of the constitutive relation of the classical Burgers model.

## *3.2 Parameter determination and model verification*

In order to verify the rationality and adaptability of the damage-mechanism-based model proposed in this paper, the experimental data of fractured granite samples at different confining pressures were fitted. Taking granite samples with 45-degree inclination fractures for example, the strain curves fitted by the damage-mechanism-based model and Burgers model are presented in Figure 5. The model parameters are given in Table 1. When $\sigma_3$=10MPa, elastic modulus $E_0$ can be calculated by Eq. (11). It is found that the damage-mechanism-based model is capable of describing the full creep regions of fractured granite, i.e., the transient creep stage, the steady creep stage and the accelerated creep stage. Compared with the classical Burgers model, the proposed model can better reproduce the nonlinear creep characteristics by introducing the damage variable into the creep constitutive model. The samples with 30-degree inclination fractures were also fitted, and the fitting result was similar to that of the samples with 45-degree inclination fractures.




(a) $\sigma_3$ = 0MPa, $\sigma_1$ = 87.20 MPa (b) $\sigma_3$ = 10MPa, $\sigma_1$ = 199.30 MPa (c) $\sigma_3$ =10MPa, $\sigma_1$ = 226.20 MPa

Figure 5. Experimental data and fitting curves of granite samples with 45-degree inclination fractures.

Table 1. Parameters of the damage-mechanism-based model of fractured granite (fracture angle: 45°).

| $\sigma_3$ [MPa] | $\sigma_1$ [MPa] | $E_0$ [GPa] | $E_1$ ($G_1$) [GPa] | $\eta_1^\gamma$ [GPa·h$^\gamma$] | $\eta_2$ [GPa·h] | $\gamma$ | $\alpha$ [h$^{-1}$] |
|---|---|---|---|---|---|---|---|
| 0 | 87.20 | 29.80 | 824.49 | 3.03E+11 | 8.54E+10 | 0.567 | 3.748 |
| 10 | 199.30 | 33.75 | 1.14E+03 | 824.37 | 1.51E+03 | 0.368 | 0.002 |
| 10 | 226.20 | 31.77 | 121.20 | 3.71E+08 | 7.94E+06 | 0.721 | 19.473 |

# 4 CONCLUSIONS

In this study, creep tests of fractured granite under multi-step stresses have been performed to characterize the time-dependent behavior of fractured granite. Experimental results indicate that the creep deformation of fractured granite exhibits typical three stages, i.e., transient creep stage, steady creep stage and accelerated creep stage. The creep strain rate has presented a rapid growth tendency along with the increase of axial stress. On this basis, a damage-mechanism-based creep model for fractured granite has been proposed through introduction of damage evolution in the constitutive relations. The parameter determination and model verification have been performed by fitting the experimental results of the time-dependent deformation of fractured granite. The fitting results show that the creep model proposed in this paper is capable of describing the full creep process in fractured granite under different stresses.

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