A study on the correlation between Hoek–Brown m_i constant and Brinell hardness of intact rocks

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ABSTRACT: Conventional triaxial compression tests were conducted on dry cylindrical samples of three silicate and three carbonate rocks with porosities in the range 0.3%-24.0%, and confining pressures 0-70 MPa, in order to determine the constant m_i of the Hoek-Brown criterion for intact rock. The same rock types were used to determine the Brinell hardness number (BHN) using a spherical indenter with 10 mm diameter and a vertical indentation force of 500 kgf. The values of BHN ranged from 8 to 150 and those of m_i from 5 to 24. The experimental results show that the constant m_i increases linearly with Brinell hardness and that both BHN and constant m_i depend on mineralogy and porosity. The results may be proved useful in estimating the Hoek-Brown m_i constant from Brinell hardness which can be determined using small and easily prepared rock samples and sufficiently compact and easy to operate indentation testers.

Keywords: constant m_i, Brinell hardness, triaxial tests, Hoek-Brown criterion, porosity.

1 INTRODUCTION

Indentation hardness is a measure of material's resistance to permanent deformation, closely related to friction (Bowden & Tabor 1985 and Tabor 1956). According to Brace (1968) it has the same meaning as one point on the stress-strain curve in compression and, as such, measures the compressive strength of rocks. Brinell hardness tests have been widely used in laboratory investigations of processes leading to rock fragmentation in drilling, mechanized tunneling, and mining (Yang et al. 2022). It is an important mechanical property that governs wellbore stability and proppant embedment. Brinell hardness tests are used for characterization before and after treatment using various techniques to establish changes in rock mechanical properties (Zheng et al. 2020 and Samarkin et al. 2022). The main advantages of indentation hardness tests are that (a) the samples can be small and their preparation simple compared with other mechanical tests, such as the uniaxial or triaxial deformation experiments and (b) indentation testers are sufficiently compact and easy to operate. Several empirical relations have been suggested between hardness and mechanical rock properties such as the modulus of elasticity, the unconfined compressive strength and the yield stress (e.g. Geertsma 1985 and Brace 1960). Teymen (2021) found a strong correlation between Brinell hardness and both Young' modulus (E) and unconfined compressive strength (UCS).

The Brinell hardness number can be calculated using the following relationship:

$$HB = \frac{P}{\frac{\pi D}{2}\left[D - \sqrt{D^2 - d^2}\right]} \tag{1}$$

where HB is the Brinell hardness number (kg/mm^2), P is the applied test force (kgf), D is the diameter of the spherical indenter (mm) and d the diameter of the indentation (mm).

Relations between results from different methods of measuring hardness (Brinell, Vickers, Rockwell, Scleroscope, etc.) as well as between different scales of Brinell hardness (i.e. different combinations of ball diameter and applied force) used to perform the Brinell hardness tests on metallic materials have been suggested by ASTM E140-07.

The Brinell hardness method was selected over other methods because it is commonly used in practice and referenced as a proxy to modulus of elasticity, the unconfined compressive strength and yield strength for various rocks and also because of the larger-diameter sphere used, which is better suited for larger-grain as well as for polymineralic rock materials.

The non-linear Hoek-Brown failure criterion (Hoek & Brown 1980, 1997 and 2019) is the most widely used criterion for intact rock and jointed rock masses. For intact rock the criterion has the form:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_i \frac{\sigma_3}{\sigma_{ci}} + 1\right)^{0.5} \tag{2}$$

where σ_1 is the major principal stress, σ_3 is the minor principal stress, σ_{ci} is the unconfined compressive strength of the intact rock material and m_i is a material constant which depends on the frictional characteristics of the component minerals and the degree of particle interlocking (Marinos & Hoek 2000 and Carter 2022).

Brinell hardness and constant m_i are both closely related to brittleness (e.g. Ohnaka 1973, Hucka & Das 1975 and Hoek & Martin 2014). This study was focused on the experimental investigation of the relation between m_i and Brinell hardness, using the results of conventional triaxial compression tests for the determination of m_i and Brinell hardness tests for the determination of hardness.

2 EXPERIMENTAL PROCEDURE

Samples of three silicate and three carbonate rocks with porosities 0.3% - 24.0%, dry density 2925 - 2709 kg/m^3 and unconfined compressive strength 12.5 - 82.5 MPa were collected from quarries and natural exposures from Greece. The petrophysical properties and the modal analysis of the investigated rocks are depicted in Table 1.

The conventional triaxial compression testing program included at least eight specimens per rock type raising the total number of triaxial compression tests to 77 including 15 uniaxial compression tests. The specimens had a nominal diameter of 54 mm (NX core size) and a length of 108 mm, their ends were ground flat and parallel and were tested under confining pressures in the range 0 - 70 MPa, as per ASTM guidelines. The specimens were oven-predried at 70 °C and tested to varying strains at a constant displacement rate of $5x10^{-5}s^{-1}$ in a conventional Hoek triaxial cell at room temperature and humidity. The confining pressure was applied by an automatic electric constant-pressure pump. All tests were carried out according to ASTM D7012-14A and were performed using an external servo-controlled actuator using a stiff 4000kN INSTRON servo-controlled compression testing machine. The constant m_i was determined from the triaxial σ_1-σ_3 data sets, using RSdata and Levenberg-Marquardt algorithm.

Table 1. Material properties of investigated rocks.

Property	UCS	Mean grain size	Dry density	Open porosity	Mineral composition
Units	[MPa]	[μm]	[kg/m^3]	[%]	[%]
Rhodope Sandstone (RS1)	82.5	134	2517	3.88	Q: 56. K:18 Ms: 4 Pl: 22
Rhodope Sandstone (RS2)	78.5	411	2513	4.29	Q: 67, K: 14, Ms: 3, Pl:14, Kln: 2
Xanthi Graywacke (XG)	32.5	376	2029	17.42	Q: 81, Kln: 15, K:3, Ms:1
Giannitsa limestone (GLM)	80.8	8	2682	0.75	Cal: 99, Q: 1
Kefalonia limestone (KL)	12.5	149	2025	23.95	Cal: 98, Chl: 1, Q: 1
Thasos marble (TKM)	55.2	1433	2709	0.32	Cal: 98, Do: 1, Q: 1

*Q: Quartz, K: K-feldspar, Ms: Muscovite, Pl: Plagioclase, Kln: Kaolinite, Cal: Calcite, Chl: Chlorite, Do: dolomite

NX core size (54mm) specimens, with a height to diameter ratio of at least 1:1 from the same rocks were prepared for Brinell hardness testing. Top and bottom of the specimens were saw-cut. In order to avoid premature lateral cracking, the test specimens were encased in low expansion dental plaster and special steel rings 110 mm in diameter (Figure 1b). The top of the rock specimens was exposed and could be positioned perpendicular to the indenter. With the surface diameter of 54 mm and indenter diameter of 10 mm, indentations were performed on a number of locations with a certain distance between and from the edges in order to eliminate the mutual interference between indented points and the effect of possible heterogeneities. Figure 1a shows the Brinell tester used for testing and Figure 1b shows a cylindrical rock specimen after hardness testing.

The Brinell hardness testing involved the penetration of a high-hardness steel ball 10 mm in diameter to specimen by applying a constant load of 187.5 kgf (1.8 kN) on KL and XG and 500 kgf (4.9 kN) on other rocks for 10-15 s (Figure 1a). The total number of Brinell tests was 186 (on average 31 tests per rock type). To ensure accuracy, the tested surfaces were photographed and each dented area was quantified using a high-resolution NIKON SMZ25 stereo microscope with an image processing software. Indicative photographs of the investigated rocks after indentation are given in Figure 2.

3 RESULTS AND DISCUSSION

The Brinell hardness number of investigated rocks ranged between 8 and 131 with an average value of 60. More specifically the average value of Brinell hardness were 56 for TKM, 12 for XG, 131 for GLM, 8 for KL, 71 for RS1 and 80 for RS2 (average 60, standard deviation 46). The variation of BHN with porosity (shown in Figure 3a) is described by the equation:

$$BHN = 101e^{-0.11n} \tag{3}$$

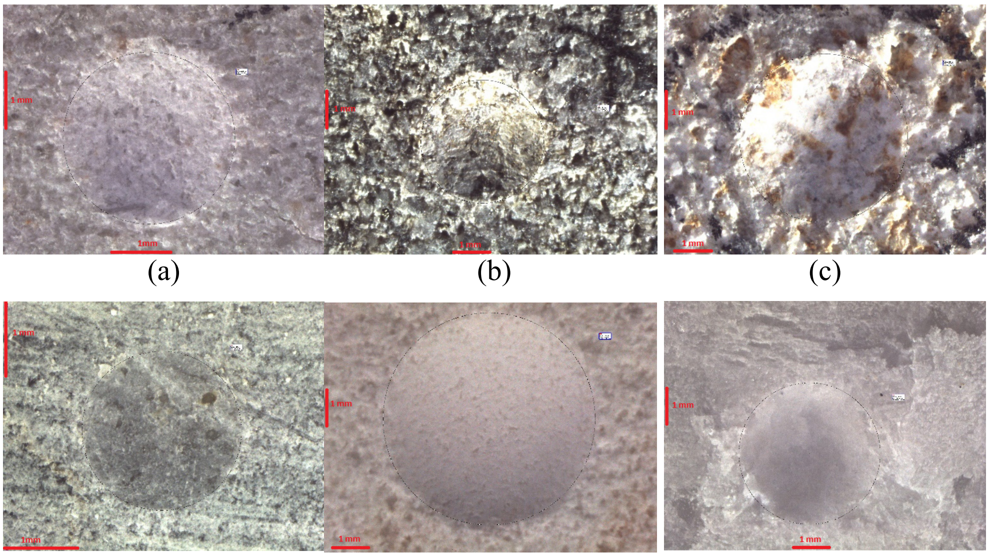
For a reliable determination of the constant m_i of the Hoek-Brown criterion sound laboratory data covering the entire brittle field, from σ_3=0 to the brittle–ductile transition, is needed (Hoek & Brown 2019). Given the limitation of the maximum confining pressure used in our study (max σ_3=70 MPa), it was possible to carry out tests up to the brittle–ductile transition only for XG, KL and TKM that had a brittle ductile transition less than 70 MPa. For GL, RS1 and RS2, that exhibited a brittle-ductile behavior at confining pressures higher than 70 MPa, it was not possible to carry out triaxial tests covering the entire brittle field. However, the requirement of the previous (1997) version of the criterion (Hoek & Brown 1997) regarding the number of at least 5 tests covering the range 0-0.5σ_{ci} was fulfilled. The constants m_i and σ_{ci} of the Hoek-Brown criterion for each rock as well as the brittle-ductile transition pressure σ_{3T} and the range of the confining pressure σ_3 used for their determination are given in Table 2.

Table 2. Values of m_i, σ_{ci}, σ_{3T} and range of confining pressure used in the triaxial tests of the investigated rocks.

Rock	m_i	σ_{ci}	Brittle-ductile transition pressure σ_{3T}	Range of confining pressure
		[MPa]	[MPa]	[MPa]
Rhodope Sandstone (RS1)	18.5	98.7	283	0-0.9σ_{ci}
Rhodope Sandstone (RS2)	22.4	78.5	218	0-0.7σ_{ci}
Xanthi Graywacke (XG)	5.3	33.9	12	0-12
Giannitsa limestone (GLM)	24.0	79.6	94	0-0.9σ_{ci}
Kefalonia limestone (KL)	8.8	13.4	6	0-6
Thasos marble (TKM)	14.3	55.2	42	0-42

(a) (b)

Figure 1. Photographs of the Brinell hardness tester used (a) and a rock specimen cast in a steel ring using dental plaster. Note the number and size of the indentations (b).



(a) (b) (c)

(d) (e) (f)

Figure 2. Indicative photographs of samples after indentation. (a) Sandstone RS1, (b) Sandstone RS2, (c) Graywacke, (d) Limestone GLM, (e) Limestone KL and (f) Marble.

The values of m_i ranged between 5.3 and 24.0. With the exception of limestone, these values are broadly consistent with the values suggested by the guideline chart published by Marinos & Hoek (2000). The value of m_i for limestone was determined from confining pressures in the range 0-0.9σ_{ci} and found to be 24, a noticeable higher value compared with the suggested range of 12±3. However, this result falls within the range given by Richards & Reed (2013) and is in good agreement with the results of Tsikrikis et al. (2022) for Mesaio limestone (23.5) and the mean value of 24.6 found by Sabatakakis et al. (2018) for a number of Greek micritic limestones.

The constant mi increases linearly with the value of the BHN, according to the relation:

$$m_i = 0.15BHN + 6{,}45 \tag{4}$$

The correlation between BHN and constant m_i of Hoek & Brown criterion is presented in Figure 3. The results indicate that the constant m_i and BHN depends on mineralogy and porosity (Figure 3a). This is perceived with the difference between the sandstones (RS1, RS2) and the graywacke (XG) porosities. The XG has a higher value of porosity (17.42%) than the sandstones (3.88% and 4.29% respectively). Although, the graywacke has a higher percentage of quartz, it has a significantly lower value of BHN and constant m_i. The same reduction of hardness and constant m_i is confirmed by the two limestones (KL and GLM) and shows the critical role of porosity. Brinell hardness tests depends on mean grain size of rock sample. The indenter ball has 10 mm diameter. The rocks with coarser grains and varied mineral composition have a higher value of coefficient of variation.

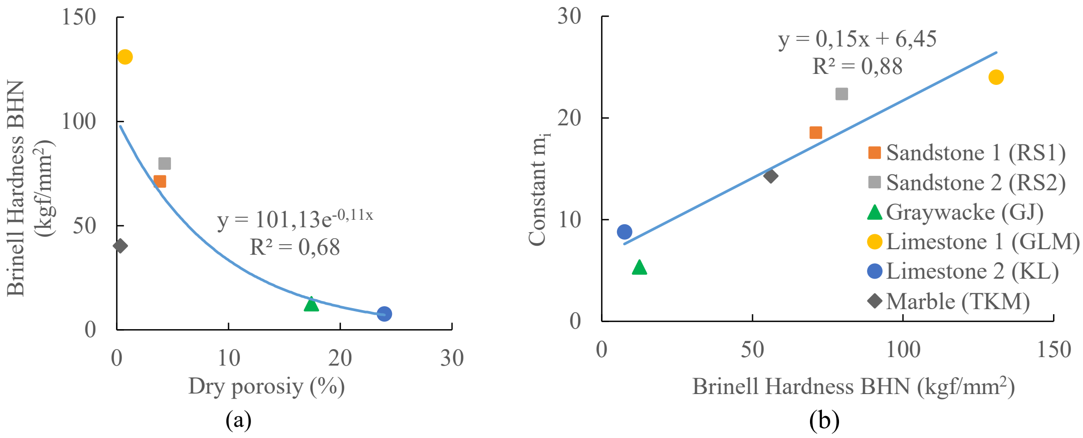


Figure 3. Correlation between and porosity (a) and constant m_i and Brinell Hardness Number (b).

An important advantage of Brinell hardness testing is that it gives the value of hardness and indirectly the values of unconfined compressive strength UCS and deformation modulus E of the superficial layer of a rock, which may be significantly lower than the corresponding values determined by traditional UCS tests based on core samples representing the underlying unweathered rock. Relation (4) is based on the experimental results from tests on fresh samples of six different rocks with UCS ranging from 13.4 to 98.7 MPa. Given that the samples required for the determination of hardness are smaller than those required for triaxial or uniaxial compression testing, more specimens from the same available rock samples may be prepared for hardness testing than those for compression testing and therefore more information may be acquired.

4 CONCLUSIONS

The results indicate that both the constant m_i of the Hoek & Brown criterion and Brinell penetration hardness depend on mineralogy and porosity and that m_i increases linearly with the value of the Brinell hardness.

The main utility of this methodology is the indirect estimation of the constant m_i using a series of Brinell hardness tests on pieces from the same rock. Triaxial compressive tests under a wide range of confining pressure capable to define the m_i experimentally, are not often preferred in practice.

Hardness is a surface material property and the effect of weathering, alteration degree or porosity is reflected in its value. Therefore, the decrease in the value of m_i for a weathered intact rock sample can be indirectly estimated by the Brinell hardness method.

The results of this study suggest that Brinell hardness, besides its use as a means of obtaining indirectly the unconfined compressive strength and the modulus of elasticity of intact rock, may be also used in estimating the Hoek-Brown constant m_i from simple tests, using small and easily prepared rock samples and sufficiently compact and easy to operate indentation testers.

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