Mathematical model to describe the movement of rocks due to gravity in block caving mining

Rodolfo Morales
Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Santiago, Chile
JRI Ingeniería, Santiago, Chile

Sergio Palma
Complex Fluids Laboratory, Geomechanics Research Group, Department of Metallurgical Engineering and Materials, Universidad Técnica Federico Santa María, Santiago, Chile

ABSTRACT: Chuquicamata, located in northern Chile, has recently started operating as an underground mine using the Block Caving method. In this work, a new mathematical model is proposed to explain the phenomenology observed in the gravitational flow of fractured rocks in block caving mining. We propose that the tensional state and gravity originate a driving force greater than the resistance developed at the boundary of the group of fluidized rocks. The dynamics is controlled by a dimensionless number Y obtained as the ratio between the driving and resisting forces. If the frictional force at the interface has a viscous component proportional to the relative speed of the fluidized rocks, an analogous equation to Darcy's law for the flow of a Bingham fluid in a porous medium is recovered. The model was numerically solved using FEM, achieving an adequate reproduction of the results of Fullard et al. (2019), for the velocity field.

Keywords: Block caving mining, fractured rocks, granular, velocity field.

1 INTRODUCTION

Granular matter is a conglomerate of visible rigid particles in which friction is the predominant interaction force. One of the phenomena of growing interest in this area is the flow in silos with a basal opening, where the particles discharge under gravity, also known as gravity-driven granular flows. This phenomenon is of particular interest in chemical and farming industries, where silos are widely used to store granular materials. Its applications extend to nuclear reactors, and underground block & panel caving mines. There are several approaches used to understand the characteristics of gravity-driven granular flows, the main result of which has been the description of the velocity field. Leaving aside numerical methods like DEM that require large processing capacities and simulation times, there are two widely used approaches in the scientific community. The first provides a kinematic description of grains motion, ignoring the stress state, and leading to diffusion-type equations. The original discussion is due to Litwiniszyn (1958, 1966), who introduced a stochastic model in which grains describe random paths through the available spaces. Later, Mullins (1972) provided an equivalent description in which particle downward movement, towards or between empty spaces, was equivalent to the random movement of a void released in discharge. As an alternative view, Nedderman & Tüzün (1979) proposed the relationship $v_x = -d(\partial v_y/\partial x)$ between the horizontal and vertical velocity components, with d a diffusive length for which they observed $d \approx 2.24D_{50}$ for various grains sizes. Here, D_{50} is the average particle size. Using this relationship together with the null divergence condition, the diffusion equation is obtained $\partial v_y/\partial y = d\,\partial^2 v_y/\partial x^2$. This is the well-known kinematic model. The main problem of this models is that the diffusion length takes values between 1 and 4 times D_{50}, depending on the distance to the extraction point. As an alternative to the Mullins description, the spot model developed by Bazant (2004, 2006) arises, which presupposes that there is a spatially extended entity called spots (composed of interstitial volume), whose diffusive rise originates the movement of the grains in according to the speed predicted by the kinematic model, but with a non-local stochastic equation. The second approach assumes a constitutive relationship between stress and strain based on the $\mu(I)$ rheology proposed by Jop, Forterre, Pouliquen and co-workers. In this model, the mathematical description of granular media is made through the usual continuum media conservation laws, with a stress tensor composed of a hydrostatic pressure P and a shear part τ that follows the Coulomb friction law dependence: $\tau = \mu(I)P$. The friction coefficient is a function of the inertial number I, defined as the ratio between macroscopic deformation time scale $1/\|\dot{\gamma}\|$ and inertial time scale $\sqrt{d^2\rho/P}$ (da Cruz et al. 2005). Unfortunately, despite numerous studies carried out to understand the dynamics of gravity-driven granular flows, there is still no commonly accepted theory to explain the observed phenomenology. To find a solution to this problem, and provide an explanation based on first principles for the origin of the kinematic model, in this work, part of Fullard's experimental measurements is analyzed, and a novel mathematical model is proposed. This paper is structured as fallows: in Section 2, Fullard's results are described and correlations for diffusion length are presented. In Section 3, the mathematical model is elaborated, and numerical results are presented and compared with its experimental counterpart. Finally, conclusions are drawn in Section 4.

2 METHODOLOGY

To make a phenomenological description of the gravity-driven granular flows, we make use of the experimental velocity field measured by Fullard et al. (2019) through the Particle Image Velocity (PIV) method. The experimental system used by Fullard et al. (figure 1a) consisted of a rectangular silo of width W = 200 mm, height H = 350 mm and depth D = 15 mm, with two basal openings of size D_0 = 14 mm symmetrically located on both sides of the silo axis, spaced at varying distances L. For the granular media, mustard seed were selected with a Sauter mean particle diameter of D_{50} = 2.15 mm. The averaged steady velocity fields calculated from PIV are shown in Figure 1 for the single opening cases of D = 14 mm and D = 28 mm, and 4 cases where there are two openings separated by a distance of L = 2 − 8 − 32 and 80 mm. Results of these experiments illustrated in figure 1b, show that for L ≤ 2 mm spacings, the velocity fields are quantitatively similar to the single aperture case with D_0 = 28 mm. For 2 mm < L < 32 mm values, the effect of separation of the openings becomes evident, emerging a zone of constructive interference that disappears for L > 32 mm. Additionally, in figure 2c and figure 2d both components of the velocity are observed, referred to different heights above the extraction opening for the case of single opening with D = 28 mm. Qualitatively, both components are symmetrical around the axis of the opening and the vertical speed resembles a Gaussian curve up to a height in the order of 0.145 mm, after which it flattens in the central zone. The Fullard et al. results illustrate the typical Gaussian velocity profiles as well as the existence of an interaction between the flows originated by spaced openings. In principle, Gaussian profiles are an expected result in diffusive processes, in which the entity in motion follows a random trajectory with mean square displacements proportional to time. This leads to think that the grains follow a diffusive behavior, however, experimental results have been shown that a ballistic tendency is exhibited ($\langle \Delta x \rangle^2 \sim t^2$) at time shorter than it takes for a grain to fall its own diameter, becoming diffusive ($\langle \Delta x \rangle^2 \sim t$) at greater distances. Furthermore, in a diffusive behavior, interaction effects between openings would not be measured because of the linearity of the equation.

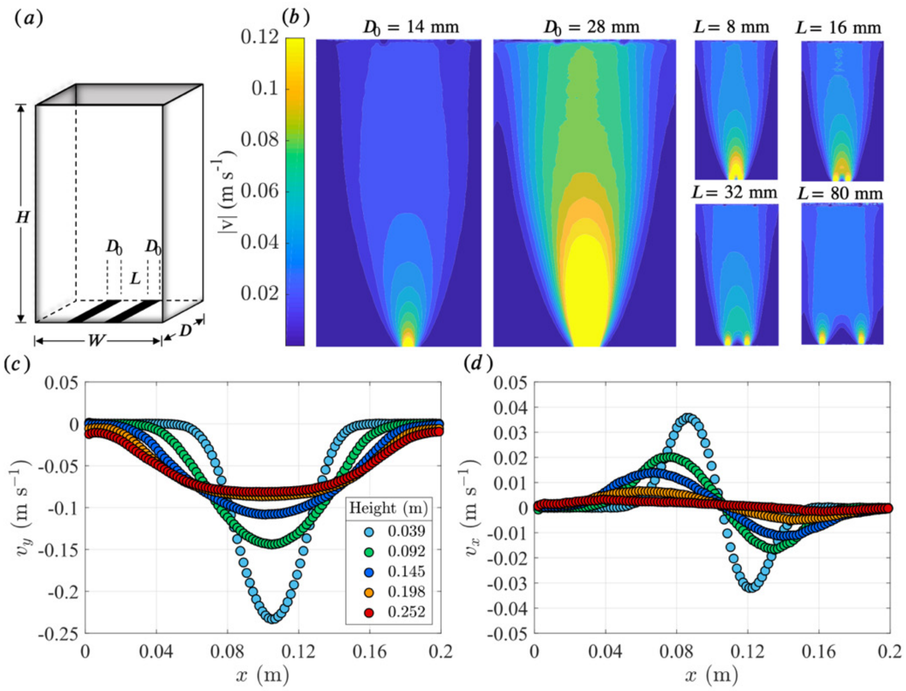

Figure 1. (a) Schematic of the experimental set-up used by Fullard et al. (2019), silo dimensions are W = 200 mm, H = 350 mm and D = 15 mm. (b) Velocity magnitude for one basal opening of size D_0 = 14 mm and D_0 = 28 mm, and two basal openings of size D_0 = 14 mm separated by a distance of L = 8−16−32−80 mm. (c-d) Vertical and horizontal velocity profiles drawn at heights ranging from 0.0039 to 0.252 m, for the opening size of D_0 = 28 mm.

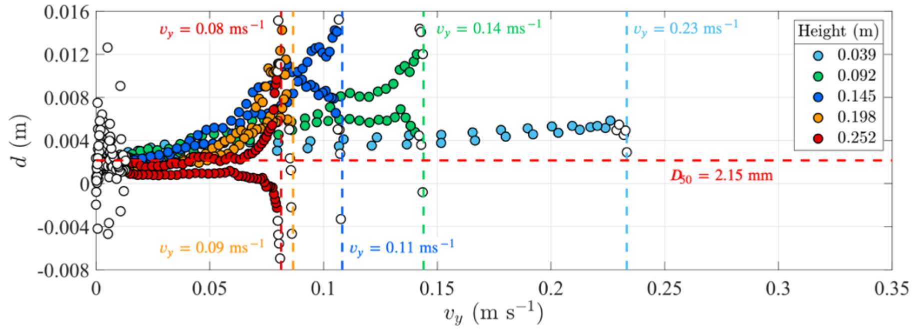

Figure 2. Diffusion length d as function of vertical velocity v_y at heights ranging from 0.039 to 0.252 m, for the Fullard et al. (2019) experiments with an opening size of D_0 = 28 mm. Diffusion length was obtained by the kinematic relation $d = -v_x/(\partial v_y/\partial x)$ using central differences for the partial derivative. White dots represent discarded data due to the small value of $\partial v_y/\partial x$. Dashed lines indicates the maximum vertical velocity at the respective height, and Dash red horizontal line correspond to mean particle size D_{50} = 2.15 mm of mustard seeds.

However, sub or over diffusive regimes can be explained through anomalous diffusion using nonlinear diffusion coefficients, a situation that motivates the study of more complex relationships. For this purpose, the Figure 2 presents a graph of the diffusion length parameter defined by the kinematic model according to $d = -v_x/(\partial v_y/\partial x)$ as a function of the vertical velocity v_y for different heights h, in which it is possible to visualize a clear dependence between the variables, with a notable collapse of the diffusion length towards the mean diameter of the grains for zero vertical velocities. Qualitatively, it is observed that d is increasing with vy and, to some extent, also with h. Although it is possible to find a functional relationship between these variables, it must be taken into consideration that if the final objective is to find equations to model the gravity-driven granular flows, at least in principle these must be autonomous i.e., not depend on spatial variables or time, therefore such a relationship would not be of general validity. From this hypothesis, it is inferred that a dependence on the height is implicitly originated in a dependence on the pressure, since it is the only scalar quantity that can transmit information about the height of the grains column.

3 RESULTS

Notably, the kinematic model is a good first approximation to the description of the phenomenon, thus, to some extent, it captures the essence of the physics of the gravity-driven granular flow. However, its functional relation has the disadvantage of relating two components of velocity in a non-reciprocal way (i.e, $v_y \neq v_y(\partial v_x/\partial y), v_x$) while establishing an equality between a vector magnitude v_x and a tensor one $\partial v_y/\partial x$. Conceptually, this represents a difficulty, given that the expected dependence is between magnitudes of the same type. A simple way to overcome this aspect is to suppose that such identity emerges from a motion equation of the type, $\rho dv_x/dt = -\partial P/\partial x - \rho v_x/\tau$ and $\rho dv_y/dt = -\partial P/\partial y - \rho g + P/d - \rho v_y/\tau$. Thus, neglecting the acceleration term, we recover the kinematic relation:

$$v_x = -\frac{\tau}{\rho}\frac{\partial P}{\partial x} = -\frac{\tau}{\rho}\frac{\partial}{\partial x}\left(d\frac{\partial P}{\partial y} + d\rho g + \frac{d\rho v_y}{\tau}\right) = -d\frac{\partial v_y}{\partial x} + O(d^2) \qquad (1)$$

Therein to arrive at this result, we have introduced the volumetric forces $f_s = P/d$ and $f_r = -\rho v/\tau$, which follow the Coulomb and Stokes friction laws respectively. Although this formulation appears to be satisfactory in appearance, it presents a problem inasmuch the first force is not a vector quantity and must be zero in a static equilibrium. Similarly, it is difficult to comprehend why the second force must be linear in velocity, instead of depending on a difference between velocities of neighbouring particles, and therefore correspond to second order velocity derivatives. To solve the problems associated with the first force, we can propose a vector form as $\boldsymbol{f_s} = -d^{-1}P\boldsymbol{v}/\|\boldsymbol{v}\|$. However, this velocity dependency is not Galilean invariant, so perhaps a better choice is available by $\boldsymbol{f_s} = min(1, P/d\|\boldsymbol{\nabla}\phi\|)\boldsymbol{\nabla}\phi$ where the *min* function ensures that the friction force does not exceed the non-equilibrium force $\boldsymbol{\nabla}\phi = \boldsymbol{\nabla}P + \rho\mathbf{g}$. Indeed a more general form of this expression is given by $min\left(1, (P/d\|\boldsymbol{\nabla}\phi\|)^k\right)\boldsymbol{\nabla}\phi$, or by its continuous approximation (valid for large *n* values):

$$\boldsymbol{f_s} = \left(1 + \left(\frac{R}{R_0}\right)^{kn}\right)^{-n} \boldsymbol{\nabla}\phi \quad \text{with } \boldsymbol{\nabla}\phi = \boldsymbol{\nabla}P + \rho\mathbf{g} \qquad (2)$$

Where we have defined the following dimensionless numbers:

$$Y = D_{50}\frac{\|\boldsymbol{\nabla}\phi\|}{P} \quad \text{and } \mathrm{Y}_0 = \frac{\mathrm{D}_{50}}{d} \qquad (2)$$

Finally, the proposed motion equation to explain the phenomenology of dense and dry gravity-driven granular flows is,

$$\rho \frac{d\boldsymbol{v}}{dt} = -\left[1 - \left(1 + \left(\frac{Y}{Y_0}\right)^{kn}\right)^{-\frac{1}{n}}\right] \boldsymbol{\nabla}\phi - \frac{\rho \boldsymbol{v}}{\tau} \quad (2)$$

Neglecting the acceleration term and therein assuming the incompressibility condition, the velocity field is given by,

$$\boldsymbol{v} = -u\left[1 - \left(1 + \left(\frac{D_{50}}{Y_0}\frac{\|\boldsymbol{\nabla}\phi'\|}{P'}\right)^{n}\right)^{-1/n}\right] \boldsymbol{\nabla}\phi' \text{ and } \boldsymbol{\nabla}\cdot\boldsymbol{v} = \boldsymbol{0} \quad (3)$$

Here, $u = \tau g, P' = P/\rho g$ and $\phi' = \phi/\rho g$. These equations were numerically solved using the finite element method software, COMSOL Multiphysics. Both isolated Fullard et al. (2019) extraction experiments were designed to study the advantages of the model. The parameters were iteratively modified seeking to adjust the geometry of the Isolated Extraction Zone (IEZ), corresponding to the original position of the particles that exited the aperture within a given time. Experimental IEZ was obtained by the integration of Fullard et al. (2019) isolated velocity fields through the 4th order Runge Kutta method. The result of this iteratively procedure is seen in the Figure 3 in terms of the IEZ for D_0 = 28 mm and D_0 = 14 mm. In general, it is evident that the proposed model achieves a better description compared to the kinematic model, adequately capturing the geometry of the IEZ, however except for the lower zone, a sector in which the modelled IEZ is located below the experimental data. The IEZ reproduced by the kinematic model resemble a drop, unlike the experimental IEZ or the IEZ obtained from the proposed model, which are more like an ellipsoid or an inverted-drop, in agreement with other studies. Finally, the third row shows the fracturing process of the rock mass in a simplified way when the block caving method is used (g - h). The movement ellipsoids originated due to the extraction of rocks can be observed. (i) In the near future it is expected to have tools based on differential equations such as those presented in this work for the creation of abacuses for the design and operation of underground mines

4 CONCLUTIONS

In this study, we have developed a novel mathematical approach for gravity-driven granular flows in silos, based on Newton's second law. And within this study, the mechanics of continuous media were used, herein assuming a hydrostatic stress state and considering the effect of four volumetric forces: pressure gradient; force of gravity; friction force; and a memory-induced reaction force. It was evident that given these forces it was possible to recover the kinematic model and subsequently explain the dependence of diffusion length on height (through its dependence on pressure). Herein the friction force was as assumed to be of the Coulomb's type, with the direction defined by the non-equilibrium force. This force is a necessary function on a new dimensionless number Y, which evidently plays a fundamental role in dense flows. As regards to the memory-induced reaction force, it was evident that its effect is equivalent to that of assuming a Stoke's type viscous force. The resulting equation was solved neglecting the acceleration term and presuming an incompressibility condition. Subsequently the results were compared with the isolated Fullard et al. extraction experiments, therein finding a good qualitative agreement between both. The main advantage of the proposed model is the reproduction of the isolated extraction zones (IEZ), which are like an ellipsoid, or an inverted drop, accordingly in agreement with that reported by other studies. Therefore, it provides a simple framework to explain the origin of the kinematic model, and of course its numerical implementation can be practiced with relative ease without significant computational demands, which is a great advantage over DEMs models.

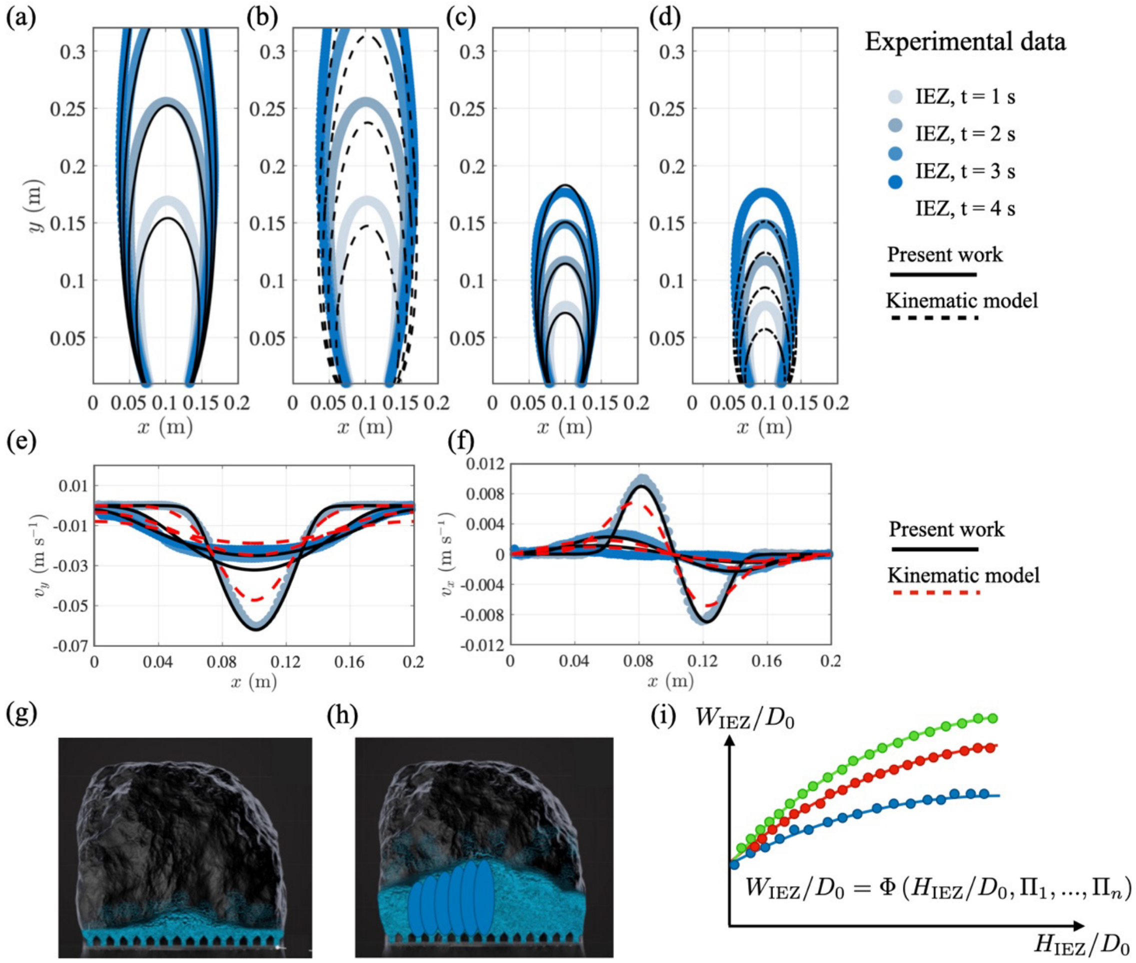

Figure 3. Fullard's experimental data for the 28 mm and 14 mm aperture IEZs. Each set of experimental data is compared with the proposed model (solid lines) and with the kinematic model (dashed lines) for different extraction times. Parameters used are: (a-c) $u = 0.8$ ms^{-1}, $Y_0 = 0.015$, $n = 20$, $k = 2$ (proposed model); (b-d) $d = 2.5 \cdot D_{50}$ (kinematic model).The second row shows the velocity components for a 14 mm aperture for different times: (e) v_x, (f) v_y. Finally, the third row shows the fracturing process of the rock mass in a simplified way when the block caving method is used (g - h). The movement ellipsoids originated due to the extraction of rocks can be observed. (i) In the near future it is expected to have tools based on differential equations such as those presented in this work for the creation of abacuses for the design and operation of underground mines.

REFERENCES

Fullard, L. A., Breard, E. C. P., Davies, C. E., Godfrey, A. J. R., Fukuoka, M., Wade, A., ... & Lube, G. (2019). The dynamics of granular flow from a silo with two symmetric openings. Proceedings of the Royal Society A, 475(2221), 20180462.

Jin, A., Sun, H., Ma, G., Gao, Y., Wu, S., & Meng, X. (2016). A study on the draw laws of caved ore and rock using the discrete element method. Computers and Geotechnics, 80, 59-70.

Melo, F., Vivanco, F., Fuentes, C., & Apablaza, V. (2008). Kinematic model for quasi static granular displacements in block caving: Dilatancy effects on drawbody shapes. International Journal of Rock Mechanics and Mining Sciences, 45(2), 248-259.