

Influence of the specimen slenderness on the direct tensile strength of rocks

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ABSTRACT: Some rock engineering design operations are mainly controlled by the tensile strength and deformability of the rock (i.e., critical span of excavations or hydraulic fracturing). The ISRM Commission on Testing Methods and the ASTM procedures recommend the use of cylindrical rock specimens, with height-to-diameter (H/D) ratio of 2.5 to 3.0 for direct tensile strength (DTS) determination, in a similar way as per UCS testing. Nevertheless, it is unclear how this ratio may affect the tensile strength of a rock when directly determined. In the present work, the authors performed some series of numerical simulations with a 3D distinct element code (3DEC), replicating DTS tests carried out on specimens presenting five H/D ratios (0.5, 1, 1.5, 2, 2.5, and 3). The study is complemented with laboratory DTS tests in granite. The results herein presented indicate that H/D ratios greater than one have almost negligible influence on DTS.

Keywords: direct tensile strength, slenderness, suggested method, 3DEC.

1 INTRODUCTION

The study and understanding of rock strength and deformability represented one of the main topics in the development of rock mechanics as a discipline in the beginning of the 20th century. Although somewhat overlooked, the tensile strength of rock materials attracted the attention of some researchers at mid-century (Obert et al. 1946) leading them to even attempt to standardize some rock-mechanics laboratory tests, particularly, the direct tensile strength test (DTS). The relevance of rock tensile strength was also remarked by some researchers (Hawkes et al. 1973; Hoek, 1964), who recommended its direct determination through DTS tests under controlled conditions.

Some years later, the International Society for Rock Mechanics and Rock Engineering (ISRM, 1978) published the *Suggested Method for Determining Tensile Strength of Rock Materials*, where some guidelines on the determination of direct tensile strength through cylindrical rock core testing are contained. Aspects like the dimensions of the metal caps to be used, specimen geometry, linking system or loading rate are provided. Following the line of the ISRM document, the last version of the ASTM International Standard Test Method for DTS (ASTM, 2020) presents also the main features for this type of test.

An aspect addressed in the two methodologies is the slenderness ratio (H/D) of the rock specimens to be tested, which in both documents is recommended to be within 2.5 to 3.0. Despite being a relatively well-studied parameter for the case of UCS tests, scarce attention has been paid in the literature to the effect of the height-to-diameter ratio on DTS results (Hashiba et al. 2017; Wijk et al. 1978). The existing studies basically coincide on the negligible effect that specimen slenderness may have on DTS.

The present paper explores, through a combination of numerical models carried out with 3DEC and an experimental laboratory program involving DTS tests in granite, the influence of height-to-diameter ratio on results.

2 METHODOLOGY

2.1 Numerical modelling

The influence of the slenderness was first analysed through numerical modelling. In this case, the distinct element method (DEM) coded in 3DEC v5.2 (Itasca Consulting Group, 2019) was resorted to. A crystalline rock (as granite) was simulated by generating a 3D-random Voronoi tessellation (Figure 1). This method creates geometries that replicate those resulting from the growth process of polycrystalline nucleus in brittle rock (Wang et al., 2021). For this purpose, the geometries were generated by the open-source code Neper (Quey & Kasemer, 2022) and were directly imported from 3DEC.

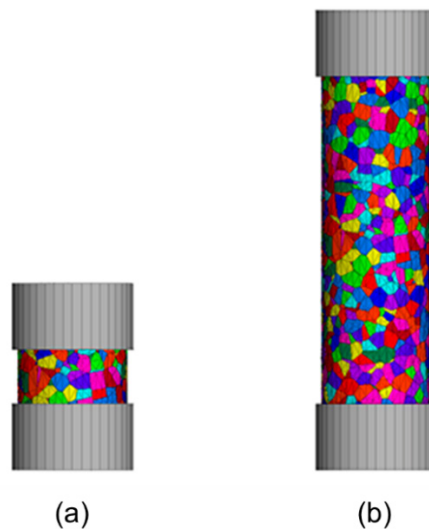


Figure 1. Example of the models tested: (a) model with $D = 54$ mm and $H/D = 0.5$; (b) model with $D = 54$ mm and $H/D = 3$.

A total of 60 DTS-test simulations were performed on samples with different H/D ratios (0.5, 1.0, 1.5, 2.0, 2.5, and 3.0). For each ratio, 10 samples were considered. To ensure the homogeneity among the samples and the randomness of the crystal orientation and shape, all the samples were extracted at a random location from a single cylinder (1.0 m height and 54 mm in diameter) created by a tessellation of 20.000 Voronoi polyhedra. After importing the Neper's geometry into 3DEC, each sample was cut and meshed (Figure 1). Each of the bases were "glued" to a rigid cylinder, intended to replicate the metal caps used in the DTS test. A vertical velocity was imposed to the upper cap, in order to ensure a constant stress generation within the sample, being the lower one fixed. This velocity was maintained until the sample failed (Figure 2). This loading rate was kept for all tests.

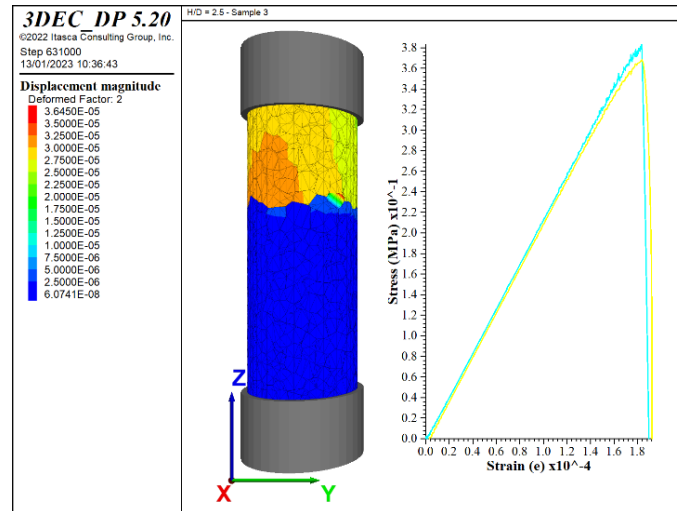


Figure 2. Example of one of the DTS tests simulated for a specimen with $H/D = 2.5$ by DEM in 3DEC.

Since the main objective of the study was to compare the influence of geometry and the number of models to be run was large, the physical parameters (strength, deformability, density, etc.) were selected within reasonably realistic ranges, prioritising feasible calculation times over realistic modelling of a particular rock type. The properties used in the model are shown in Table 1.

Table 1. Main properties used in the 3DEC simulations.

Rock	Model	-	-	Elastic
	Density	ρ	[Mg/m ³]	2700
	Young's modulus	E	[GPa]	100.0
	Poisson's ratio	ν	[-]	0.20
Steel	Model	-	-	Rigid
	Density	ρ	[Mg/m ³]	7850
Contact crystal - crystal	Model	-	-	Mohr-Coulomb
	Normal stiffness	k_n	[MPa/m]	$7.0 \cdot 10^5$
	Shear stiffness	k_s	[MPa/m]	$1.5 \cdot 10^5$
	Cohesion - peak	c	[MPa]	0.4
	Shear angle - peak	ϕ	[°]	20
	Tension strength – peak	σ	[MPa]	1.0
	Cohesion - residual	c_R	[MPa]	0
	Shear angle - residual	ϕ_R	[°]	0
	Tension strength - residual	σ_R	[MPa]	0
Contact rock - steel ("glue")	Model	-	-	Mohr-Coulomb
	Normal stiffness	k_n	[MPa/m]	$7.0 \cdot 10^5$
	Shear stiffness	k_s	[MPa/m]	$1.5 \cdot 10^5$
	Cohesion - peak	c	[MPa]	20
	Shear angle - peak	ϕ	[°]	40
	Tension strength – peak	σ	[MPa]	10

2.2 Laboratory testing

A preliminary experimental study was carried out in order to check if the main conclusions derived from the numerical simulations could also be drawn from DTS tests performed in actual rock specimens.

For this purpose, two H/D ratios (1 and 2.5) were selected, and 9 specimens of a medium-grained granite (*Blanco Mera* type) per ratio prepared. The specimens were adhered to the metal caps with an epoxy resin *Loctite EA 9483* and connected to the loading press through two roller chains placed at right angles (Figure 3a). Valid tests were considered those where the failure occurred through the rock material (Figure 3b). A more detailed description of the loading frame and DTS procedure can be found elsewhere (Muñiz-Menéndez & Pérez-Rey, 2023).



Figure 3. (a) Specimen (H/D = 1) mounted in the loading frame prior to testing; (b) Same specimen after a valid test.

3 RESULTS

In this section, DTS results as obtained from the numerical simulations, and those determined from the experimental program are presented.

3.1 Numerical modelling

As mentioned above, 60 test simulations were carried out (10 per H/D ratio), from which representative values of the DTS were obtained. These results are presented in Table 2 and in Figure 4.

Table 2. DTS values as obtained from the numerical models for each H/D ratio.

H/D		DTS [MPa]									Mean	Std. dev.
0.5	0.40	0.42	0.42	0.38	0.40	0.41	0.40	0.42	0.40	0.40	0.41	0.013
1.0	0.36	0.40	0.40	0.40	0.39	0.40	0.40	0.39	0.40	0.40	0.39	0.013
1.5	0.39	0.40	0.37	0.40	0.41	0.39	0.37	0.39	0.39	0.40	0.39	0.013
2.0	0.40	0.38	0.38	0.38	0.38	0.39	0.39	0.39	0.38	0.40	0.39	0.008
2.5	0.39	0.39	0.38	0.39	0.39	0.39	0.39	0.40	0.39	0.38	0.39	0.006
3.0	0.38	0.38	0.39	0.40	0.40	0.38	0.39	0.38	0.39	0.38	0.39	0.008

To compare the statistical significance of the obtained values, an analysis of variance (ANOVA test) was performed to determine the influence of the slenderness in the results. The results of the test show that there is a clear influence (F value 4.11; $p = 0.003$) of the slenderness in, at least, one group.

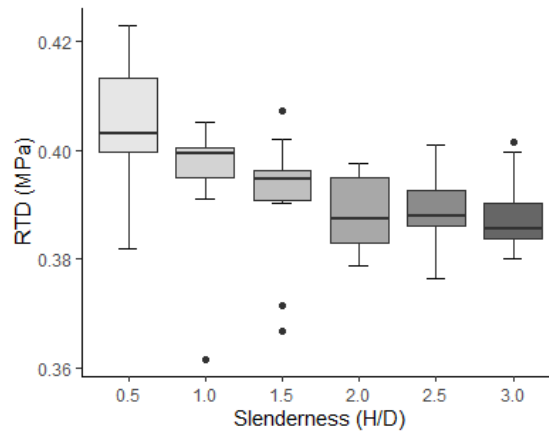


Figure 4. Boxplots of the DTS values as obtained from the numerical models for each H/D ratio.

After this result, the same test removing the group with H/D = 0.5 that, according to the boxplot (Figure 4), appeared to be clearly different both in terms of median value and scattering was performed. In this case, the ANOVA test did not show statistically significant differences (F value = 0.985; p = 0.425) between the rest of the groups (H/D from 1.0 to 3.0).

3.2 Laboratory testing

To corroborate the results of the numerical models, a preliminary testing program consisting of 18 DTS tests performed on a medium-grained granite (*Blanco Mera* type). Tested were two groups of samples, one with H/D = 1 and another one with H/D = 2.5 to check that there is no significant influence of slenderness for H/D ratios ≥ 1 . The results are presented in Table 3 and plotted in Figure 5.

Table 3. DTS results determined from the laboratory testing program (invalid tests indicated with dash (-)).

H/D	DTS [MPa]									Mean	Std. dev.
1.0	5.94	5.88	6.42	5.34	5.33	6.53	4.91	3.94	5.83	5.57	0.80
2.5	5.66	6.61	4.36	5.55	6.20	6.19	4.54	-	-	5.59	0.86

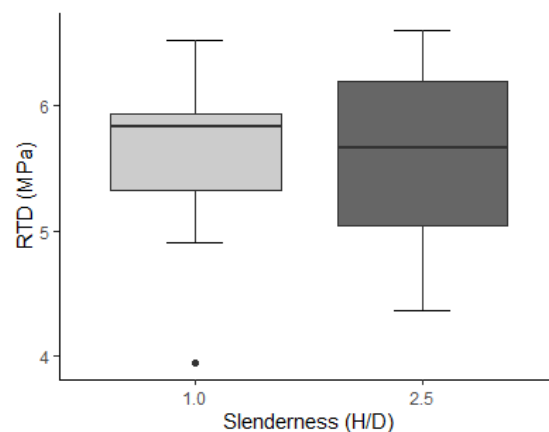


Figure 5. Boxplot for DTS results obtained from the laboratory testing program for H/D values 1.0 and 2.5.

After checking the normality (Shapiro-Wilk test) and homoscedasticity (F Test to Compare Two Variances) of the two groups of results, the mean values were compared by the Student's t-test. The

test did not show significant differences (p -value = 0.8407) between the test results performed on samples with $H/D = 1$ and those performed on samples with $H/D = 2.5$.

4 CONCLUSIONS

The comparison of the numerical models carried out allows concluding that in direct tensile strength tests there is only a statistically significant influence in slenderness when considering specimens with H/D ratios less than 1. This conclusion was also supported by laboratory DTS tests.

In the light of these results, it can be proposed that direct tensile strength tests, at least for the rock type studied (medium-grained crystalline rock) can be carried out on samples with lower height-to-diameter ratios than those suggested by the ISRM's SM and ASTM procedures. A lower limit for the H/D ratio of 1.0 can be proposed.

Although the results seem clear, this study should be, on the one hand, improved by performing DTS laboratory tests including the rest of H/D ratios selected and, on the other hand, extended to different rock types.

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