# The impact of rock strength on the measurement of shear modulus from cavity expansion testing

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ABSTRACT: The cavity expansion test is frequently used to determine the shear stiffness of rock. When stiffness is derived from an unload/reload event (cycle) then the process is repeatable and will give consistent values that are unaffected by drilling disturbance.

The simplest interpretation of rock stiffness data is to assume the cycle is a linear-elastic event. In practice the response of the rock mass can be complicated by tensile failure and the level of shear stress. If the tensile strength is overcome and failure occurs, fracture growth can give cycles of reducing stiffness, unrepresentative of the rock mass at the insitu state. Material showing shear failure is likely to give a non-linear reduction of stiffness with strain. This includes decomposed and weathered materials. If the material remains largely intact, increasing the applied stress level will often produce a stiffer response with a power law trend.

*Keywords: Pressuremeters, Shear Stiffness, Tensile Failure, Cavity Expansion.*

## 1 INTRODUCTION

The cavity expansion test is frequently used as a means of determining the shear stiffness of many different types of ground, including rock. The experimental data are presented as a plot of cavity expansion (displacement or strain) against total pressure. A large volume of material contributes to the result, over 1000 times greater than a standard laboratory sample. Stiffness is derived with minimal assumptions and without empiricism.

Commonly, insitu testing of this kind is undertaken using a pressuremeter or dilatometer, placed in a pre-bored pocket formed by rotary tools. The cavity is completely unloaded prior to the expansion test commencing. All examples in this paper were obtained using a High Pressure Dilatometer (HPD) able to apply up to 20MPa of stress and resolve a movement of 0.3 μm.

The observed response depends on the extent to which the rock mass is an intact material. A competent, largely intact rock mass is unlikely to fail in shear at the pressures that can be applied by commercial equipment. The entire test is an elastic process, but the expansion can be influenced by tensile failure and fracture development. However, a test in a weathered or weak rock is likely to show shear failure and the response can resemble that of a dense sand. Figure 1 is an example of three tests, all conducted with a HPD, in competent mudstone, weathered mudstone and a dense sand.

The competent mudstone example shows less than 0.1 mm of radial displacement for 20MPa applied pressure, once full contact is made with the cavity wall. The response is linear-elastic, and there is no shear failure. It is a pre-requisite of such an examination that the measurements be made using high resolution instruments (Hughes & Whittle, 2023).

The weathered rock response is very different. The maximum pressure reached is 8.2MPa, resulting in 8mm radial expansion of the cavity (>30% shear strain). Shear failure starts to develop at 4MPa, and the structure of the rock starts to break down at 7MPa. Material exhibiting this kind of response can be analysed using methods developed for drained soil tests, such as the dense sand example also included in Figure 1. All three of these tests contain repeatable unload/reload events (cycles). The competent rock test includes 4 cycles. The linear stiffness shown in the competent rock test is 20 times greater than the linear stiffness shown in the weathered rock and dense sand tests. The weathered rock and dense sand tests show hysteresis in the unload/reload cycles, indicating strain dependency.

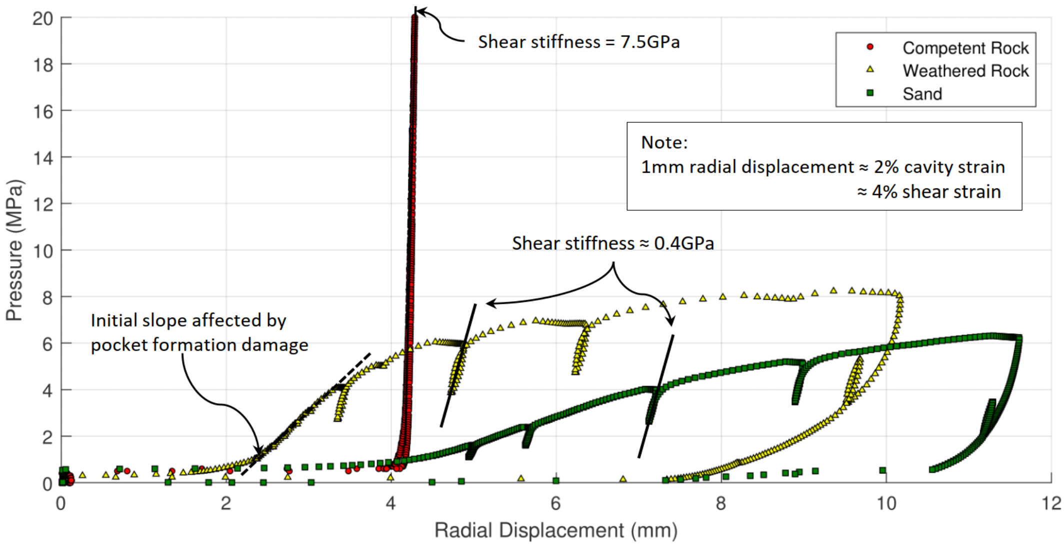


Figure 1. A comparison of HPD tests conducted in nominally 101 mm diameter pockets in different materials.

## 2 SHEAR STIFFNESS

The cavity expansion test is a shearing process. When used in rock it is more appropriate to refer to 'shear stiffness' than 'shear modulus'. Because the volume of tested material is large, the magnitude of the measured movement will be controlled by the weaker parts of the rock mass. This may be a bedding plane or discontinuity. The test is therefore the response of a structure and may itself introduce additional weakness by causing tensile failure and possibly shear failure.

Stiffness can be obtained in two ways from the cavity expansion test:

- From the initial response of the cavity during expansion or contraction ($G_i$).
- From the rebound response of a small cycle of unloading and reloading ($G_{ur}$).

The simplest approach is to assume the response is linear-elastic. If the material is deforming elastically then changes of shear stress ($\tau$) are directly proportional to changes of shear strain ($\gamma$). Equation 1 shows how linear stiffness can be determined from observed changes in stress and strain:

$$G = \frac{\Delta\tau}{\Delta\gamma} = \frac{\Delta p_c}{2\Delta\varepsilon_c} \tag{1}$$

Where $\varepsilon_c$ is the *current* cavity strain (given by $(r_c - r_o)/r_c$ where $r_c$ is the current cavity radius and $r_o$ is the initial cavity radius) and $p_c$ is the current cavity pressure.

The initial response is generally impacted by drilling induced disturbance and cavity relaxation. If stiffness is derived from an unload/reload cycle then the process is controlled by material beyond the immediate cavity surface. It can be repeated and will give consistent values that are normally unaffected by any drilling disturbance. For the unload/reload cycle to be valid, the stress at the point where the cycle is initiated must be the greatest stress that the material has experienced and in particular must be greater than the largest insitu stress. This paper focuses on the stiffness obtained from unload/reload cycles.

Figure 2 shows three tests in sandstone from the same borehole, influenced by differing degrees of fracturing during expansion. All three tests show an initial stiffness smaller than the stiffness obtained from any of the unload/reload cycles. Below 3MPa the tests exhibit a curved response. This is assumed to be a consequence of pocket formation, subsequent unloading, and the probe deforming to fit the pocket. The ratio $G_i/G_{ur}$ is a rough index of the degree of fracturing. Generally, this ratio is lower when testing in a weaker material or more fractured rock mass. This is most apparent in the weathered rock example presented in Figure 1, where the material has insufficient strength to support a fully unloaded cavity and the cavity wall has moved beyond its elastic range. This is irrecoverable, so the slope of the initial expansion for this case will always be unrepresentative of the true stiffness.

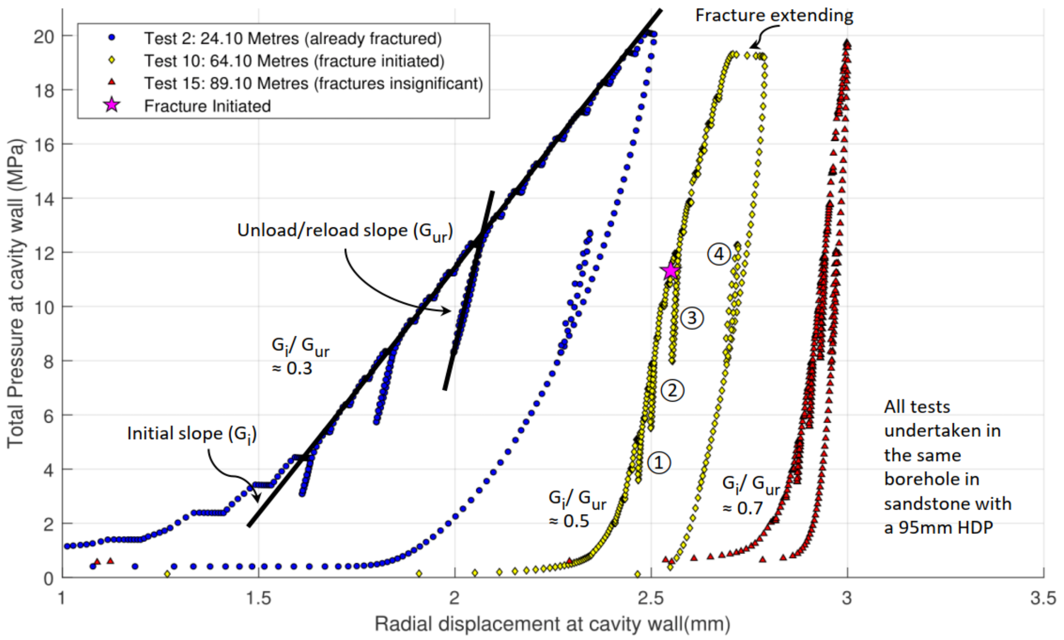


Figure 2. Three tests from the same borehole showing differing extent of fracturing (Test 2 and 15 offset to allow for comparison).

In many rock tests a linear approach is applicable, even where in practice the cycles show a more complex response (such as fracture influence). However, the stiffness of weaker rock masses is typically non-linear and hysteretic. This response is related to the proportion of mobilised strength utilised in the process of implementing a cycle.

Figure 3a presents an enlarged view of a competent rock test. This test was conducted at 320mBGL, so the initial curvature (below 6.5MPa) are data below the overburden stress. The response above the overburden stress is linear-elastic, as shown by the unload/reload cycles lying on the expansion and contraction paths, where all sources for stiffness parameters give similar values.

Between the insitu stress (6.5MPa) and the maximum pressure (20MPa) the change in radial displacement is 64μm. It is vital that the instrument can reliably discriminate sub-micrometre displacement. The success of the test depends on the observed movement being that of the rock mass, and not due to random noise or instrument effects. Prior to any testing, the equipment must be calibrated, by expansion inside a calibration cylinder of known properties. The system compliance is calculated from the difference between instrument slope and the response of the cylinder. Figure 3(b) shows an example HPD calibration response which is smooth, linear and repeating.

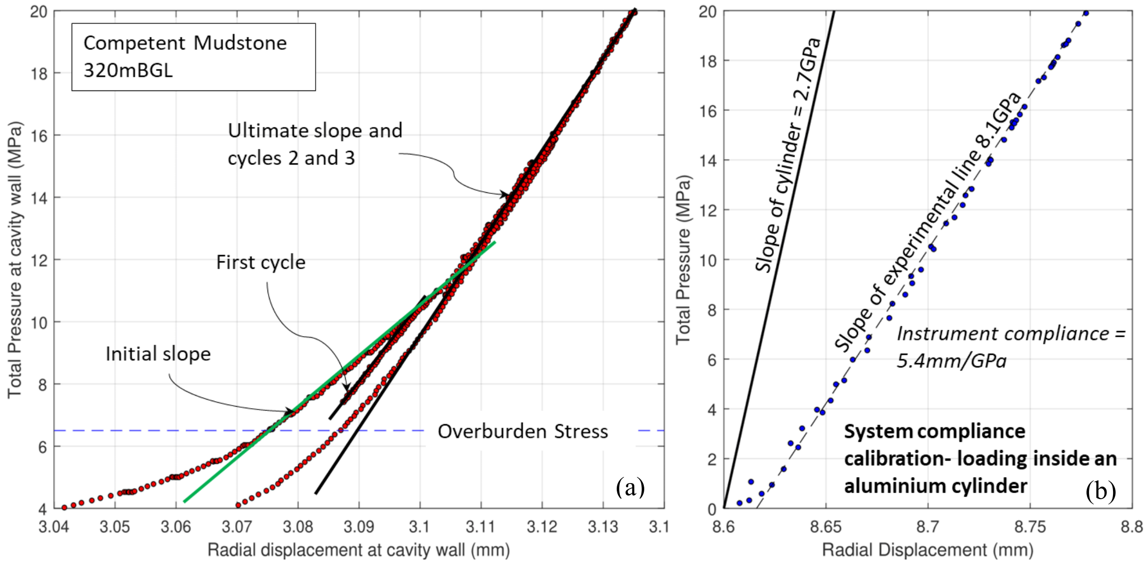


Figure 3. (a) A linear-elastic cavity expansion in mudstone at 320mBGL (b) System stiffness calibration.

## 3 INFLUENCE OF FRACTURING ON STIFFNESS

Where fracture opening and crack growth dominate the stiffness response, a reduction can be observed in stiffness, and shape alteration to the unload/reload cycles can occur. Figure 4 shows a test in sandstone, where the maximum load is 17MPa and the consequent radial movement of the rock is 0.5mm (1% cavity strain). This movement is a combination of creep, rock mass movement, and fracture development. To determine the shear stiffness (G), four unload/reload cycles have been undertaken during the expansion, and a further cycle on the contraction.

The expansion has been undertaken with a series of pressure holds, each 1 minute in duration. The displacement that takes place whilst the pressure is held is plotted in Figure 4b as creep strain.

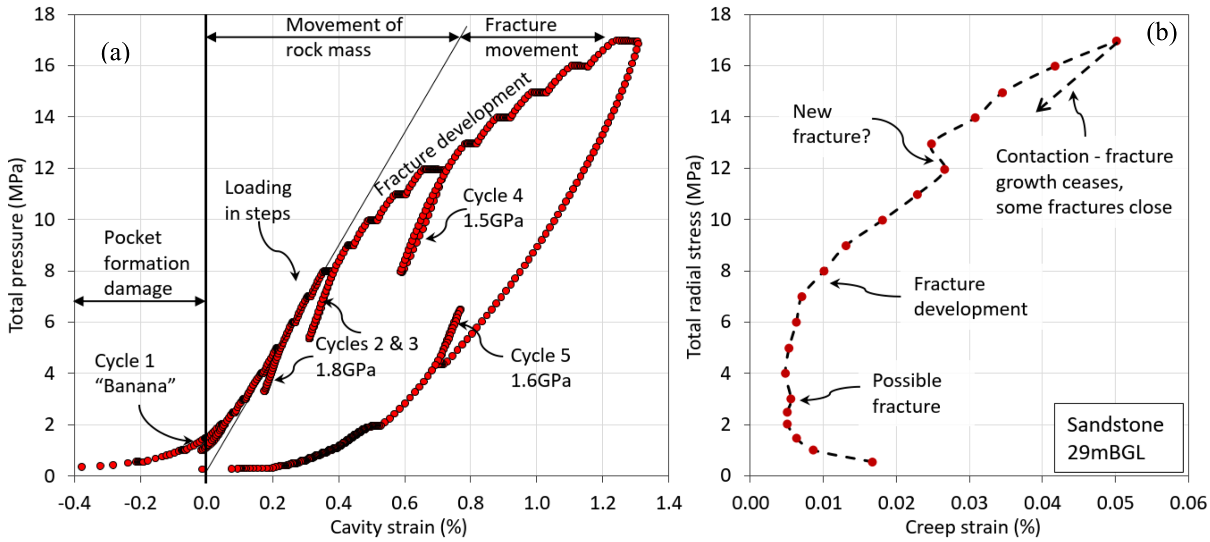


Figure 4. (a) HPD test in sandstone with multiple unload/reload cycles. (b) Creep strain against stress response.

In the zone affected by pocket formation, creep is substantial but reduces over the first 2MPa. Between 2 and 7MPa the creep is almost constant, indicating linear-elastic behaviour. Beyond 7MPa, fracture development and crack growth result in a consistent rate of stress dependent creep increase.

Cycle 1 was taken below the insitu stress and hence is invalid for deriving stiffness (Byrne 2022). Cycles 2 and 3 give similar stiffness values of 1.8GPa. The creep strain plot shows that both cycles were taken before significant creep occurred. Following cycle 3 the loading path appears to curve. This is not shear failure but is cumulative creep strain as a consequence of fracture growth. When the material is cycled, the response is a combination of induced fracture movement and elastic rock mass movement, with an apparent decrease in stiffness. Cycle 4 is hysteretic and less stiff than cycles 2 and 3 due to these effects. When the cavity is unloaded, some fracture closure occurs, and the stiffness partly recovers. Cycle 5 gives a value slightly greater than cycle 4 despite being initiated at a much lower stress level.

## 4 SHEAR FAILURE AND NON-LINEARITY

In a soil, non-linear stiffness is generally attributed to the loss of intergranular contact as shear strain increases, an effect that can be represented as a power curve of reducing stiffness with strain. In rock, non-linearity can be modelled and represented in a similar way but a different physical process is responsible if shear failure has not been achieved. This is shear stress dependent, and is principally a measure of increasing discontinuity in the rock mass. The strain dependent stiffness of soils can be calculated from the power law in Equation 2 as suggested in Bolton & Whittle (1999)

$$G = \alpha\gamma^{\beta-1} \tag{2}$$

Here $\alpha$ is the shear stress constant and $\beta$ is the exponent of non-linearity, valid between 0.5 and 1. If linear elastic $\beta = 1$ and therefore $G = \alpha$.

In soils, use of this expression can provide numerical solutions for cavity expansion testing, since integration with respect to shear strain gives the current shear stress. However, in rock, $\alpha$ and $\beta$ are simply curve fitting factors and cannot be translated into engineering parameters. There are limits as to whether a response should be considered as non-linear. If the linear interpretation (Equation 1) of an unload/reload cycle gives values for shear stiffness in excess of 1GPa, then it is not appropriate to consider non-linear interpretation. Likewise values of $\beta > 0.9$ are too close to linearity to merit a strain dependent description. To analyse for strain dependent stiffness above these limits requires a level of confidence in the measurements which no current calibration process can justify.

In addition to strain dependency, rock masses can develop significant stress dependency, where successive cycles give a stiffer trend. Figure 5(a) shows a test in sandstone with 5 cycles and 5(c) is the stiffness/strain response from those cycles. Equation 3 gives one possible method for describing this stress relationship. It is based on a formulation proposed by Janbu (1963):

$$\alpha_{ref} = \alpha(\sigma_{ref}/\sigma_{av})^n \tag{3}$$

$\alpha_{ref}$ is the shear stress constant at a reference state. To calculate the strain dependent stiffness, $\alpha$ is replaced by $\alpha_{ref}$ in Equation 2. $\sigma_{ref}$ is a designated reference stress and an appropriate value would be the average insitu horizontal stress. $\sigma_{av}$ is the mean stress at the initiation of the cycle, which can be approximated by halving the current total radial stress at the start of the cycle. The exponent of stress dependency ($n$) is obtained by plotting $G$ against $\sigma_{av}$ as shown in Figure 5(c). In this example $n \approx 0.36$ with good correlation. The stress dependency curve also has a constant term, the shear stiffness of the rock at the insitu state, which in this example is 577MPa.

The relevant stress for cycles taken during expansion is the current shear stress at the cavity wall. This reaches a maximum at the end of expansion and does not change whilst the cavity contracts elastically. In the example, this condition applies to cycle 5 because it was taken within the elastic range of the contraction. If the procedure outlined above is applied to a material where stiffness is reducing with increasing stress, $n$ will be low. For the data in Figure 4, $n \approx -0.1$ and this result expresses fracture growth and is not a material property.

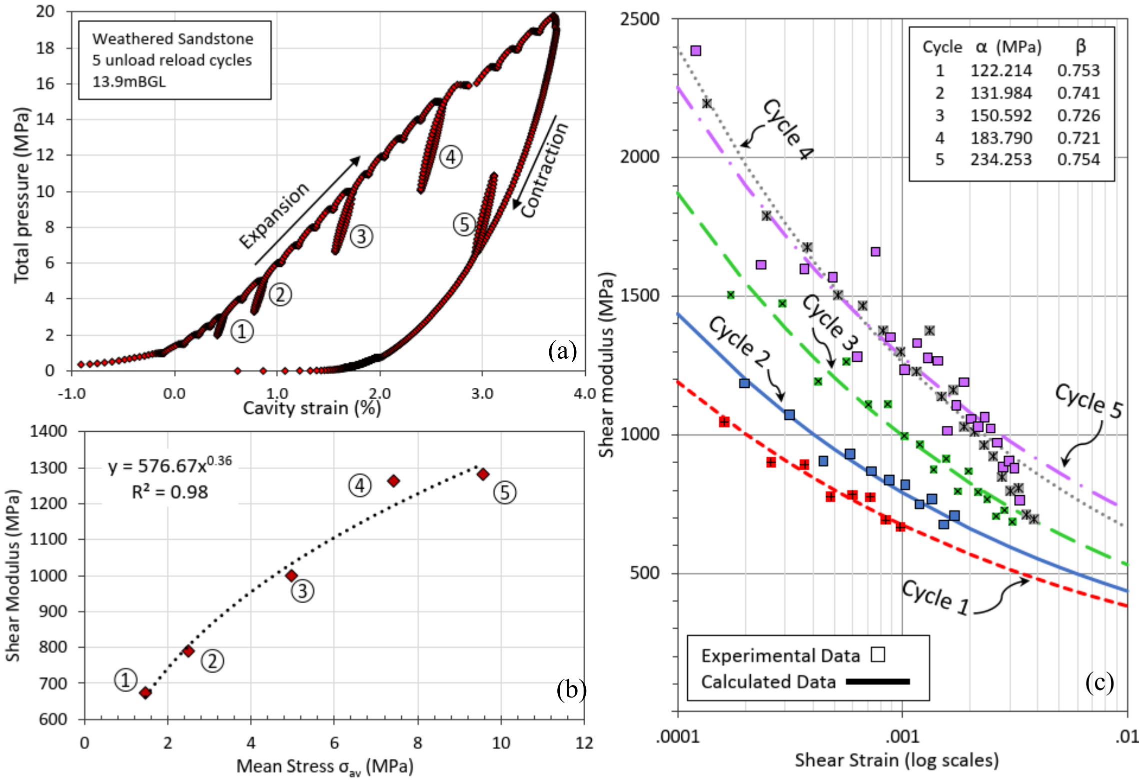


Figure 5. (a) HDP test in sandstone with multiple cycles. (b) Stress dependency relationship based on the 5 cycles (c) Stiffness against shear strain curves derived from the 5 cycles in the test.

## 5 CONCLUSIONS

This paper presents a brief introduction to the process of extracting stiffness data from cavity expansion tests in rock. The measurement of the shear stiffness of a rock mass can be impacted by both shear and tensile failure. Cycles, which utilise a small proportion of the available shear stress, are the most reliable way of determining shear stiffness from an expansion test. Typically, when testing in competent rock, a simple elastic assessment is all that is required. In more complex material, using unload/reload cycles, it is possible to identify fracture growth rates and stress dependency. When the rock is failing in shear it is also possible to derive strain dependent stiffness parameters. Additionally, the cavity expansion test in rock is reliant on rigorous calibration of the equipment. Accurate measurement of very small displacements is implicit. Any uncertainty in the calibration process must be demonstrably less than the stiffness of the material.

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