A numerical modelling study of the effect of pillar shape on pillar strength

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ABSTRACT: Bord and pillar layouts are typically designed using square or rectangular pillars. Pillar cutting is poor in many hard rock mines and many pillars have an irregular shape. This may affect pillar strength. The “perimeter rule” is commonly used for rectangular pillars to determine an “effective width”, but its applicability to pillars with irregular shapes has never been tested. This paper describes numerical modelling to investigate the effect of pillar shape on pillar strength. A limit equilibrium model, implemented in a displacement discontinuity code, was a valuable approach explored in this study. Preliminary evidence indicates that the perimeter rule should not be used for irregularly-shaped pillars. For rectangular pillars of increasing length, the numerical model correctly predicts an increase in pillar strength for an increasing in length. However, the increase in strength predicted by the modelling is higher than that predicted by the perimeter rule.

Keywords: Pillar strength, Perimeter rule, Limit equilibrium model, Irregular pillar shape.

1 INTRODUCTION

Extraction of sub-horizontal, tabular reefs is typically done using a bord and pillar layout. This involves mechanised mining methods to mine the ore whereby square or rectangular pillars are left behind as local and regional support. Owing to the difficulties of mining in hard rock and the inherent blast damage associated with the use of explosives, the pillar shapes are often irregular. Pillar spalling along major joint planes or at the corners of the pillars may exacerbate the problem. Figure 1 illustrates the typical differences between planned pillar layouts versus the actual mined layouts. Note the many highly irregular pillar shapes. Pillar shape affects pillar strength (e.g. Wagner, 1974 and Maritz, 2017), but no clear methodology exists to determine the strength for different pillar shapes. The strength of pillars with irregular shapes is particularly difficult to estimate.

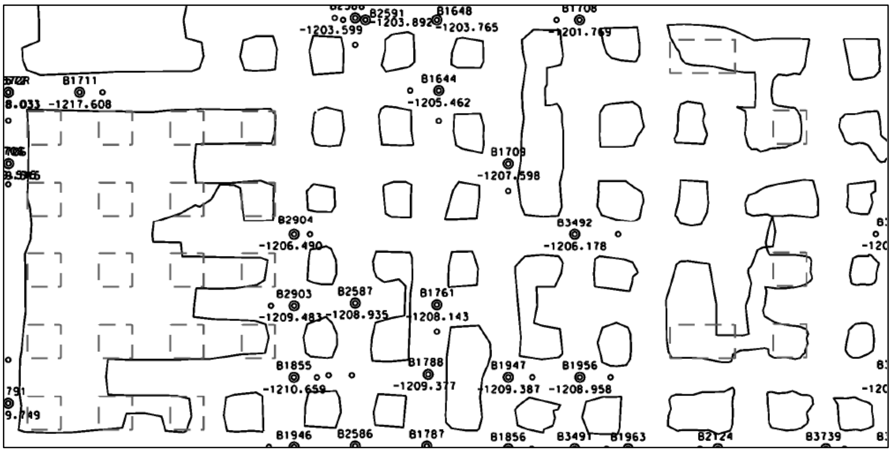


Figure 1. Example of planned versus actual pillar layouts. The planned shapes are indicated by the square boxes (dotted lines) in the large pillar on the left.

1.1 The "perimeter rule"

To account for the strength of elongated pillars, Wagner (1974) proposed the concept of an "effective width" for these pillars and this can be calculated from the area and perimeter of a pillar:

$$w_{\text{eff}} = \frac{4A}{C} = \frac{2wL}{w + L} \tag{1}$$

where;
A = cross-sectional area of the pillar
C = perimeter of the pillar
w = minimum lateral dimension of the pillar
L = maximum lateral dimension of the pillar

To determine pillar strength, this effective width is typically used in the empirical power-law pillar strength formulae. Examples of these strength formulae are given in Malan and Napier (2011), and it is not discussed in this paper.

It is not clear if the perimeter rule given above is appropriate as no experimental work was conducted to verify this equation. Ryder and Ozbay (1990) suggested a strengthening factor with values f = 1.0/1.1/1.2/1.3 for pillars having w/L ratios of 1/2/4/∞. In the mining industry, equation (1) is also used for pillars with an irregular shape and this is probably not correct. For example, several different pillar shapes are shown in Figure 1 and rock engineering practitioners may use the perimeter rule to estimate the strength of these various pillars. Its applicability needs to be carefully assessed; however, this paper is preliminary study of this important question.

2 NUMERICAL SIMULATION OF THE EFFECT OF PILLAR SHAPE

To simulate the effect of pillar shape on pillar strength, a novel approach of using a displacement discontinuity boundary element code, TEXAN, was explored (see Napier & Malan 2007) for a description of the code). The displacement discontinuity codes typically do not simulate the failure of the pillars, but the use of a limit equilibrium constitutive model allows for the modelling of on-reef pillar failure. The details of the model are not described in this paper and the reader is referred to the papers available on this topic (e.g. Napier & Malan 2018 and Couto & Malan 2023). The code is particularly well suited to simulate the shallow bord and pillar layouts in the Bushveld Complex in South Africa as it can easily represent the irregular pillar shapes.

For this limit equilibrium model, it assumes that the pillar is delineated by frictional parting planes at the contacts with the hangingwall and footwall. By considering the force equilibrium of a slice of rock in the fractured edge of the pillar, it is possible to construct a differential force balance for the reef-parallel and reef-normal tractions. The solution of the governing differential equation implies that the tractions increase in an exponential fashion towards the centre of the pillar. For a tabular layout problem, with irregular pillar shapes discretized using triangular elements, a "fast marching solution" to determine the reef-parallel stress is implemented (Napier & Malan 2021). A number of assumptions is made in the TEXAN program, for example that it is assumed that the reef-parallel stress gradient direction is perpendicular to the adjacent element edge at the excavation boundary.

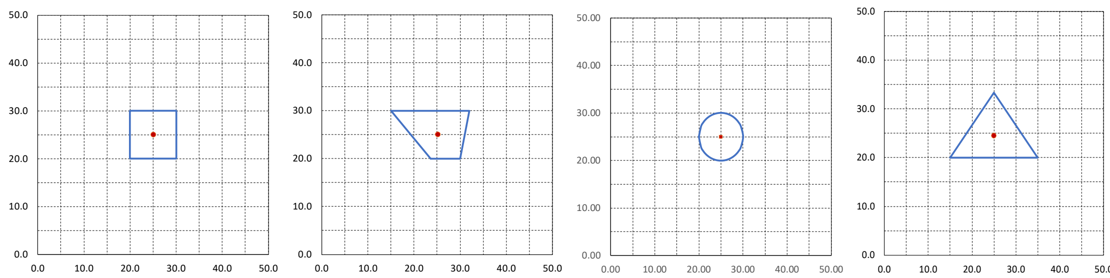
In terms of the limit equilibrium model parameters, Table 1 lists the values used for the modelling described in this paper. The model parameter values given in Table 1 were selected arbitrarily as the objective was to investigate the effect of shape on strength and the only requirement was that the pillars failed in the simulations. The same parameters were used to simulate the different pillar shapes. The reader can consult Couto & Malan (2023) for a description of these various parameters.

Table 1. Parameters used for the limit equilibrium model.

Parameter		Value
Intact strength intercept	[MPa]	12.0
Intact strength slope	[-]	6.0
Residual strength intercept	[MPa]	2.8
Residual strength slope	[-]	2.0
Effective seam height	[m]	2.0
Intact rock Young's modulus	[MPa]	70 000
Intact rock Poisson's ratio	[-]	0.2
Fracture zone interface friction angle	[°]	20
Seam stiffness	[MPa/m]	2 000
Pillar width – square pillar	[m]	10

2.1 Numerical modelling geometries

For the initial studies, the numerical model was used to determine if different shapes with identical w_{eff} values will have similar peak strengths. Four pillar shapes were generated. The pillar shapes are shown in Figure 2. The w_{eff} parameter was calculated using equation (1). Interestingly, the triangular pillar has a significantly larger area compared to the other shapes to give the required constant w_{eff} value. The effect of elongation of the square pillar was studied as a second set of simulations. The geometry of these rectangular pillars is shown in Figure 3.



a) Square: A = 100 m^2, P = 40 m   b) Trapezoid: A = 117 m^2, P = 47 m   c) Circle: A = 79 m^2, P = 31 m   d) Triangle: A = 133 m^2, P = 53 m

Figure 2. The different pillar shapes simulated using the TEXAN code. The dimensions were selected to ensure a constant effective width of w_{eff} = 10 m. The symbols A = area and P = perimeter.

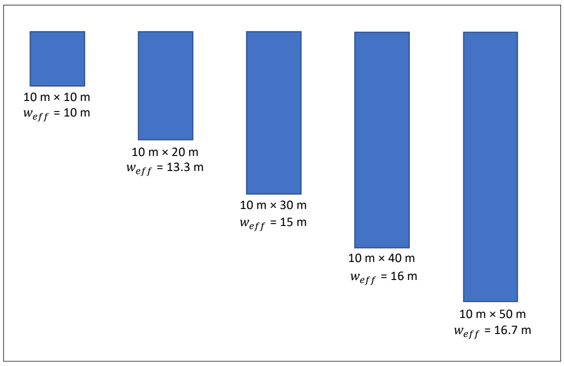


Figure 3. The geometries used to investigate the effect of an increase in pillar length on strength for a rectangular pillar.

The pillar height used in the simulations shown in Figure 3 was 2 m. This gives a w:h ratio of 5 for the square pillar. These specimens can therefore be considered as "squat" pillars and it is expected that the pillar shape will make a difference in terms of strength when considering the information discussed above.

The geometries were discretised using triangular elements of a size ≈ 0.08 m^2. As this was simulated using a displacement discontinuity code, the pillars had to be positioned in a "mined stope" and an arbitrary overall excavation size of 50 m × 50 m with the pillar in the centre was simulated. As a crude method to gradually increase the stress on the pillars, the depth of the excavation was increased in successive runs and this enabled a stress-strain curve of the pillars to be generated.

3 NUMERICAL MODELLING RESULTS

The first modelling results presented are for the different pillar shapes with a similar w_{eff} (see Figure 2). Figure 4 illustrates the failed sections of the various pillar shapes. The intact core for each pillar assumed the original outline shape of the pillar. The load-deformation curves for the pillars are presented in Figure 5. The circular pillar is stronger (also observed by Du et al. 2019 in the laboratory), but the peak strength of the other three pillars are almost identical. According to equations (1), the circular pillar should not be stronger as the w_{eff} of the simulated pillars are identical. It is speculated that the absence of sharp corners delays the onset of fracturing and hence the greater load bearing capacity of the circular pillar.

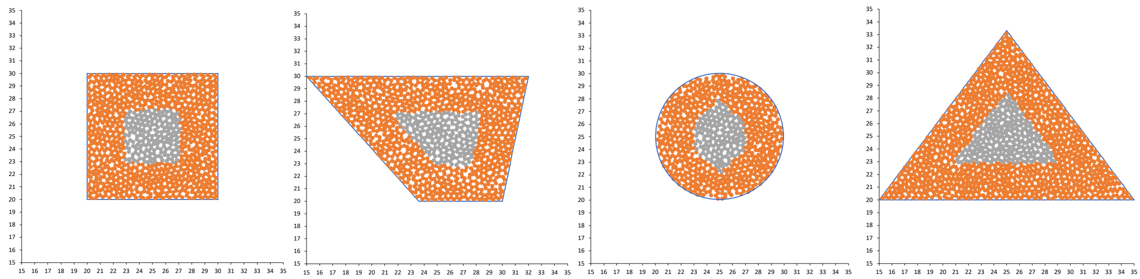


Figure 4. Failed portions of the various pillar shapes. This is presented for the peak stress. The orange colour denotes the failed elements and the grey colour the intact elements.

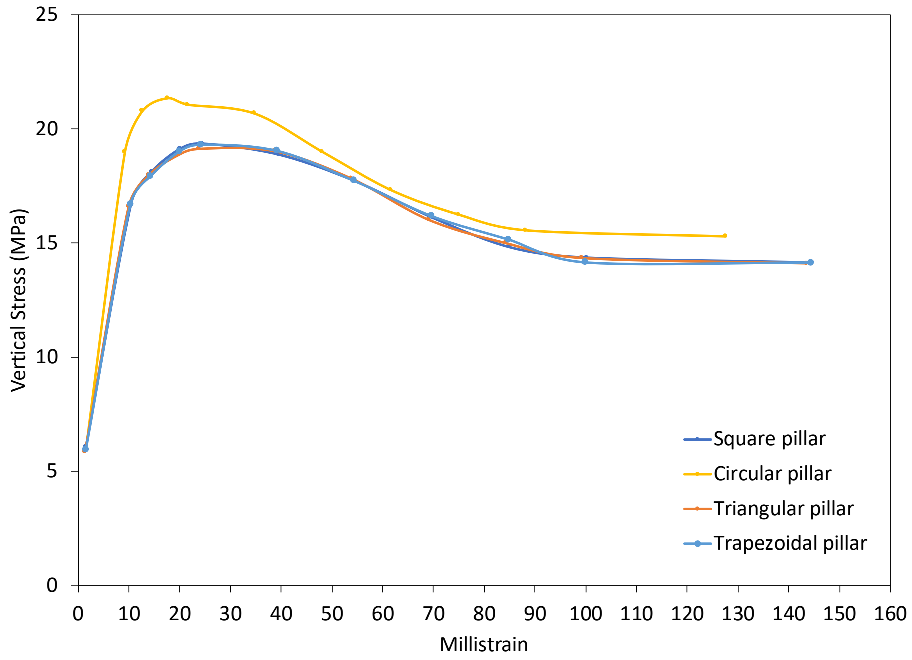


Figure 5. Simulated pillar strength for the various pillar shapes. This was for a constant w_{eff} = 10 m.

In contrast to the constant w_{eff} illustrated in Figure 5, the increase in length for the rectangular pillar resulted in an increase in w_{eff}. The results of the simulations are presented in Figure 6. The peak strength increases with an increase in w_{eff}. This is an important finding as the limit equilibrium model mimics the expected increase in strength for the rectangular pillars. For the model parameters used, the modelling predicts an increase of peak strength from 19.4 MPa for the square pillar to 38.9 MPa for the 50 m long pillar. This is an increase in strength of approximately 2 for the rectangular pillar and it is higher than the 1.414 predicted by the perimeter rule for an infinitely long pillar.

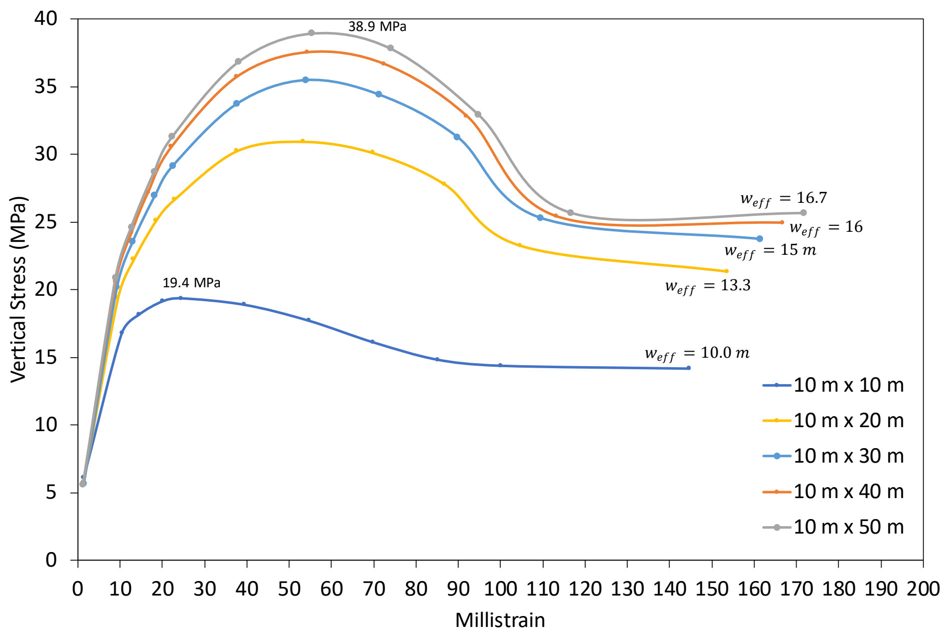


Figure 6. Simulated pillar strength for the various lengths of rectangular pillars.

4 CONCLUSIONS

This paper is a preliminary study of the effect of pillar shape on pillar strength. The "perimeter rule" is widely adopted for non-square pillars, but its applicability for arbitrary pillar shapes has never been tested.

The study indicated that the displacement discontinuity modelling approach, using a limit equilibrium failure model, is well suited to simulate the effect of pillar shape. It can for example predict the increase in strength for elongated rectangular pillars and the results qualitatively agrees with the increase in strength predicted by the perimeter rule. This study nevertheless highlighted that the perimeter rule should be used with caution for pillars with a complex or irregular shape. For example, the models indicated that a circular pillar is stronger than other shapes with a similar effective width.

The limit equilibrium constitutive model is a valuable addition to displacement discontinuity modelling. As illustrated in this paper, this can be used to simulate the effect of shape on pillar strength, but careful calibration of the model (not explored in this paper) is required. Assigning material properties to the failed rock on the pillar edges is particular challenging.

In terms of future work, additional laboratory studies of the effect of pillar shape is required to confirm the results obtained from the numerical modelling studies.

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