Numerical modeling of cracking process in partially saturated porous media and application to rainfall-induced slope instability analysis

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ABSTRACT: Rainfall-induced landslides are one of the major natural catastrophes causing heavy economic and human loses. In this study, a new numerical model is proposed by considering crack initiation and propagation and hydromechanical coupling. For the description of cracking process, a new phase-field model is developed for porous media with hydromechanical coupling process. In particular, a new evolution law is proposed by considering both tensile and shear cracks as well as mixed-mode. The effects of pore pressure and capillary pressure on cracking evolution are further taken into account. Moreover, the intrinsic permeability of rock is also modified by the induced cracks. The proposed model is implemented in the framework of finite element method. It is applied to the analysis of rainfall-induced landslides. An example based on real case is considered. Progressive deformation and cracking process is investigated and analyzed.

Keywords: Cracking, Porous rocks, Phase-field method, Hydromechanical coupling, Rainfall-induced landslides.

1 INTRODUCTION

Between 1995 and 2014, a total of 3,786 landslides were reported worldwide, resulting in 163,658 deaths and 11,689 injuries (Haque et al. 2019). Over 50% of these landslides occurred in areas with a high risk of heavy rainfall. Various empirical models, such as those proposed by Knighton (1998) and McDonnell (1990), have been developed to assess the instability of rainfall-induced landslides. These models typically assume that slope stability is affected by triggering factors such as rainfall infiltration and are compared against limit conditions. However, the hydromechanical coupling and progressive cracking processes are often inadequately considered in such models (Kukemilks et al. 2018 and Zhang et al. 2005).

On the other hand, there are now lots of interest in using the numerical simulation to study the development of cracks which usually causes the instability of slopes. Although significant progress has been made in numerical methods, such as the enriched finite element method (EFEM) proposed by Oliver (1996) and the extended finite element method (XFEM) proposed by Moës et al. (1999), most of the methods mentioned above are primarily used to model the propagation of pre-existing cracks, and the transition from diffuse damage (micro-cracks) to localized macroscopic cracks is still an open issue. One method that addresses this issue is the phase field method, which is based on the variational brittle fracture mechanics model proposed by Francfort & Marigo (1998) and numerically implemented by Bourdin et al. (2000). This method approximates sharp crack surfaces with a volumetric crack surface density function of an auxiliary damage variable and its gradient. The main advantage of the phase field method is its ability to describe the continuous transition from diffuse damage to localized cracks.

For these reasons, a novel phase-field model with two independent damage variables is first introduced to better describe the tensile, shear and mixed cracks. The effective elastic properties of cracked materials are affected differently by open and closed cracks. The proposed phase-field method is further extended to partially saturated porous media in order to account for hydromechanical coupling. The proposed phase-field method is applied to a typical example of natural hazards, rainfall induced landslides.

2 PHASE-FIELD METHOD FOR PARTIALLY SATURATED MEDIA

This study concentrates on a cracked porous medium, which is partially saturated and comprises a solid skeleton (indexed as s), pore water (indexed as w), and pore air (indexed as g). The target here is to determine, throughout the loading history, the displacement fields of $\boldsymbol{u}$ (including strains and stresses), the pore pressure (for both pore water and air), as well as the initiation and propagation of cracks inside. The current investigation is conducted in isothermal conditions.

2.1 Regularized crack fields

Indeed, most of geo-materials are subjected to complex loadings, where mixed cracks can be observed and generated by combined tensile and shear strains or stresses. In order to distinguish these complex cracking processes and effects of induced cracks, the phase-field (or damage field) is decomposed into d^t and d^s respectively representing the tensile and shear crack variables. Therefore, it is easily to approximate the total crack area as:

$$A_{\Gamma^\alpha} = \int_{\Gamma^\alpha} \mathrm{d}A \cong \int_{\Omega} \gamma^\alpha(d^\alpha, \nabla d^\alpha)\mathrm{d}V \tag{1}$$

Two scalar-valued functions $\gamma^\alpha(\alpha = t, s)$ denote the tensile and shear crack density. A common form was introduced in (Ambrosio & Tortorelli 1990) and it is adopted here:

$$\gamma^\alpha(d^\alpha, \nabla d^\alpha) = \frac{(d^\alpha)^2}{2l_d} + \frac{l_d}{2}|\nabla d^\alpha|^2 \ ; \quad \alpha = t, s \tag{2}$$

2.2 Energy functionals for damaged partially saturated media

2.2.1 Constitutive relations of undamaged porous media

In landslide scenarios, the change in air pressure has generally less impact than that in water pressure. As a result, the air pressure change is here neglected. Water pressure can be positive in saturated conditions or negative in unsaturated conditions. Accordingly, the capillary pressure is equal to $-p_w$. With this assumption, the poroelastic constitutive model for undamaged materials can be expressed as follows (Coussy 2010):

$$\begin{cases} d\boldsymbol{\sigma}^0 = \mathbb{C}^{b0}: d\boldsymbol{\varepsilon} - bS_w dp_w \mathbf{I} \\ dp_w = \mathrm{M}_{ww}\left[-bS_w d\boldsymbol{\varepsilon}_v + \left(\frac{dm_w}{\rho_w}\right)\right] \end{cases} \tag{3}$$

Here S_w is the saturation degree of pore water, which is defined by van Genuchten (1980) as:

$$S_w = S_r + S_e(1 - S_r), \qquad S_e = [1 + (\beta p_c)^n]^{-m} \tag{4}$$

S_r is a residual value of degree of saturation, and $\beta(1/\text{kPa})$, n and $m(= 1 - 1/n)$ are curve fitting parameters of the soil water characteristic curve (SWCC).

2.2.2 Energy functionals for damaged partially saturated materials

Due to the presence of fluid, the total energy for damaged partially saturated materials should include two parts, one is the stored energy which is conventionally seen as sum of elastic strain energy of porous medium and that related to fluid mass change, and the other is that used for cracks creation.

$$E(\boldsymbol{\varepsilon}, m_w, d^t, d^s) = \int_\Omega \left[\psi^{eff}(\boldsymbol{\varepsilon}, d^t, d^s) + \psi^{fl}(\boldsymbol{\varepsilon}, m_w, d^t, d^s)\right]\mathrm{d}V + D^{crack} \tag{5}$$

Based on constitutive relation of undamaged materials, the stored elastic energy of porous medium ψ^{eff} can be expressed as a function of the Bishop's effective stress tensor. Meanwhile, in order to define physically based criteria for the growth of tensile and shear cracks, the effective elastic stored energy is decomposed into a positive (tensile) part and a negative (compression) part and degraded by a specific degradation function. Consequently, the effective stored energy density for damaged materials is rewritten as:

$$\psi^{eff}(\boldsymbol{\varepsilon}, d^t, d^s) = \frac{1}{2} g(d^t)\boldsymbol{\sigma}_+^b : \boldsymbol{\varepsilon} + \frac{1}{2} g(d^s)\boldsymbol{\sigma}_-^b : \boldsymbol{\varepsilon} \tag{6}$$

The degradation function $g(d^\alpha) = (1 - d^\alpha)^2$ proposed in (Miehe et al. 2010) is adopted here. The effective stress tensor is decomposed into $\boldsymbol{\sigma}_+^b$ the tensile and $\boldsymbol{\sigma}_-^b$ the compressive stress tensors, respectively:

$$\boldsymbol{\sigma}_\pm^b = \sum_{a=1}^{3} \langle \sigma_a \rangle_\pm \boldsymbol{n}_a \otimes \boldsymbol{n}_a \tag{7}$$

Where σ_a is the principal stress and $\boldsymbol{n}_a$ the principal stress direction. The operator $\langle\cdot\rangle_\pm$ is defined as: $\langle\cdot\rangle_\pm = (\cdot \pm |\cdot|)/2$. On the other hand, in prior research (Aldakheel et al. 2021), it has been revealed that fluid free energy plays a limited role in the cracking process. Therefore, the contribution of the fluid energy to the damage evolution can be neglected. Thus, one has:

$$\psi^{fl}(\boldsymbol{\varepsilon}, m_w, d^t, d^s) \equiv \psi^{fl}(\boldsymbol{\varepsilon}, m_w) = \frac{1}{2} M_{ww} \left[bS_w \boldsymbol{\varepsilon}_{\boldsymbol{w}} - \left(\frac{m}{\rho}\right)_w\right]^2 \tag{8}$$

The dissipation for crack creation should include both of that used for tensile and shear cracks:

$$D^{crack} = \int_\Omega g_c^t \gamma^t(d^t, \nabla d^t) + g_c^s \gamma^s(d^s, \nabla d^s)\mathrm{d}V \tag{9}$$

2.3 Governing equations

The evolution of each damage process is described using the variational approach introduced by Francfort & Marigo (1998), where the evolution of each damage field is governed by minimizing the total energy functional with respect to each damage variable. However, in rock materials, the shear cracking is physically driven by the maximum shear stress, and prevented by the compressive mean stress. In order to better reflect this mechanism, a hybrid formulation is adopted. Inspired by the classical Mohr-Coulomb criterion, an alternative driving energy based on the generalized shear stress is here introduced for the shear crack growth:

$$W_-^{eff} \Rightarrow W_-^s = \frac{1}{2G}\left\langle \frac{\langle\sigma_3^b\rangle_- - \langle\sigma_1^b\rangle_-}{2\cos\varphi} + \frac{\langle\sigma_1^b\rangle_- + \langle\sigma_3^b\rangle_-}{2}\tan\varphi - c \right\rangle_+^2 \tag{10}$$

Where c and φ denote the cohesion and frictional angle of material. On the other hand, to ensure irreversibility condition $\dot{\gamma}^\alpha \geq 0$ and $\dot{d}^\alpha \geq 0$, the concept of energy history function introduced in (Miehe et al., 2010b) is here adopted, which reads:

$$\mathcal{H}^t(t) = \max_{\tau\in[0,t]} W_+^{eff}(\tau), \qquad \text{and} \quad \mathcal{H}^s(t) = \max_{\tau\in[0,t]} W_+^s(\tau) \tag{11}$$

Consequently, the governing equation of phase-field variable is modified to:

$$-2(1-d^\alpha)\mathcal{H}^\alpha - g_c^\alpha\left[\frac{d^\alpha}{l_d} - l_d \operatorname{div}(\nabla d^\alpha)\right] = 0, \quad \alpha = t, s \tag{12}$$

The fluid flow in porous media is described by the Darcy's conduction law and the mass balance equation. Together with the constitutive relations, it gives:

$$\frac{k_r k_p}{\mu_w} div(\nabla p_w - \rho_w \mathrm{g}) = \frac{1}{M_{ww}}\frac{\Delta p_w}{\Delta t} + bS_w\frac{\Delta\varepsilon_{ii}}{\Delta t} \tag{13}$$

where k_p is the saturated permeability, μ_w is the dynamic water viscosity , g the gravitational acceleration and. k_r is the relative permeability which is related to the saturation degree:

$$k_r = \sqrt{S_w}\left[1 - (1 - S_w^{1/m})^m\right]^2 \tag{14}$$

3 NUMERICAL MODELING EXAMPLES AND RESULTS

3.1 Description of numerical model

The example of slope instability studied here is located along the Renbo Expressway in mid-eastern Guangdong Province, China. The tropical climate in this region has led to frequent landslide disasters caused by rainfall, as reported by geological surveys. A typical cross-section from Li et al. (2020) has been chosen as an example to study the mechanism of rainfall induced slope instability.

The numerical simulation domain, after excavation, has a length of 130m and a height of 102m, as depicted in Figure 1(a). The mechanical parameters are listed in Table 1, and the hydraulic characteristics of the unsaturated material are shown in Figure 1(b). The model consists of 163522 triangular elements, with a region of refinement around the slope surface. The minimum element size is approximately 0.1m, which results in a reasonable value of l_d=0.2m.

The left and right boundaries have been fixed for horizontal displacement, while the bottom is fixed as well. The initial distribution of pore water pressure is assumed to be linear with the groundwater level located at an elevation of 133m above sea level. A constant rainfall infiltration is simulated as an extreme condition where a rainfall intensity of 12mm/day is sustained for 20 days on the slope surface.

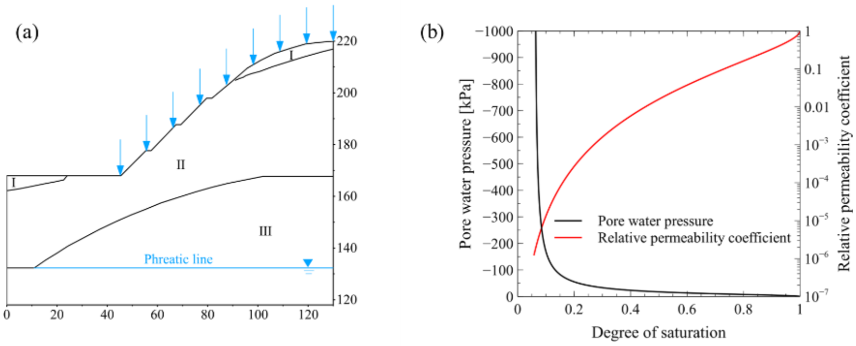

Figure 1. (a) Geometrical domain; (b) Hydraulic characteristics curves.

Table 1. Mechanical parameters input.

Section	Material	Unit weight [kN/m^3]	Young's Modulus [MPa]	Poisson's ratio	Permeability [m/day]
I	Silty clay	21	30	0.35	0.0866
II	Highly weathered siltstone	22	100	0.32	0.0168
III	Moderately weathered siltstone	24	200	0.30	0.000648

3.2 *Main results*

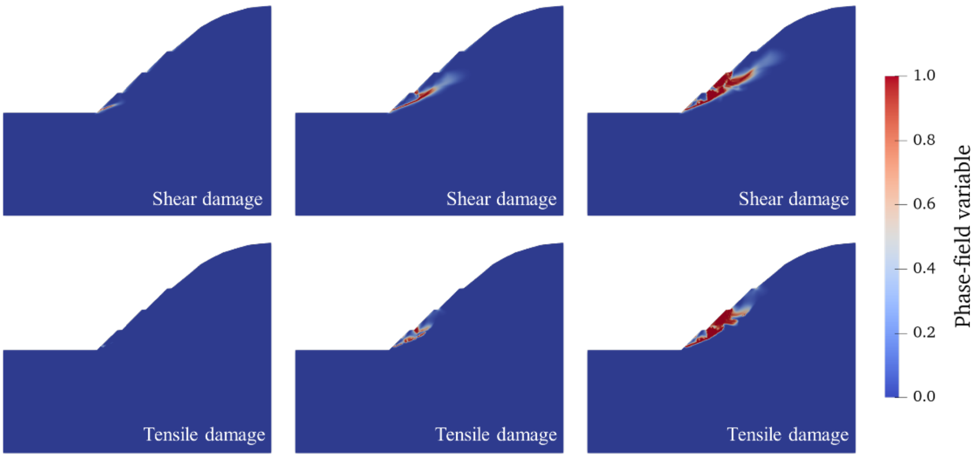

Figure 2. Distribution of shear and tensile failure within the slope with different rainfall infiltration.

Presented in Figure 2 are the onset and propagation of shear and tensile cracks in this slope, which undergoes three main stages of failure. In the initial stage, shear cracks emerge primarily around the foot of the slope and the toe bulging due to concentrated stress from gravity. Shear cracks develop faster at the foot of the slope than the toe at other excavation depths, while tensile damage remains moderate. Subsequently, as localized shear damage propagates, some tensile cracks arise in the crest of the first excavation slope surface, connecting to shear cracks from the toe of the first excavation slope surface and leading to the first slope failure. With continuous rainfall infiltration, both tensile and shear cracks further develop, potentially leading to a second failure of the second excavation slope surface.

4 CONCLUSION

In this paper, a novel phase-field method has been introduced to model the onset and propagation of cracks in partially saturated rock materials. Two independent damage variables have been incorporated, which evolve via two independent processes and are solved using two coupled boundary value problems. The new phase-field method is capable to describe the onset and growth of tensile, shear, and mixed cracks under varying loading conditions. The proposed method has been successfully applied to analyze landslides induced by rainfall in partially saturated media. It can describe the initiation and propagation of localized damage zones and cracks resulting from rainfall, with shear cracking being the primary failure mechanism of landslides. The numerical results are in good qualitative agreement with the field observations.

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