Relations between elastic waves speeds and densities

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ABSTRACT: For civil engineers, working on rocks, it is important to understand how density varies in the rock mass volume. Direct in-situ measurements are not easy, because of very often coring difficulties. The in-situ measures of elastic waves speeds can be interesting, but we need strong correlations between speeds and rock densities. Many researchers have worked on the subject but none has given physical relationships explaining the relations they had observed. Working on Nafe and Drake empirical curves, it is possible first to find an empirical relationship between Poisson ratio and density. Then, will be presented physical relationships existing between these two types of speed and rock density. Based on them, a relationship between rock mass elastic modulus Emr and density will be proposed. And finally comparing it to the one given by Hoek, linking Emr and the Geological Strength Index GSI, a relationship between GSI and density will be deduced.

Keywords: Waves, speeds, densities, elasticity, modulus.

1 INTRODUCTION

Numerous researchers have published collections of measured values of densities compared to compressional and shearing waves speeds. Aiming to synthetize their results, they very often have tried to find through the scatter graph the best calibration curve, by the least squares method (Brocher 2005). It is not satisfactory, because the proposed formulae are note explained by rock's physical behavior.

First of all, we have to stress on two difficulties. The first one is the type of density, we work on. We will suppose for this work, that the rock mass is dry and then we will mention γd. Secondly, in our analysis, γd will be greater than 22.5 kN/m^3. Under this value, the behavior of the material is that of a soil, for which the relationships between speed and density are totally different (4).

The two Nafe and Drake curves (Nafe & Drake 1957), shown on Figure 1, are the basis of this work.

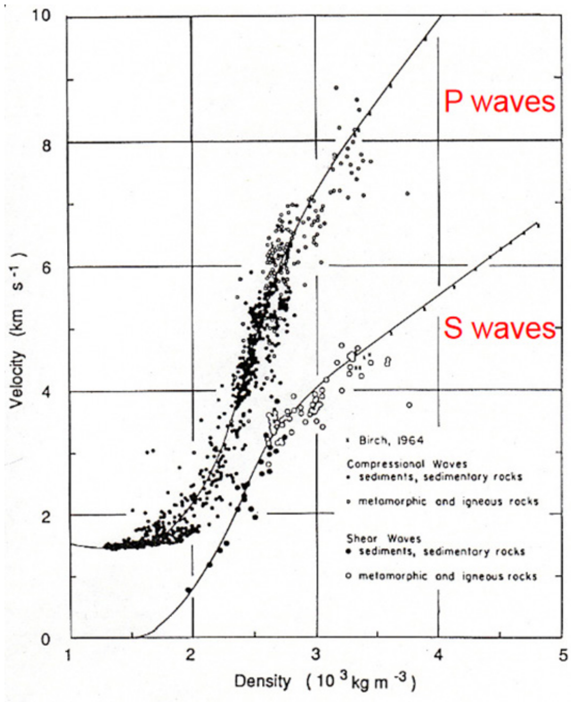


Figure 1. Nafe and Drake curves (1957).

2 POISSON RATIO ν VARIATION WITH DENSITY

Poisson ratio is linked to the ratio Vp/Vs through the relationship hereunder:

$$\nu = 0.5 \cdot \left[\frac{\left(Vp/Vs\right)^2 - 2}{\left(Vp/Vs\right)^2 - 1} \right] \tag{1}$$

Considering mean speed's values for Vp and for Vs, for densities comprised between 15 kN/m^3 and 40, calculating the corresponding values of Vp/Vs ratios and then Poisson ratios through Formula 1, it appears that a sigmoid function explains the results, as shown on Figure 2.

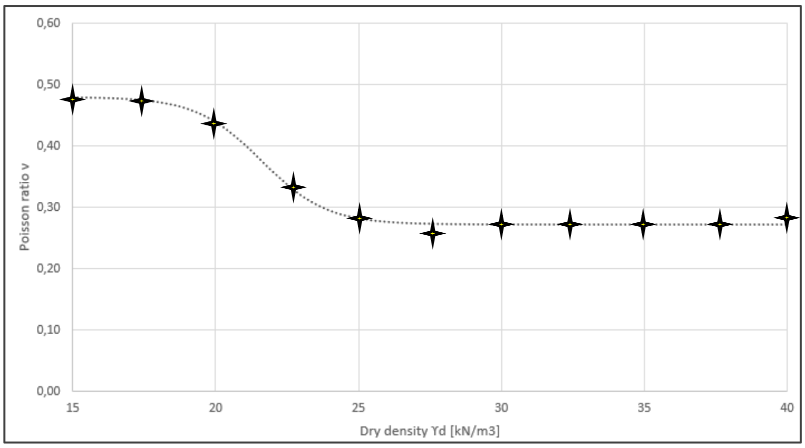


Figure 2. Calculated Poisson ratios as functions of densities ✦.

Table 1. Poisson ratio as a function of density.

γd	[kN/m³]	15	17.5	20	22.5	25	27.5	30	32.5	35	37,5	40
Vs	[m/s]	427	583	714	1642	2714	3643	4143	4571	4893	5286	5643
Vp	[m/s]	1571	1785	2178	3286	4928	6286	7357	8178	8750	9500	10214
Vp/Vs	[-]	3.68	3.06	3.05	2	1.81	1.726	1.766	1.789	1.788	1.797	1.81
ν	[-]	0.48	0.47	0.44	0.333	0.282	0.247	0.268	0.273	0.272	0.275	0.280

The sigmoid function obeys to the Relationship 2:

$$\nu = 0.208 \cdot \left[\frac{1}{1+e^{(-19+0.88\gamma d)}} + 1.308 \right] \quad (2)$$

γd in kN/m³.

3 RELATIONSHIP LINKING Emr AND γd THROUGH Vp CURVE

As a first step, we will work on the Vp curve, aiming to find a relationship between Emr elastic modulus and γd dry density.

The relationship linking Vp and Emr is:

$$Vp^2 = \left[\frac{(1-\nu)}{(1+\nu)(1-2\nu)} \times \frac{g}{\gamma d} \right] \times Emr \quad (3)$$

g gravity acceleration.

We propose to write: $Vp^2 = \text{Term N° 1.1} \cdot Emr$ (4)

And Emr can be written as:

$$Emr = 2 \cdot 10^8 \cdot \text{Term 2.1 [in kPa]} \quad (5)$$

Assessing the values of Term 1.1 in Table 2, the values of Poisson ratios being given by Relation 1, the calculated values of Term 2.1 are given.

Table 2. Calculation of Term 2.1 leading to Emr.

γd	[kN/m³]	15	17.5	20	22.5	25	27.5	30	32.5	35	37.5	40
ν relation 1		0.480	0.474	0.439	0.376	0.282	0.273	0.273	0.273	0.273	0.273	0.273
Vp NAFE-DRAKE	[m/s]	1571	1785	2178	3286	4928	6286	7357	8178	8750	9500	10214
$[\text{Term 1.1}]^{0.5}$		2.42	1.98	1.264	0.820	0.717	0.676	0.648	0.622	0.599	0.579	0.561
Term 2.1 with 2.10^8 [kPa]		0.0021	0.0046	0.0148	0.0810	0.236	0.432	0.644	0.864	1.067	1.346	1.657

4 RELATIONSHIP LINKING Emr AND γd THROUGH Vs CURVE

As a second step, we now work on the Vs curve.

The relationship linking Vs and Emr is:

$$Vs^2 = \frac{1}{2(1+\nu)} \cdot \frac{g}{\gamma d} \cdot Emr \quad (6)$$

We can then write:

$$Vs = (\text{Term } 1.2)^{0.5} \,.\, (2 \,.\, 10^8)^{0.5} \,.\, (\text{Term } 2.2) \tag{7}$$

With:

$$\text{Term } 1.2 = \frac{1}{2(1+\nu)} \cdot \frac{g}{\gamma d} \tag{8}$$

In Table 3, Term 2.2 is calculated and compared to Term 2.1.

Table 3. Calculation of Term 2.2 and comparison with 2.1.

γd	[kN/m³]	15	17.5	20	22.5	25	27.5	30	32.5	35	37.5	40
ν relation 1	[-]	0.480	0.474	0.439	0.376	0.282	0.273	0.273	0.273	0.273	0.273	0.273
Vs NAFE-DRAKE	[m/s]	427	583	714	1642	2714	3643	4143	4571	4893	5286	5643
$[\text{Term } 1.2]^{0.5}$	[-]	0.474	0.440	0.417	0.408	0.395	0.378	0.362	0.348	0.335	0.324	0.313
Term 2.2	[-]	0.0045	0.0088	0.0147	0.0810	0.236	0.464	0.655	0.863	1.067	1.331	1.625
Term 2.1	[-]	0.0021	0.0046	0.0148	0.0810	0.236	0.432	0.644	0.864	1.067	1.346	1.657
Term 2	[-]	/	/	0.0147	0.081	0.236	0.448	0.649	0.863	1.067	1.336	1.641

It appears that Term 2.1 and Term 2.2 have the same order of magnitude, when γd is comprised between 20 and 40 kN/m³, and we will name them 2.

Term 2 is well displayed by the function:

$$\text{Term } 2 = \left[2.92 \cdot \frac{\gamma d - 19.53}{\gamma d + 7.96}\right] \tag{9}$$

when γd is comprised between 22.5 and 37.5 kN/m³.

We can then write:

$$Emr = 2 \cdot 10^8 \cdot \left[2{,}92 \cdot \frac{\gamma d - 19.53}{\gamma d + 7.96}\right]^2 \text{ kPa} \tag{10}$$

Or:

$$Emr = 1705 \cdot \left[\frac{\gamma d - 19.53}{\gamma d + 7.96}\right]^2 \text{ GPa} \tag{11}$$

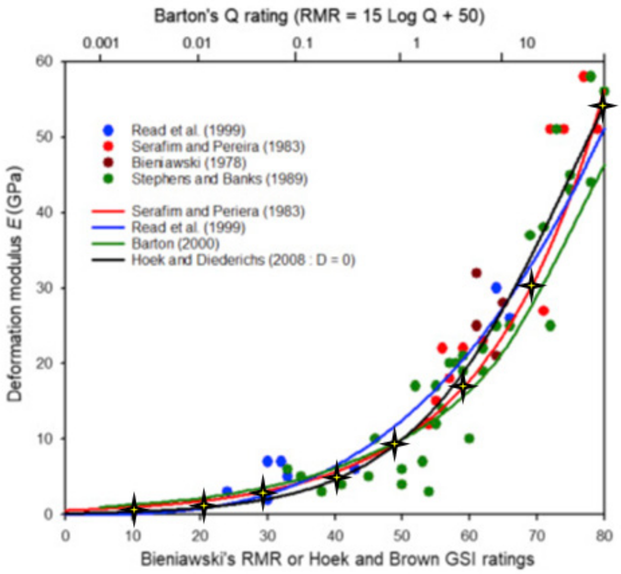
With γd in kN/m³.

Table 4. Values of Emr in GPa compared to γd values.

γd	[kN/m³]	22.5	25	27.5	30	32.5	35	37.5	40
Emr	[GPa]	16,2	47	86.1	129.7	175.2	221.1	266.4	310.6

5 COMPARISON WITH EMR AS A FUNCTION OF GSI.

When we look at Figure 3, different authors have proposed relationships to explain the position of the points on the graph, like Serafim and Pereira, Bieniawski, Hoek and Diedrichs (2006), and Barton.



✦ : Relationship 12.

Figure 3. Comparison between four authors approach and Gress Relationship.

We propose another formula:

$$\text{Emr} = 24.47 \cdot \left[\frac{\text{GSI} + 3.54}{136.20 - \text{GSI}}\right]^2 \text{ in GPa} \qquad (12)$$

Table 5 shows its relevance.

Table 5. Comparison between mean Emr values and those calculated through Relationship 12.

GSI	[-]	10	20	30	40	50	60	70	80	90	100
Figure 3 Mean values	[GPa]		1.25	2.9	5	10	19.2	33.3	55.8		
Relation 12	[GPa]	0.28	1	2.44	5	9.44	17.01	30.2	54.1	100.3	200

6 INDUCED RELATIONSHIP BETWEEN GSI AND DRY DENSITY

Comparing Relationships 11 and 12, we can write:

$$\text{GSI} = 122\left[\frac{\gamma d - 19.44}{\gamma d - 16.59}\right]^2 \qquad (13)$$

γd in kN/m^3.

Table 6 shows how the two parameters match.

Table 6. Comparison between geological strength index and dry density.

γd	[kN/m^3]	22.5	25	27.5	30	32,5	35	37.5	40.0	45	46.6
GSI	[-]	32.7	53.32	66.58	75.65	82.20	87.15	91	94.1	98.75	100

7 CONCLUSION

Using Nafe and Drake curves, we have proposed Relationships between Poisson's ratio, Emr in-situ rock elastic modulus, formulae linking Vp and Vs and dry density γd and as complementary proposals, derived a new formula linking Emr and GSI and GSI with γd. We have now to make complementary work on the explanation of the scatter around the Nafe and Drake curves, with Ei, intact rock modulus.

REFERENCES

Barton, P.J. 1989. The relationship between seismic velocity and density in the continental crust — a useful constraint? *Geophysical Journal International*, Volume 87, Issue 1, October 1986, Pages 195–208.

Bieniawski, Z.T. 1989. Engineering Rock Mass Classifications: A Complete Manual for Engineers and Geologists in Mining, Civil, and Petroleum Engineering. Wiley. New York.

Birch, F. 1966. Compressibility: elastic constants, in S.P. Clark, Jr. (ed.). *Handbook of physical constants*, Geol. Soc. Am. Mem.97, pp. 97–113.

Brocher, T. M. 2005. Empirical relations between elastic wave speeds and density in the earth 's crust. *Bulletin of the Seismological Society of America*, Vol. 95, No. 6, pp. 2081–2092, December 2005, doi: 10.1785/0120050077.

Gardner, G.H.F., Gardner, L.W. & Gregory, A.R. 1974. Formation Velocity and Density—The Diagnostic Basics for Stratigraphic Traps. *Geophysics*, 39, 770-780.

Gress, J.C. & Ferreira, M. 2020. Shear wave velocity Vs for soils and rocks correlated to dry density. *4th International Conference on Geotechnical Engineering (ICGE20)*, Hammamet, Tunisia.

Hoek, E. & Diederichs, M.S. 2006. Empirical Estimation of Rock Mass Modulus. *International Journal of Rock Mechanics and Mining Sciences*, 43, 203-215.

Nafe, J.E. & Drake, C.L. 1957. Variation with depth in shallow and deep-water marine sediments of porosity, density and velocities of compressional and shear waves. *Geophysics*, 22, 523-552.

Serafim, J.L. and Pereira, J.P. 1983. Considerations on the Geomechanical Classification of Bieniawski. *Proceedings of International Symposium on Engineering Geology and Underground Openings*, Lisbon, Portugal.

Tanzini, M. 2014. La relazione geologica e geotechnica. Dario Flaccovio Editore.