Simple and multiple regression study of physical-mechanical properties of ignimbrites

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ABSTRACT: The objective of this study is to analyse the relationship between uniaxial compressive strength (UCS) with physical properties, such as apparent density (AD) and ultrasound velocity (PWV), and with other mechanical properties, such as point load test (PLT) and indirect tensile strength (BTS). This analysis was carried out on a group of ignimbrites, consisting of a total of 45 samples, from the Canary Islands and consisted of applying simple regression and multiple regression methods. Applying the simple regression method, it was determined that the ultrasound velocity (PWV) is the property that explains the variability of the compressive strength (UCS) to the greatest extent. Using the multiple regression method, a model capable of increasing the proportion of variability of the (UCS) is obtained, using the apparent density (AD), the ultrasound velocity (PWV) and the point load test (PLT) as independent variables.

Keywords: Canary Island, ignimbrite, boreholes, geomechanical properties, regression.

1 INTRODUCTION

Although all properties are important and necessary for a correct physical-mechanical characterisation of the rock mass, the uniaxial compressive strength (UCS) is one of the most relevant properties when estimating its load bearing capacity. This parameter is obtained in the laboratory by the unconfined compression test performed according to the procedure described in the corresponding standard. Usually, is one of the most expensive test on rocks, due to the characteristics of the equipment needed and the time required to prepare the samples, as well as being a destructive technique that makes it impossible to reuse the specimen after testing. This has generated a lot of research, in which the authors propose different indirect methods for estimating UCS and whose predictive functions are derived from simpler tests.

Most commonly, in this type of research, several properties are included in order to determine the best relationship for predicting UCS among all of them. In other studies, in addition to the simple regression method the multivariate regression method is applied in order to estimate UCS using two or more properties as independent variables. The interest in this topic is well demonstrated in the work of Aladejare et al. (2021) a magnificent database which compiles the information available in the literature on the estimation of UCS. The review covers more than 250 references in which authors apply simple and/or multiple regression methods. However, it should be noted that the vast majority of this kind of studies have been carried out on sedimentary or metamorphic rocks, with very few literature on igneous rocks, especially on those of volcanic origin.

2 MATERIAL

From the point of view of geology and volcanology, the Canary Islands is one of the most interesting volcanic regions on the planet, as practically all possible volcanic processes have taken place there, where different types of volcanic structures and a great lithological diversity can be found. According to Hernández, L.E. (2014), the volcanic rocks of the Canary Islands can be classified, according to lithological criteria, into four main groups: basalts, trachytes, phonolites and ignimbrites.

The rocks used in this study belong to the ignimbrite group, which includes both welded and unwelded ignimbrites. Samples were obtained by core drilling or block sampling. The cores samples were obtained by the traditional system of rotary drilling with continuous extraction of rock cores, with drilling diameters of 76, 86 and 101 mm. The blocks sampling consisted on obtaining a single mass, approximately 50 kg in weight, from which the cylindrical specimens were subsequently extracted. A total of 45 samples were analyzed, 10 coming from the island of Tenerife and 35 from the island of Gran Canaria. The points in Figure 1 show the location from where the samples were taken.

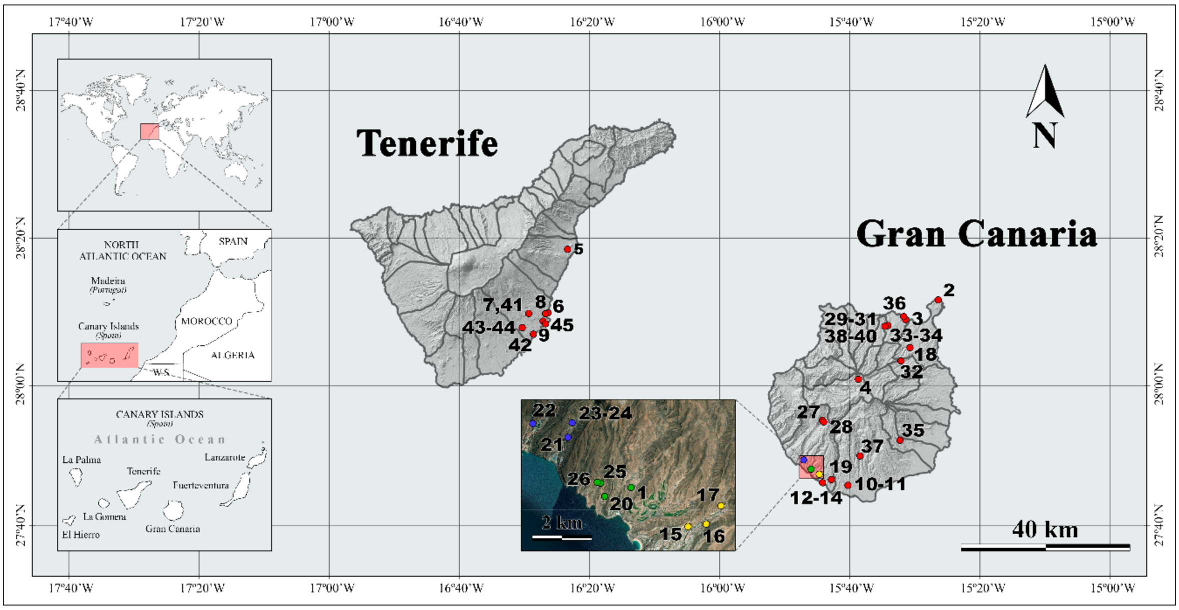

Figure 1. Geographical distribution of the samples.

The lithology of this type of volcanic rocks is typical of a breccia rock with eutaxitic or flow texture, marked by the orientation of the vitreous fragments in the form of small flames (1 - 3 cm). It contains dispersed angular fragments of trachytic or phonolytic composition (1 - 5 cm) and abundant feldspar crystals. The ash matrix can present a wide variety of colours. In outcrops they appear in powerful packages of massive rock with few fractures (Hernández, L.E. & Santamarta, J.C. 2015)

Projecting the contents of alkaline minerals (Na_2O y K_2O) and silicates (SiO_2), obtained by chemical analysis, onto a TAS diagram (Bas et al. 1986), it is concluded that most of the ignimbrites in this study have a trachyte and trachydacite composition.

3 METHODS

The number of specimens available for each test was determined by the recovery capacity of the core samples and the dimensions and geometry of the block, obtaining the maximum number possible for each sample. The specimens were dried to constant mass before testing (weighting each 24 hours until less than ± 0.1 % variation in mass), in accordance with the procedure described in the standards.

3.1 Apparent density (AD)

The apparent density was determined according to UNE - EN 1936 standard (AENOR 2007), as the ratio between the mass of the dry specimen and its apparent volume. Alternatively, the apparent volume was based on the dimensions of the specimen. Cylindrical specimens of dimensions 50 × 100 mm (diameter × height) were tested.

3.2 Ultrasound velocity (PWV)

The ultrasonic velocity was determined using ASTM D2845-08 standard (ASTM 2008). A wave transmitter is placed at one end of the specimen and a receiver at the other end. The time it takes, in this case the P-waves, to travel from the transmitter to the receiver is measured. PUNDIT plus (Portable Ultrasonic Non-destructive Digital Indicating Tester) of CNS Farnell, with transducers of nominal frequency 54 kHz, was used. Cylindrical specimens of dimensions 50 × 100 mm (diameter × height) were tested.

3.3 Diametral test (Point Load test, PLT)

The point load resistance was determined according to UNE 22950-5:1996 standard (AENOR 1996). The specimens are subjected to a concentrated load using a pair of spherically truncated conical punches until breakage occurs. Core fragments with a length/diameter ratio equal to the unit were tested (50 × 50 mm).

3.4 Indirect tensile strength (Brazilian tensile strength, BTS)

The indirect tensile strength test was carried out in accordance with the UNE 22950-2 standard (AENOR 1990b). The specimens are placed between two jaws so that the axes of rotation of both (specimen and jaws) coincide and an axial load is applied until breakage occurs. Cylindrical specimens, 50 mm in diameter and 25 mm thick (disc type) were used.

3.5 Uniaxial compressive strength (UCS)

This test as defined by the UNE 22950-1:1990 standard (AENOR 1990a) consists on subjecting the rock cores to compression without confinement. The load is applied uniformly distributed over the faces of the specimens, increasing continuously until failure occurs. Cylindrical specimens of dimensions 50 × 125 mm (diameter × height) were tested.

4 RESULTS AND DISCUSSIONS

The results and statistical parameters obtained corresponding to the pooled values of the 45 samples are shown in Table 1. The AD values ranged from 1.14 to 2.91 g/cm^3, with a mean value of 2.07 g/cm^3, while the mean PWV value was 3482.5 m/s with a variation between 2135.94 and 5864.47 m/s. The dispersion of these two properties is close to 20 %, which is relatively low, compared to the rest of the properties that exceed 40 % (even UCS reaches 70 %). PLT values fluctuate between 0.59 and 7.81 MPa, with a mean value of 3.13 MPa. The BTS has a mean value of 20.73 MPa, in a range between 1.11 and 41.63 MPa and the UCS, with a minimum and maximum of 4.66 and 132.68 MPa, respectively, a mean value of 40.36 MPa.

Table 1. Results and basic descriptive statistics of the properties.

		Nº	Min.	Max.	Mean	Std. deviation	CV
AD	[g/cm^3]	45 (110)	1.14	2.91	2.07	0.36	0.17
PWV	[m/s]	45 (45)	2135.94	5864.47	3482.51	767.95	0.22
PLT	[MPa]	45 (396)	0.59	7.81	3.13	1.63	0.59
BTS	[MPa]	45 (172)	1.11	41.63	20.73	9.19	0.44
UCS	[MPa]	45 (110)	4.66	132.68	40.36	27.96	0.69
( )	values in brackets represent the number of measurements made						

4.1 Simple Regression Method (SRM)

Table 2 shows the results obtained by applying SRM. With a coefficient of determination ($R^2 = 0.77$), PWV property, used as independent variable, that manages to explain with a higher proportion the variation of UCS. In fact, given the simplicity and speed of the test, PWV is one of the most frequently used properties as a predictor variable in the estimation of UCS. For example, Yagiz (2011) obtains in his study of different rock types (sedimentary and metamorphic) from southwestern Turkey an ($R^2 = 0.89$) and Azimian (2017) in his investigation of a group of sedimentary rocks (limestone) from Iran ($R^2 = 0.87$). Figure 2 shows the linear relationship between the UCS and the other properties, the blue region represent the confidence regions, with a probability range of 95 %.

Table 2. Results of SRM analysis.

No.	Dependent parameter		Independent parameter		Equation	Coefficient of determination (R^2)
1	UCS	[MPa]	AD	[g/cm^3]	UCS = -88.35 + 62.07 (AD)	0.63
2	UCS	[MPa]	PWV	[m/s]	UCS = -70.76 + 0.032 (PWV)	0.77
3	UCS	[MPa]	PLT	[MPa]	UCS = -4.37 + 14.30 (PLT)	0.69
4	UCS	[MPa]	BTS	[MPa]	UCS = -9.5 + 2.39 (BTS)	0.61

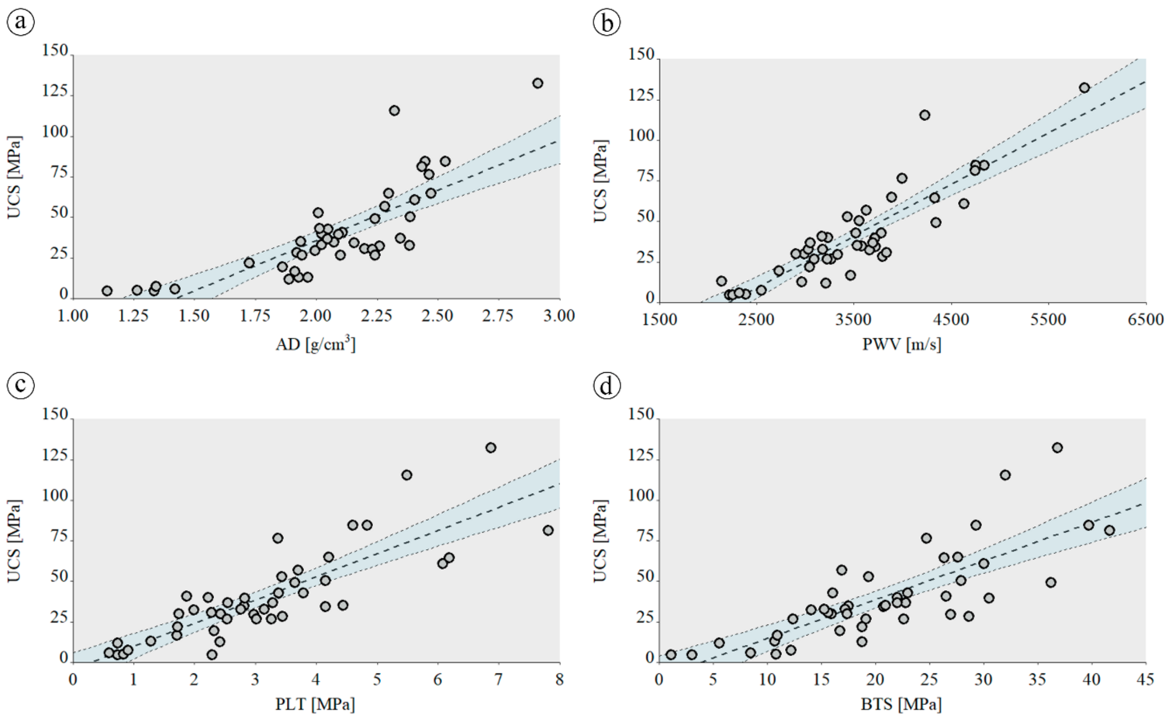

Figure 2. Relationships between UCS and (a) AD, (b) PWV, (c) PLT and (d) BTS.

4.2 *Multiple Regression Method (MRM)*

The number of properties that can be used as independent variables (AD, PWV, PLT and BTS) allows for 15 possible combinations (4 correspond to the SRM). Table 3 shows the four models with the highest coefficients of determination. In most models where BTS was an independent variable, it was not statistically significant (p-value ≥ 0.05), so these models were omitted. On the other hand, variance inflation factor (VIF) has been taken into account to quantify multicollinearity. The model formed by the variables AD, PWV and PLT is the one that manages to explain the variability of UCS with the highest proportion ($R^2 = 0.82$), followed by the one formed by PWV and PLT, with an $R^2 = 0.80$. Using these same parameters (PWV and PLT), Çobanoğlu & Çelik (2008) obtain a similar result ($R^2 = 0.85$) for sedimentary rocks (sandstone and limestone) from Turkey, as does Diamantis et al. (2009) ($R^2 = 0.88$), although in this case, their research focuses on metamorphic rocks (serpentine) from Greece. Figure 3 plots the measured values against the predicted values using the different models selected.

Table 3. Results of MRM analysis.

No.	Dependent parameter	Independent parameter	Equation	R^2
1	UCS	AD PWV PLT	UCS = -69.79 + 18.18 (AD) + 0.016 (PWV) + 5.00 (PLT)	0.82
2	UCS	PWV PLT	UCS = -53.72 + 0.022 (PWV) + 5.48 (PLT)	0.80
3	UCS	AD PWV	UCS = -87.03 + 20.31 (AD) + 0.025 (PWV)	0.79
4	UCS	AD PLT	UCS = -55.02 + 31.78 (AD) + 9.42 (PLT)	0.78

UCS [MPa], AD [g/cm^3], PWV [m/s], PLT [MPa], BTS [MPa]

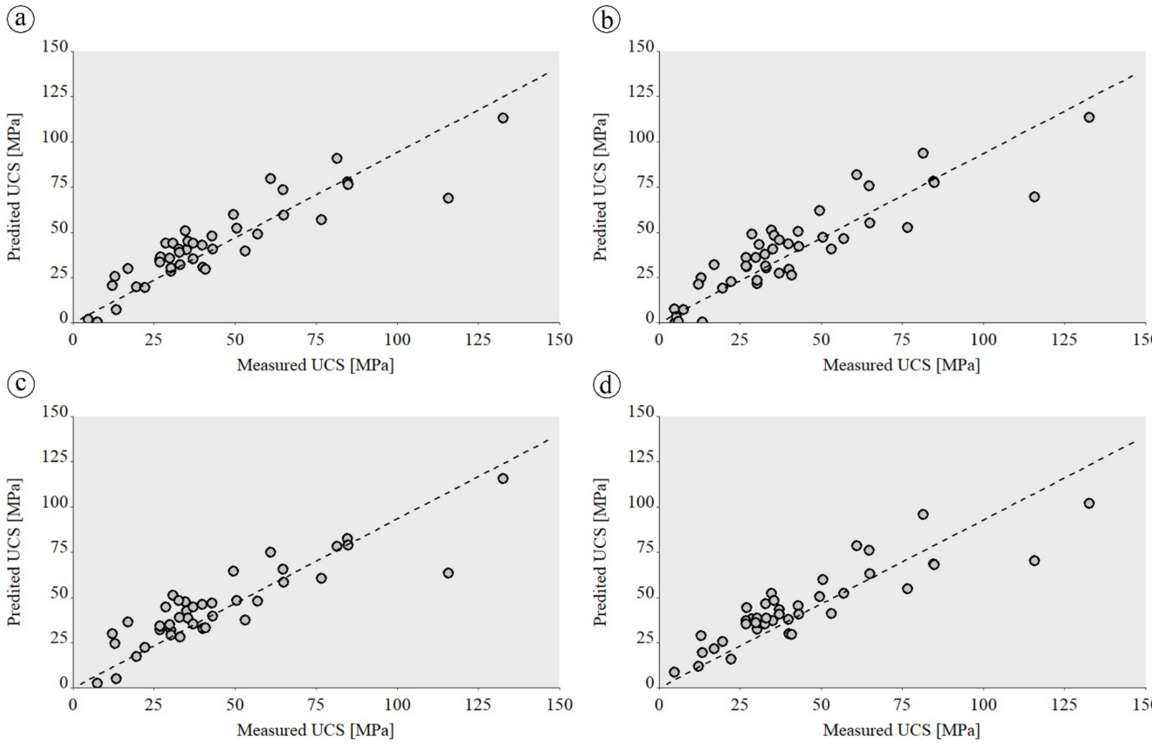

Figure 3. Predicted UCS versus measured UCS (a) Model 1, (b) Model 2, (c) Model 3 and (d) Model 4.

5 CONCLUSIONS

Applying the simple regression method (SRM) it has been obtained that the ultrasonic velocity (PWV) is the property with the highest linear correlation with the uniaxial compressive strength (UCS). The coefficient of determination obtained (R^2 = 0.77) is somewhat lower than would be expected from the literature. However, despite the high dispersion of the data, the fit is able to explain practically 80 % of the variability of UCS. Using the multiple regression method (MRM), the best model obtained was the one that used the following independent variables: apparent density (AD), ultrasound velocity (PWV) and point load test (PLT). Taking into account the difference in proportion explained, between the simple regression method (R^2 = 0.77) and the multiple regression method (R^2 = 0.82), which is 5 %, it would be necessary to evaluate the need to determine all the properties that constitute the multiple regression model or it would be sufficient to obtain the ultrasound velocity (PWV) and apply the simple regression method (SRM) to estimate the uniaxial compressive strength (UCS) of the ignimbrites.

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