Temperature-dependent strength of ice-filled discontinuities in frozen and thawing rock masses

Greg Gambino
University of Toronto, Toronto, Canada; now at WSP Canada Inc., Mississauga, Canada

John P. Harrison
University of Toronto, Toronto, Canada

ABSTRACT: This paper presents new results regarding the strength of ice-filled discontinuities. Careful laboratory testing using prepared core specimens containing ice in artificial discontinuities allowed the indirect tensile strength of ice-rock interfaces to be determined at different temperatures. These results were combined with those for shear strength published by others in 2000, to extend the shear strength data into the tensile regime. The combined results suggest a linear Coulomb criterion with no tensile cut-off is appropriate for both the shear and tensile regimes at −2.5 and −5.0°C. Two regression models were investigated, one extending the shear data into the tensile regime, and the second combining the indirect tensile and shear data with the shear data being used to inform the relation through Bayes' rule. These analyses confirm the robustness of the indirect tensile testing and provide a predictive strength criterion for rock engineering design in frozen and thawing discontinuous rock masses.

Keywords: ice-filled discontinuities, tensile strength, strength criterion, Bayesian regression.

1 INTRODUCTION

Rock masses containing ice-filled discontinuities (IFDs), particularly at high altitudes, are becoming more susceptible to instability caused by thawing as global temperatures continue to rise (Savi et al., 2021). Underground mining operations in permafrost rock are facing similar challenges as mining machinery produces heat causing the exposed rock mass warm and IFDs within it to thaw. The warming and eventual thawing of IFDs can significantly affect their overall strength (Gambino, 2023).

This paper presents a new methodology developed to test the tensile strength of the ice-rock interface of an IFD, as this component of IFD strength has not previously been investigated. As an IFD warms, a detachment crack can form along the ice-rock interface (Gambino 2023), and thus the temperature-dependent strength along the ice-rock interface requires characterization. Moreover, this work combines the existing knowledge of IFD shear strength (Davies et al., 2000) with the new tensile strength measurements through a Bayesian analysis framework to produce a new comprehensive strength criterion.

2 LABORATORY TESTING

2.1 Material selection and specimen preparation

Specimens were carefully prepared using two lengths of 37 mm diameter rock core with parallel polished ends. As shown in Fig. 1, the two lengths of core were joined by freezing a segment of ice between their ends. Nominally, the gap between the two lengths of core and hence the ice thickness was set at 10 mm for ease of experimentation. Further details are given in Gambino (2023), including a discussion on why accurate aperture control is not necessary to ensure test repeatability.

Since the rock has a heat capacity greater than water, within the gap between the two lengths of rock core, ice first forms on the rock surfaces and freezes inward. As the ice freezes, the inward freezing direction causes impurities in the water to be concentrated along the last-to-freeze boundary, which is a plane in the centre of the ice segment. The impurities trapped in this last-to-freeze plane create a weakness plane, and this prevents the strength of the ice-rock interface being determined. As shown in steps 3a and 3b (see Fig. 1), if the ice is thinly layered above a base layer such that the last-to-freeze boundary is horizontal, this avoids development of a vertical weakness plane through the specimen midspan with the result that the ice-rock boundary strength can be determined.

Following preparation of the ice segment, the specimen is placed in a temperature-controlled chamber set to −2.5°C to −3.0°C for approximately six hours to allow the ice to undergo annealing and thus reduce interstitial point defects in the crystalline structure (Petrenko & Whitworth, 1999). After annealing, the specimen is brought to and maintained at the desired testing temperature for at least 15 hours to ensure the specimen reaches thermal equilibrium.

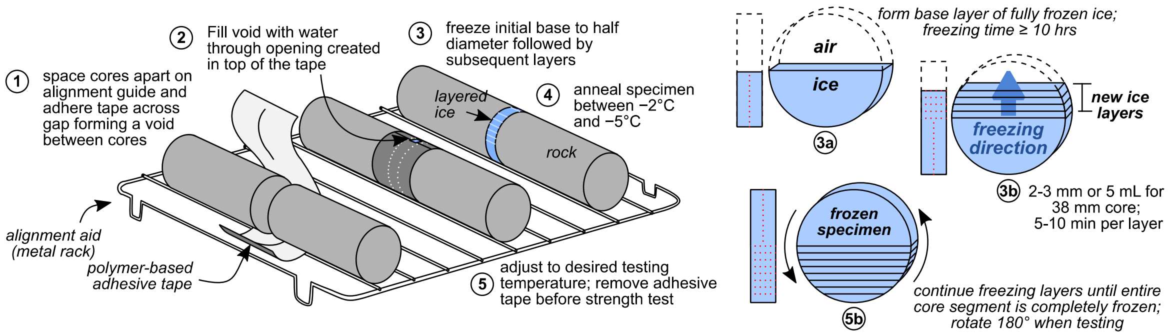

Figure 1. Specimen freezing schematic (from Gambino, 2023); detailed images of layering process shown by steps 3a and 3b; final ice layering profile used in strength testing shown by step 5b.

2.1.1 Testing technique

The indirect tensile strength testing incorporated a four-point bending test configuration, as shown in Fig. 2. Two rollers supported the specimen while the loading points were connected from the top of the specimen, through a yoke to a suspended stress-controlled load below the support frame. An initial load was gradually applied using fixed weights followed by the addition of sand to give a tensile stress rate of approximately 6.7 kPa/s until the specimen achieved peak strength. The elapsed time for the entire testing procedure (i.e., removal from the freezer to attainment of peak strength) was 1 to 3 minutes. This short period of time did not allow significant warming of the test specimen. Additional details of the testing procedure are given in Gambino (2023).

3 DEVELOPMENT OF A STRENGTH CRITERION

3.1 Previous test results

Testing of IFD strength has previously been undertaken by Davies et al. (2000), although this work assessed only the shear strength of IFDs. The shear strength data collected in this work at temperatures of −0.5°C, −2°C, and −5.0°C, together with Mohr-Coulomb (M-C) strength criteria ($\tau = c + \sigma_n \tan \phi$) are illustrated in Fig. 3. These three criteria are indicative of the temperature dependent strength properties of ice.

As shown by the shear strength test results and Coulomb criteria in Fig. 3, the shear strength increases as the temperature decreases. Also, the friction angle is seen to slightly increase with decreasing temperature. Davies et al. (2000) also concluded that at low temperatures and normal stress, the IFD strength can be significantly greater than that of unfrozen discontinuities. However, at low normal stress it may also be significantly less than that of an unfrozen discontinuity.

The characterization of the shear strength of IFDs using the M-C strength relation is used here to further extend the strength criterion into the tensile regime. Setting $\tau = 0$ and solving for σ_n, the tensile strength may be estimated as:

$$\sigma_t \approx c/\tan \phi. \quad (1)$$

This approximation will be used in §3.2 as the relation between the shear and tensile regimes in the Bayesian analysis.

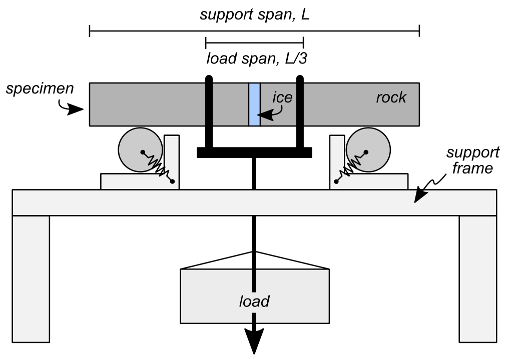

Figure 2. Four-point testing configuration and apparatus (after Gambino, 2023); schematic not drawn to scale.

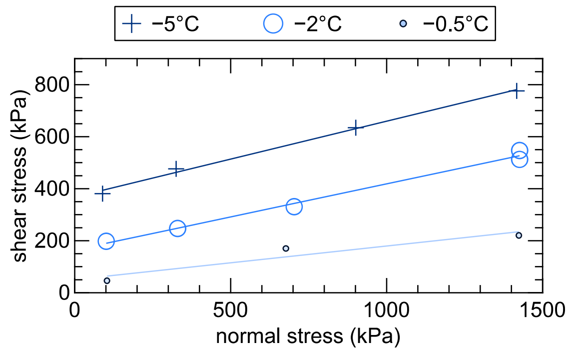

Figure 3. Shear strength results from tests on IFDs; relationship between shear strength and normal stress of an IFD, after Davies et al. (2000).

3.2 *Indirect tensile test results and Bayesian model fitting*

The indirect tensile tests were grouped by temperature, with a total of eight valid tests performed at −5°C and six valid tests at −2.5°C being obtained. Specimens that did not fracture along the ice-rock interface were excluded from the analysis. The peak load for each test was recorded and the peak tensile stress induced computed; this is summarized in Table 1 using a tension positive sign convention. Like the previous shear test results, the tensile strength of an IFD was found to increase with decreasing temperature.

Table 1. Indirect tensile test results grouped by test temperature.

Temperature (°C)	Tensile strength (kPa)							
−5.0°C	1195.5	1333.1	1449.1	1512.4	1443.3	1357.5	1325.9	1368.0
−2.5°C	933.2	796.3	821.9	809.2	682.8	718.4	-	-

A Bayesian normal model with non-informative (i.e., vague) priors on the mean and standard deviation of tensile strength (Gelman et al., 2013) is fitted to each group of tensile strength data (Table 1) using a Markov Chain Monte Carlo (MCMC) method using 15,000 iterations. Note that the regression combines tensile strength values obtained at −2.5°C data with shear strength values obtained at −2.0°C. This temperature discrepancy will induce a small inaccuracy in the analysis, but one that is insignificant for the purposes of this paper. The histograms for the posterior mean are shown below in Fig. 4 for both test groups, and are also summarized at the top of each plot as a point estimate and a 95% credible interval (CI). Simulations from the posterior mean and standard deviation are then used to draw random samples of tensile strength from the normal likelihood (i.e., distribution of data). This forms the posterior predictive distribution (Fig. 4), of which the 95% CI is highlighted. This is referred to as the predictive interval (PI). The 95% PI represents a likely range for data that is yet to be observed. These PI bounds can be used to establish a conservative design value. For example, using the available indirect tensile strength results (Fig. 4), an IFD tensile strength value for designing an excavation in a frozen rock mass at −5.0°C may be 1.1 MPa.

A practical limitation to choosing a normal distribution for tensile strength is apparent when the temperature approaches 0°C, as the credible and predictive intervals extend into the compressive region. Further work is required to appropriately characterize the tensile strength distribution at temperatures near 0°C.

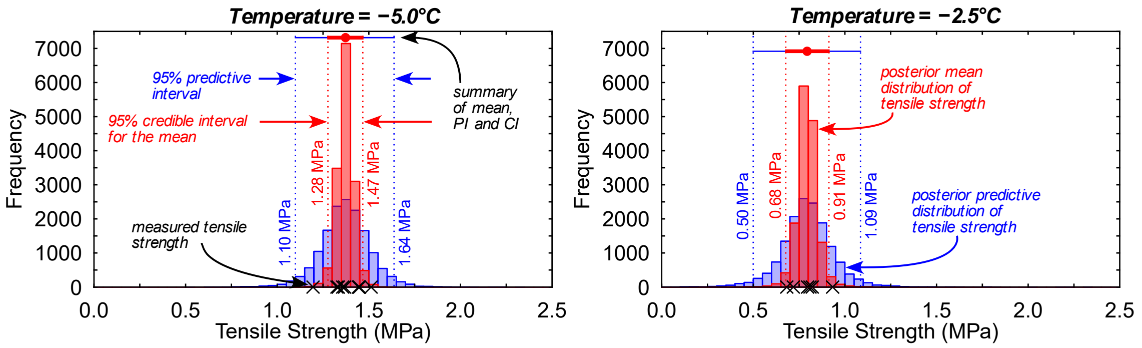

Figure 4. Tensile strength data at −5.0°C (left) and −2.5°C (right) with 95% credible interval of the mean and 95% predictive interval; data presented with posterior mean and predictive distributions of tensile strength.

3.3 *Analysis of combined shear and tensile strength results*

Classical linear regression methods do not allow tensile and shear strength data to be combined as these data have different dependent variables (i.e. shear stress for shear data, and the minor principal stress for tensile data), but Bayesian regression models offer a framework that permits their simultaneous analysis. Here, we use a Bayesian model that assumes a linear M-C criterion with no tensile cut-off. The general procedure for carrying out an analysis that simultaneously combines both shear and tensile data is detailed by Bozorgzadeh et al. (2018) and Bozorgzadeh & Harrison (2019).

Following the approach by Bozorgzadeh et al. (2018), a statistical summary of the combined analysis is shown in terms of the 95% CI and PI as illustrated in Fig. 5. This figure shows that: (i) the 95% CI for the mean tensile strength lies almost centrally within the tensile data; (ii) for mean tensile strength determined using a linear M-C criterion fitted to the shear strength data, the 95% CI is seen to extend significantly beyond the measured tensile data; (iii) the 95% CI of mean tensile strength determined from tensile and shear data closely reflects the measured tensile data; and (iv) the 95% PI of the combined data is seen to closely encompass the measured tensile data. The final

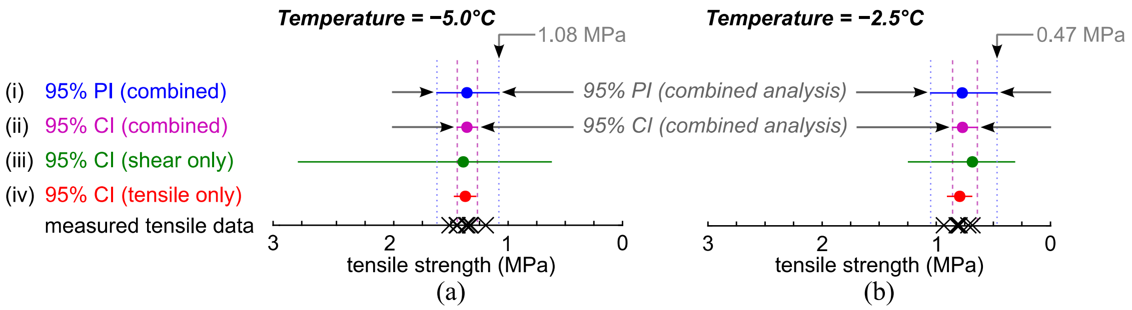

Figure 5. Mean tensile strength and resulting 95% credible and predictive intervals of combined Bayesian analysis considering only tensile data, only shear data, and combined data for (a) −5.0°C and (b) −2.5°C.

two of these observations verify the applicability of both the Bayesian approach for combined data and a linear M-C criterion with no tensile cutoff for IFD strength.

The improved CI and PI given by the combined analysis can be used to select design values depending on the volume of IFD affected (Orr, 2000; Bozorgzadeh & Harrison, 2019). For example, if a failure mechanism involves only a single IFD of limited extent (say, a small, isolated rock block on a slope) then a cautious design value should be based upon the lower bound of the 95% PI. In contrast, if the failure mechanism relates to a large rock mass volume containing many IFDs, then a design value should be based on a cautious estimate of the mean strength, i.e., the lower bound of the 95% CI.

A synopsis of the combined analysis is shown graphically in Fig. 6. Using the previous shear strength data together with the tensile strength test results, the Coulomb criteria for −5.0°C and −2.5°C are shown using the blue and red lines respectively. The posterior mean and 95% CIs are extended into the tensile region as dashed lines. Note that the usual compression positive sign convention is used here. Together with the statistical summary presented in Fig. 5, these results show how the strength characteristics of an IFD are affected by temperature as well as how one may obtain improved design parameters. These results show that predicting the average tensile strength of an IFD can be improved by incorporating existing shear strength and using the M-C relation to link the shear and tensile data through a Bayesian framework. Work is ongoing to augment these results with those obtained at other temperatures and thus develop a comprehensive temperature-dependent strength characterization.

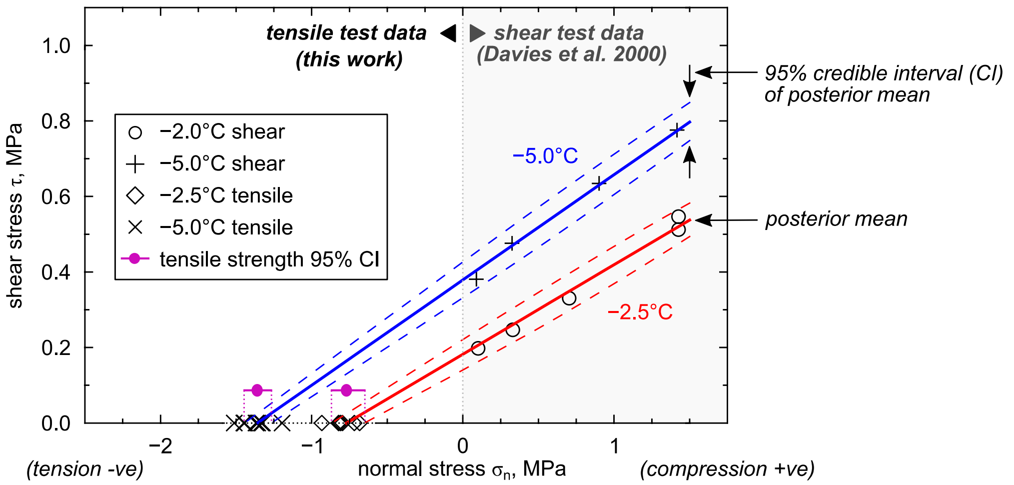

Figure 6. Combined tensile and shear results forming the IFD strength criteria for −2.5°C and −5.0°C.

4 SUMMARY AND CONCLUSIONS

In this paper, a novel specimen preparation and testing procedure for undertaking indirect tensile strength tests for IFDs has been presented. Tensile strength data obtained using this procedure have been combined with shear strength tests obtained previously by others in a Bayesian regression analysis. The simultaneous analysis of shear and tensile strength data gives improved estimates of both tensile and shear strength for engineering design. These analyses confirm the robustness of the new indirect tensile testing procedure, and provide temperature-dependent strength criteria for rock engineering design in frozen and thawing discontinuous rock masses.

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