Estimation of cutting force considering intermediate dynamic rock strength using multiple linear regression

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ABSTRACT: This paper estimates pick cutting force using multiple linear regression considering rock properties determined under intermediate dynamic loading. Existing theoretical and empirical works on rock cutting performance have focused on cutting parameters and static rock mechanical properties. Even though rock cutting is a dynamic event, prediction models that consider dynamic rock properties have not been explored. Earlier research has demonstrated that the cutting mechanism in rock operates under intermediate dynamic conditions in which the strength would improve compared to its static state. Multiple linear regression analyses were performed to estimate the mean and maximum cutting forces of unrelieved cutting tests. This study compared the performance of the suggested models to that of the models produced by Evans, Goktan, and Roxborough & Liu. Statistical analysis showed that the suggested models have a lesser mean squared error to the data population than the theoretical models.

Keywords: Rock cutting, Cutting force, Pick cutter, Intermediate dynamic loading, Multiple linear regression.

1 INTRODUCTION

In rock fragmentation via cutting, the contact between the cutting tool and the rock surface is a dynamic process for both dragging and indenting methods (Wang et al. 2019). In drag cutting, for instance, the tool strikes the rock with a quick motion, as in roadheader pick cutting. The rapidly moving pick impacts the rock surface. Moreover, because of the irregularity of the excavated surface, the tool-rock interactions during the cutting process involve several dynamic events.

Earlier theoretical and practical investigations on rock cutting performance have focused mostly on cutting parameters and static rock mechanical properties (Evans 1984, Roxborough & Liu 1995, Goktan 1997). Even though rock cutting is a dynamic operation, none of the existing prediction models account for the dynamic rock properties. According to numerous research, the strength in dynamic conditions is much greater than under static conditions (Zhao et al. 1999, Li et al. 2005 and Wicaksana & Jeon 2020). Consequently, this circumstance may lead to inaccurate performance estimation and machine selection in excavation work.

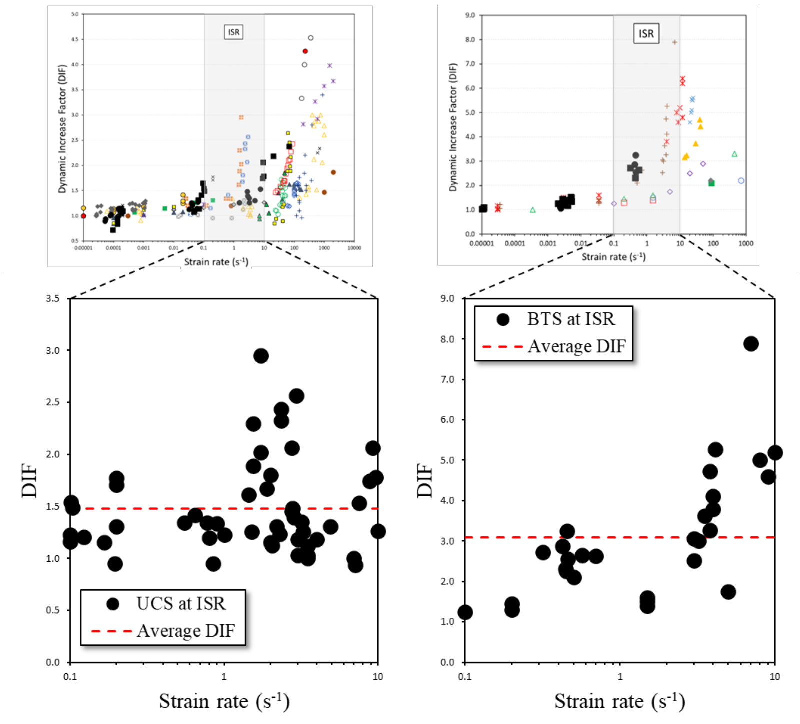
2 DATA ACQUISITION

2.1 Intermediate dynamic rock strength

In mechanical cutting, the act of a cutting tool on a rock surface is regarded as a dynamic process (Wang et al. 2019). However, little is known about how dynamic mechanical cutting is, and it is challenging to gauge the level of dynamic activity explicitly on the job site. Despite this, assuming that the tool-rock interface is less intense than rock disintegration by blasting but more progressive than a quasi-static (QS) load is plausible. Blasting or high strain rate (HSR) impacts are frequently related to dynamic rock strength. So, rock cutting is considered to have an intermediate level of dynamic loading, also called an intermediate strain rate (ISR). According to Wicaksana et al. (2021), the numerical simulation showed that a model with ISR dynamic properties fit better than static properties for estimating linear cutting laboratory experiments.

In general, no singular definition exists to classify the spectrum of ISR in terms of strain-rate-based loading categories. Nemat-Nasser (2000) claimed that the ISR classification ranges from 10^{-1} to 10^{2} s^{-1}, Ramesh (2008) asserted that the range is between 10^{0} and 10^{2} s^{-1}, and Zhang & Zhao (2014), in their thorough review paper, classified the ISR range as 10^{-1} to 10^{1} s^{-1}. This study adopted the classification by Zhang & Zhao to define the ISR range.

The dynamic uniaxial compression test (UCS) and Brazilian tensile strength (BTS) at the ISR loading were sorted from the database for various rocks in a wide spectrum of strain rates. The dynamic increase factor (DIF), described as the ratio of the dynamic values to the static one, is used to normalize the results of dynamic strength to show the increased strength under different strain rates. As shown in Figure 1, the DIFs of both strength properties were averaged only within the ISR range of 10^{-1} to 10^{1} s^{-1}. The average DIF_{UCS} was 1.48, and the average DIF_{BTS} was 3.09. The quasi-static strength can therefore be converted to the intermediate dynamic strength by applying the DIF_{UCS} and DIF_{BTS}. Notably, we hypothesized that the DIFs would represent different types of rocks because they were derived from data collected from numerous rocks.



(a) Uniaxial compressive strength (average DIF_{UCS} = 1.48)

(b) Brazilian tensile strength (average DIF_{BTS} = 3.09)

Figure 1. Dynamic increase factor (DIF) of UCS and BTS within the intermediate strain rate loading range and their representative averaged values.

2.2 Cutting forces, rock properties, and cutting configurations

A literature review was conducted to collect 80 data sets on the cutting of conical picks, including UCS, BTS, mean cutting force, peak cutting force, and some cutting configurations such as cutter tip angle and depth of cut (p). The cutting force was only selected on the unrelieved force, implying that the force is unaffected by the cut spacing. The rationale was that this study's prediction models would be compared to theoretical models based on a single cut without cut spacing parameters. Various Turkish rocks (Bilgin et al. 2006 and Copur et al. 2003) and Chinese sandstones provided the data sets (Wang et al. 2018). The UCS ranges from 6 to 174 MPa, while the BTS ranges from 0.2 to 12 MPa. Using the DIF_{UCS} and DIF_{BTS}, the quasi-static UCS and BTS (UCS_{STAT} and BTS_{STAT}) from the references were converted to the ISR dynamic strength UCS_{DYN} and BTS_{DYN}, respectively.

3 MULTIPLE REGRESSION ANALYSIS

Multiple linear regression (MLR) analyses were conducted to predict mean cutting force and peak cutting force (FC_{MEAN} and FC_{PEAK}) with a 95% (α=0.05) confidence interval. Three variables in the database could have been used as independent variables: UCS_{DYN}, BTS_{DYN}, and p. In the initial MLR analysis, all three variables were included as independent variables in the regression model using SPSS statistical software. Overall, the entire model consisting of these three independent variables is significant to the dependent variables in both instances, as evidenced by the significance F values of each model falling below the 0.05 significance threshold (significant F of 3.28×10^{-18} for FC_{MEAN} and 8.39×10^{-13} for FC_{PEAK}). Assessing the corresponding p-value is one of the most common methods for determining whether a relationship between an independent and dependent variable is significant. The result shows BTS_{DYN} and p were significant to each dependent variable FC_{MEAN} and FC_{PEAK}, given the value of p-values less than α. For UCS_{DYN}, it is not significant to the dependent variables in all cases, given the p-value greater than α (p-value of 0.603 for FC_{MEAN} and 0.318 for FC_{PEAK}). It demonstrates that using three independent variables to predict each dependent variable did not yield accurate results; therefore, the MLR analysis should be conducted using two independent variables to predict each dependent variable. Possible pairs of independent variables include UCS_{DYN} and p (regarded as Models 1 and 2), BTS_{DYN} and p (regarded as Models 3 and 4), and UCS_{DYN} and BTS_{DYN} (regarded as Model 5 and Model 6). These combinations were subjected to MLR analyses, as shown in Table 1.

Table 1. Summary of the selected results of the MLR analysis in predicting mean and peak cutting forces from two independent variables (DV: dependent variable and IV: independent variable).

Model 1 (DV: FC_{MEAN}, IV: UCS_{DYN} and p)			**Model 2** DV: FC_{PEAK}, IV: UCS_{DYN} and p		
Significance F		4.83×10^{-14}	Significance F		1.53×10^{-12}
Variables	Coefficients	p-value	Variables	Coefficients	p-value
Intercept	-1.92545	6.44×10^{-3}	Intercept	-12.1208	9.69×10^{-5}
UCS_{DYN}	0.03345	5.16×10^{-12}	UCS_{DYN}	0.10709	2.35×10^{-11}
p	0.44611	7.77×10^{-9}	p	2.44636	1.27×10^{-8}
Model 3 (DV: FC_{MEAN}, IV: BTS_{DYN} and p)			**Model 4** (DV: FC_{PEAK}, IV: BTS_{DYN} and p)		
Significance F		4.28×10^{-19}	Significance F		1.58×10-13
Variables	Coefficients	p-value	Variables	Coefficients	p-value
Intercept	-3.44832	1.04×10^{-6}	Intercept	-15.2853	2.90×10^{-6}
BTS_{DYN}	0.30269	4.04×10^{-17}	BTS_{DYN}	0.85992	2.36×10^{-12}
p	0.49167	2.86×10^{-12}	p	2.41241	5.06×10^{-9}
Model 5 (DV: FC_{MEAN}, IV: UCS_{DYN} and BTS_{DYN})			**Model 6** (DV: FC_{PEAK}, IV: UCS_{DYN} and BTS_{DYN})		
Significance F		1.96×10^{-8}	Significance F		4.70×10^{-6}
Variables	Coefficients	p-value	Variables	Coefficients	p-value
Intercept	0.82288	0.155	Intercept	2.70066	0.257
UCS_{DYN}	0.00115	0.944	UCS_{DYN}	0.02074	0.768
BTS_{DYN}	0.77407	0.006	BTS_{DYN}	1.95322	0.095

Table 1 demonstrates that all models could accurately predict FC_{MEAN} and FC_{PEAK}, with significance F values less than 0.05. Nevertheless, based on the significance test for each independent variable, the coefficients of the variables used in Models 5 and 6 were not significantly related to the response variables (p-values greater than 0.05, shaded in Table 1). Consequently, these models were eliminated from consideration. In summary, four models were constructed incorporating the combination of UCS_{DYN} and p and BTS_{DYN} and p. Models 1 and 3 predicted FC_{MEAN}, while Models 2 and 4 predicted FC_{PEAK} (equations 1-4). The FC_{MEAN}' and FC_{PEAK}' (with primes) represent the predicted cutting forces based on the suggested prediction models.

$$FC_{MEAN}{}' = -1.92545 + 0.033454 \times UCS_{DYN} + 0.446114 \times p \quad (1)$$

$$FC_{PEAK}{}' = -12.1208 + 0.107093 \times UCS_{DYN} + 2.446356 \times p \quad (2)$$

$$FC_{MEAN}{}' = -3.44832 + 0.302693 \times BTS_{DYN} + 0.491673 \times p \quad (3)$$

$$FC_{PEAK}{}' = -15.2853 + 0.859924 \times BTS_{DYN} + 2.412407 \times p \quad (4)$$

4 PERFORMANCE OF THE PROPOSED PREDICTION MODELS

The performance of the four models developed in this study was compared to that of Evans (1984), Goktan (1997), and Roxborough & Liu (1995) (equations 5-7, respectively). According to Evans, the UCS and BTS are the most influential factors in determining the cutting force of a conical pick. Goktan and Roxborough & Liu modified Evans' theory by proposing the inclusion of the friction angle between the cutting tool and rock in the model. These cutting theories can be used to estimate the cutting force of a conical cutter in an unrelieved mode (Bilgin et al. 2014).

To investigate further the correlation between measured and predicted cutting forces, a student's t-test is conducted to determine if a model is statistically significant to the measured data. The regression is statistically significant if, at the 95% confidence interval (α= 0.05), the significance F value of the suggested model is less than or the calculated t-value is greater than the t-value obtained from the distribution table (Montgomery & Runger 2010). In addition, each model's mean squared error (MSE) is calculated to estimate the models' applicability to the data population. Table 2 displays each model's coefficient of determination (R^2), mean square error (MSE), calculated t-value, and significance F value. The calculated t-values of all models are greater than the tabulated t-values of mean cutting force (1.99) and peak force (2.01). In addition, their significance F values are extremely low, below the significance level of 0.05. It indicates that all models can accurately predict the actual data.

$$FC = \frac{16.\pi.\sigma_t^2.p^2}{cos^2\left(\frac{\phi}{2}\right).\sigma_c} \quad (5)$$

$$FC = \frac{4.\pi.p^2.\sigma_t.sin^2\left(\frac{\phi}{2}+\psi\right)}{cos\left(\frac{\phi}{2}+\psi\right)} \quad (6)$$

$$FC = \frac{16.\pi.d^2.\sigma_t^2.\sigma_c}{\left[2.\sigma_t + \left(\sigma_c.\cos\left(\frac{\phi}{2}\right)\right)\left(\frac{1+tan\psi}{\tan(\phi/2)}\right)\right]^2} \quad (7)$$

where, σ_c is quasi-static UCS, σ_t is quasi-static BTS, p is the depth of cut, ϕ is tip angle, and ψ is tool-rock friction angle. It should be noted that ϕ is 80° for all case in the database, and ψ is 16°, as suggested by Roxborough & Liu (1995).

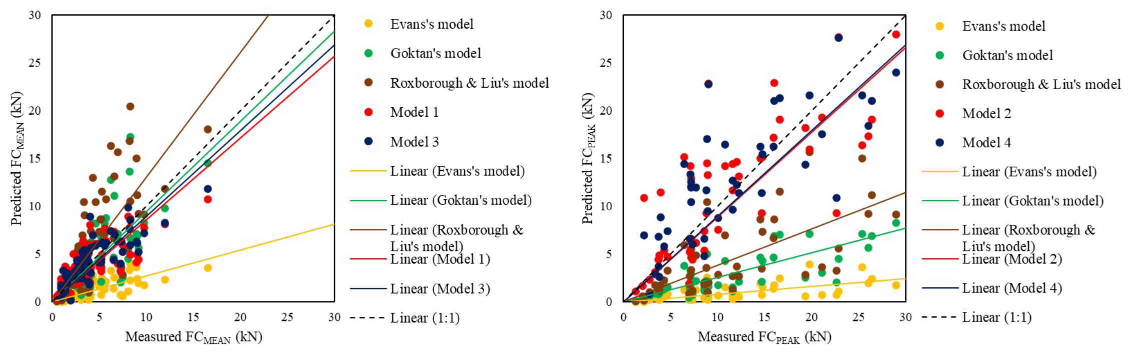
Table 2. Statistical analysis of the theoretical and prediction models.

Data	Model	R^2	MSE	Calculated t-value	Tabulated t-value (df)	Significance F
Mean Cutting Force (FC_{MEAN})	Evans	0.522	14.4	9.01	1.99 (df=73)	1.8×10^{-13}
	Goktan	0.577	28.5	9.99	1.99 (df=73)	2.7×10^{-15}
	Roxborough & Liu	0.564	41.7	9.72	1.99 (df=73)	8.3×10^{-15}
	Model 1	0.363	9.1	9.90	1.99 (df=73)	3.8×10^{-15}
	Model 3	0.600	7.4	12.78	1.99 (df=73)	2.6×10^{-20}
Peak Cutting Force (FC_{PEAK})	Evans	0.507	213.0	7.18	2.01 (df=48)	3.8×10^{-9}
	Goktan	0.769	140.8	12.67	2.01 (df=48)	6.4×10^{-17}
	Roxborough & Liu	0.614	117.9	8.73	2.01 (df=48)	1.8×10^{-11}
	Model 2	0.593	28.3	10.24	2.01 (df=48)	1.2×10^{-13}
	Model 4	0.642	25.7	10.97	2.01 (df=48)	1.1×10^{-14}

Figure 2(a) illustrates the correlation between various models' measured and predicted mean cutting forces. The model of Evans underestimates the actual mean cutting force, whereas the model of Roxborough & Liu overestimates it. The model by Goktan is closest to the 1:1 fitted line, followed by Models 3 and 1. Nevertheless, according to the MSE calculation, Model 3 has the lowest MSE among the other models (see Table 2). It suggests that Model 3 is more compatible with the provided database.

Figure 2(b) illustrates the correlation between various models' measured and predicted peak cutting forces. Only two models proposed in this study are near the 1:1 fitted line, whereas the remaining models predict a lower peak cutting force. Model 4 produces a lower MSE value than Model 2 (see Table 2). Thus, it can be inferred that Model 4 predicts the peak cutting force more accurately.

It should be noted that the suggested models were created using rocks with UCS ranging from 6 to 174 MPa and BTS ranging from 0.2 to 12 MPa. Therefore, the proposed models are hypothesized to only accurately predict cutting forces within the specified ranges.



(a) Mean Cutting Force

(b) Peak Cutting Force

Figure 2. Relationship between measured and predicted cutting forces of different models.

5 CONCLUSIONS

A number of data were gathered from literature surveys to develop a cutting force prediction model that accommodates the dynamic strength of rock under ISR. The data consists of experimental unrelieved cutting forces (mean and peak), uniaxial compression strength (UCS), Brazilian tensile strength (BTS), and other cutting configurations, including the tip angle of the conical cutter and depth of cut. The quasi-static values of UCS and BTS were then converted to dynamic values using the average dynamic increase factor (DIF) within the ISR range for different materials. Four prediction models were constructed utilizing multiple linear regression and compared to well-known theoretical cutting force models. The results demonstrate that models 3 and 4 are superior to other theoretical models for estimating the mean and peak cutting forces, respectively. According to statistical analysis, the proposed models have a smaller mean squared error than other prediction models.

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