A fast and novel hybrid technique for simulation of fluid flow in large-scale fractured rock masses using streamline method and modified Bresenham line algorithm

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ABSTRACT: The most popular numerical method for simulating fluid flow in fractured rock masses is distinct element method (DEM), which suffers from being numerically expensive. Streamline simulation (SL) is a fast method for homogeneous porous media. In this research, a MATLAB code (IUT-SL) is developed to study the flow behavior in a fractured porous media using SL. Using a Finite Volume discretization (FVM) and Discrete Fracture Network (DFN) to simulate rock matrix and fracture network, Bresenham line algorithm is used to embed the DFN into FVM cells. Pressure equations are solved with a two-point flux approximation (TPFA) scheme and Streamlines were traced in the medium via Pollock's semi-analytical method along with Time of Flight (ToF). The results show that FVM-DFN-SL hybrid technique can be used as an efficient alternative for DEM approaches, which not only can preserve the accuracy of DEM methods, but also has orders of magnitude faster runtime.

Keywords: streamline, Finite Volume, Discrete Fracture Network, Bresenham line algorithm.

1 INTRODUCTION

From small fissures to multi-kilometer faults, discontinuities exist in different sizes and shapes in rock masses. Fluid flow in discontinuous and fractured porous rock masses plays a significant role in many problems regarding earth projects such as geothermal energy reservoirs, underground hazardous waste disposal sites and oil and gas reservoirs.

With significant advances in computing capacities over the years, numerical methods were used frequently by the authors. Namdari et al. (2012) conducted a comparative study between a model without matrix permeability and a model with a very low-permeability matrix. It was shown that matrix permeability even in low magnitudes can change the whole flow regime and overall permeability of a medium. Siavashi et al. (2014) developed a 3D Streamline-based simulation of two-phase flow in heterogeneous porous media. The speed of this method was reported to be much higher than other peer methods in heterogeneous media simulation.

Wang et al. (2020) performed a tracer test and Streamline simulation for geothermal resources in Cuona of Tibet to measure the impact of injection wells in production of the sink wells in different well patterns of this geothermal reservoir. Streamline simulation of experimental studies on waterflooding under given pressure boundaries was conducted as a coupled Hydro-Mechanical (HM) process by Zhang (Zhang et al., 2021). With the help of streamline simulation, the capillary effects at the fluid front were quantified as a function of time and coordinates.

In this study, the efficient discretization of continuum methods, accuracy and detailed presentation of discrete methods and speed and visual representation of streamline simulation method has been combined in this study to construct the novel hybrid FVM-DFN-SL technique to form a discrete cell-based method. Discrete fractures are embedded through the matrix using modified Bresenham line algorithm. Pressure and flow rate equations are solved through FVM and tracing process of streamlines is carried out using Pollock's method afterwards. The distribution of Time of Flight (ToF) through the model is then obtained. Resulting from this, this hybrid technique provides additional information such as Streamlines and ToF distribution throughout whole model as against discrete methods. Streamlines and Time of Flight (ToF) contours will provide a more precise and detailed vision to hydraulic behavior of a fractured model, which are missing in discrete methods. Moreover, the simulation runtime will not increase even for large number of fractures, paradoxical to discrete methods.

2 MATERIALS AND METHODS

Firstly, the theory of hybrid FVM-DFN-SL technique is discussed. Then Bresenham line algorithm and Pollock's method for tracing process of streamlines and flow chart of the developed codes are represented.

2.1 Hybrid FVM-DFN-SL technique

The hybrid FVM-DFN-SL technique uses Two-Point Flux Approximation (TPFA) scheme from Finite Volume Method (FVM) to construct the basic discretization, applies Discrete Fracture Network (DFN) to embed the fractures into the initial model using Bresenham line algorithm and tracks the streamlines (SL) using Pollock's semi-analytical method.

2.1.1 Finite Volume Method (FVM) and Two-Point Flux Approximation (TPFA)

The key idea of Finite volume Method is to mathematically transform the integrals on the volume into partial differential equations for a finite volume. Flow rates and fluxes at the surfaces of each element (also known as Finite Volume) will be obtained using this method. Due to the clarity of physical interpretation of FVM, it acts very powerful compare to FEM and FDM in the context of fluid flow. From the Darcy's law in its integral form, it can be written (Wei et al. 2015):

$$v_{i,k} \approx A_{i,k}\vec{v}(\vec{x}_{i,k}) \cdot \vec{n}_{i,k} = -A_{i,k}(K\nabla p)(\vec{x}_{i,k}) \cdot \vec{n}_{i,k} \tag{1}$$

Where $\vec{x}_{i,k}$ is the centroid on $\Gamma_{i,k}$ and $v_{i,k}$ is the flux between two neighboring i and k elements (Jonsthovel & Stone 2019). A schematic illustration of TPFA discretization is represented in Figure 1 for two adjacent cells i and k in a Cartesian coordinate.

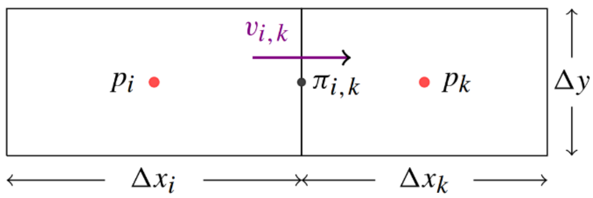

Figure 1. Two neighbouring cells used to derive the TPFA discretization for a 2D Cartesian grid (Lie 2019).

2.1.2 Discrete Fracture Network and Bresenham line algorithm

The DFN method is a special discrete approach that considers fluid flow and transport processes in fractured rock masses through a system of connected fractures (Habibi et al. 2014). As it was stated in the previous section, the TPFL discretization generates the main model with Cartesian elements. Embedding the discrete fractures into this Cartesian discretization is carried out using a graphical method called Bresenham line algorithm. Using this algorithm, a line representing a fracture can be decomposed to consecutive elements with desired size (Koopman 1987 and Kennedy 2012). Figure 2 represents this embedding procedure.

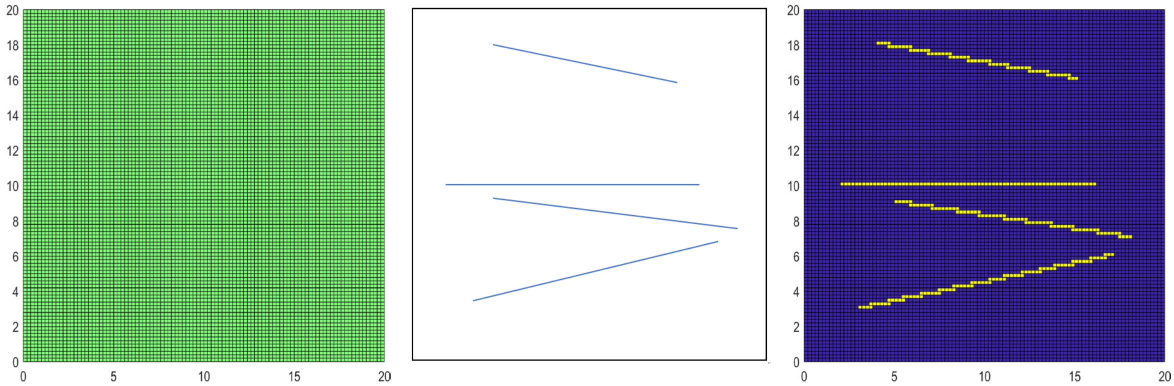

Figure 2. The schematic illustration of modified Bresenham line algorithm. A basic TPFL Cartesian discretization (left), the discrete fractures (middle), and (c) the resulting discretization with fractures introduced as consecutive elements.

2.1.3 Streamline-based flow simulation in FVM-DFN-SL approach

Streamline is a curve that is tangential to the velocity field of a flow. In can be defined as a snapshot of the path of fluid particles in a desired time. Due to the fact that each particle of the fluid cannot go in more than one direction and velocity of each point in the model is represented with a single vector, streamlines do not cross each other. Tracing the streamlines can be managed by Pollock's Method.

In Pollock's method, the total inflow and outflow for each element is calculated using Darcy's Law (Thiele & Batycky 2006). When the fluxes are known, the scheme calculate the exit point coordinates of the Streamline and after that, it computes the amount of time that it takes for a particle to travel along s Streamline from the inlet point to the exit based on velocity of the particle traveling on that streamline (Figure 3). The velocity in each direction (x or y) is calculated via a piece-wise linear approximation (Batista 2020).

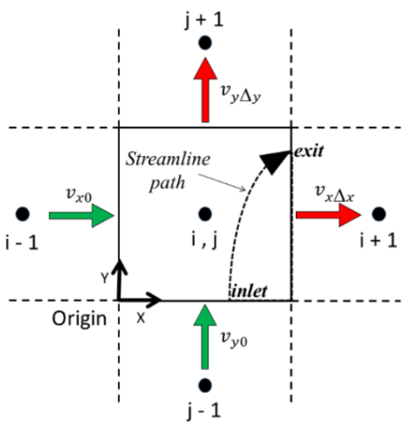

Figure 3. Pollock method for tracing streamlines in a Cartesian grid.

After rewriting the laws for kinematics and some mathematical manipulation, it can be written:

$$x_e = \frac{1}{g_x}\ln[v_{xi}exp(g_x\Delta t_m) - v_{x0}], y_e = \frac{1}{g_y}\ln[v_{yi}exp(g_y\Delta t_m) - v_{y0}] \qquad (2)$$

Where Δt_m is the minimum travel time from inlet to outlet in x or y direction, g is the gravitational acceleration and x_e and y_e are the coordination for the exit point for a known inlet point (Chen et al. 2020).

3 RESULTS

In this section, the initial model setup and hydraulic properties are represented and the results for a natural fracture pattern is discussed afterwards.

3.1 Model setup

In this study, horizontal 2D square model with 20×20 m^2 dimensions, consists of low permeability limestone with 1 mD (millidarcies) permeability and 10% porosity is simulated using IUT-SL MATLAB code. The geometry of the model is presented in Figure 4.

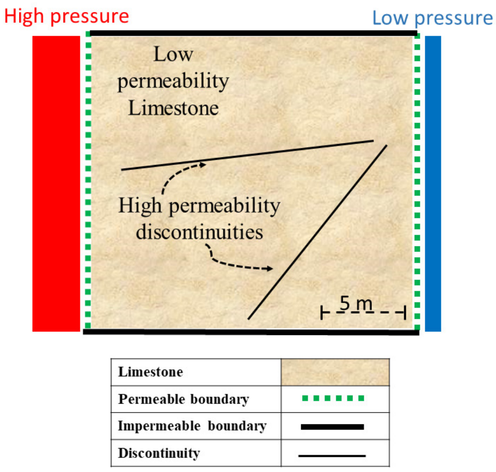

Figure 4. Hydraulic boundary conditions with impermeable top and bottom boundaries.

With this brief introduction for the model setup, the results for a natural fracture pattern is presented in the next section.

3.2 Results of IUT-SL code for complex natural fracture geometries

As it was mentioned earlier in this paper, the study area is a mineral processing facility adjacent to a copper mine in central of Iran. The geometry of the model is presented in Figure 5. Note that the discontinuities with the length less than 2 meters are omitted. Total number of 100 discontinuities are simulated in a 20×20 m^2 square model. The high-pressure and low-pressure boundaries are chosen to be identical with calibration model in Figure 4 with the same procedure.

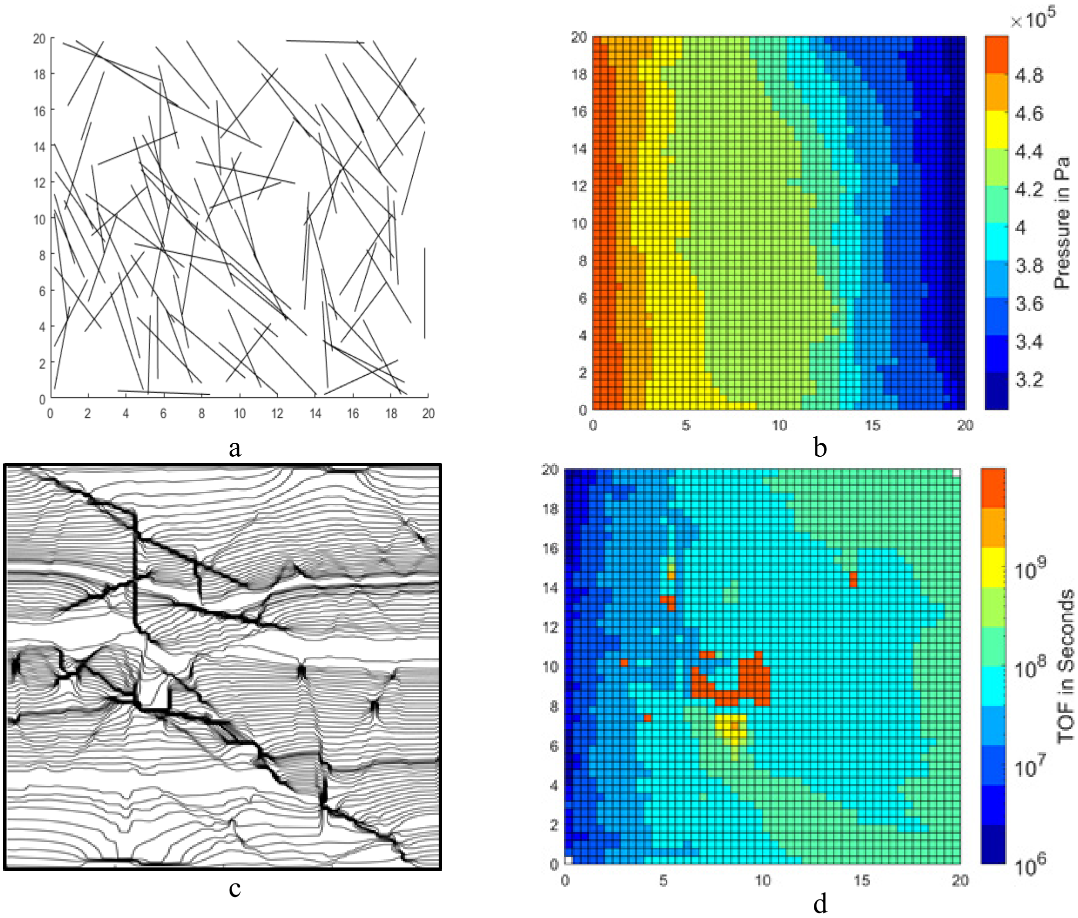

Figure 5. Simulation results of IUT-SL code for 100 fractures complex fracture geometry. (a) geometry of the model, (b) pressure distribution in Pa, (c) Streamlines and (d) ToF in seconds.

The pressure distribution plot is demonstrated in Figure 5.b. The pressure-drop, is not uniform due to the presence of discrete fractures. In the middle of the contour plot for pressure, the pressure gradient is low (the width of light-green color in the center of the model is larger compared to other regions of the model). This can be explained by a glance at Figure 5.c, where streamlines form a conduit for fluid flow, which facilitate the flow and lower the pressure gradient.

The pattern of streamlines is presented in Figure 5.c, shows that there are 4 main regions with dense streamline concentration. It is demonstrated by this figure that from 100 fractures, only a handful of fractures are dominant channels for flow. The high-concentrated paths for streamlines are often followed by sparse regions in the neighboring area. The dominant trend for flow channels is from top-left of the model to bottom-right in the diagonal direction of the square. It is shown that in general, the connected fractures have formed the main flowing paths, which is in agreement with pressure contour plot.

Time of Flight (ToF) contours are presented in logarithmic scale in Figure 5.d as an auxiliary tool to interpret the flow regime. According to the color scale in this plot, blue color denotes the short magnitudes for ToF while red color represents the long time for the fluid particles to travel to intended point. It can be observed that very high ToF are in the vicinity of the regions with highly populated streamlines, which denotes that stagnant regions are formed around regions with high fluid flow (fractures).

3.3 The simulation speed of IUT-SL code

The simulation time for UDEC and FVM-DFN-SL technique are presented in Table 1. The simulation time is divided into two model construction and simulation stages. Also, the size of elements in both techniques was varied from 0.5 % (smaller elements) to 2% (larger elements) of the size of the model.

Table 1. Comparison between runtime for model geometry construction (T_{const}) and total construction plus simulation runtime in UDEC and IUT-SL for different element size to model size ratios (all time units are in seconds).

Element size/model size	T_{const} (UDEC)	T_{const} (IUT-SL)	$T_{const+sim}$ (UDEC)	$T_{const+sim}$ (IUT-SL)	
2%	25	8.6	2425	91.6	[Sec]
1%	300	9.1	5800	121.1	[Sec]
0.5%	>3000	12.7	>73000	186.7	[Sec]

Table 1 illustrates that the increase in runtime for smaller elements for UDEC is drastic. For instance, for element size of 0.5% of the dimension of the main model, the runtime of UDEC is about 390 times of the runtime for IUT-SL code.

SUMMARY

Streamline simulation (SL) can be a good candidate for cell-based methods such as Finite Volume Method (FVM). Because FVM has been developed for continuous mediums, with the assist of Discrete Fracture Network (DFN) and modified Bresenham line algorithm, accurate outputs can be resulted where in contrast to discrete methods, the hybrid FVM-DFN-SL is not numerically expensive, even for large and highly fractured porous rock masses.

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