

# Estimation models for deformability of marlstones based on their physical and mechanical properties and for variable load range

Srđan Kostić

Jaroslav Černi Water Institute, Belgrade, Serbia

**ABSTRACT:** In present paper we establish the correlation between the field determined values of elasticity  $E$  and deformability modulus  $D$  of marlstones and their laboratory determined physico-mechanical properties. Laboratory results which are taken into consideration are the following: unit weight, moisture content, uniaxial compression strength, cohesion, friction angle, maximum and minimum pressuremeter load range, as input parameters, and elasticity and deformability modulus as output parameters. Statistically significant correlations are determined by applying multiple linear regression method and ANOVA test. Established correlations indicate that both for  $E$  and  $D$  properties for undisturbed rock samples (cohesion, unit weight, uniaxial compression strength) do not have significant effect, while friction angle has strong positive effect both on  $E$  and  $D$ . Moisture content have negative effect on  $E$  and  $D$ . Minimum/maximum load stress have opposite effect on  $E$  and  $D$ , meaning that deterioration of rock mass in situ with the increase of stress has negative effect on  $D$ .

*Keywords:* pressuremeter, marlstone, elasticity modulus, deformability modulus.

## 1 INTRODUCTION

Deformability properties of rock masses in situ are of crucial impact for adequate calculation of rock mass stress-strain behavior in interaction with designed constructions. Correct determination of elasticity and deformability moduli is of primary importance during the design and construction of capital structures, i.e., dams and tunnels. In such cases, concerning the scale of the influence of the structure on surrounding rock mass, in situ experiments are commonly used, since they provide the most reliable results. These experiments could be divided into two major groups: invasive and non-invasive. Non-invasive testing methods commonly include application of various geophysical methods, while invasive testing techniques commonly assume some additional work in the rock mass, like drilling (dilatometer/pressuremeter tests, PM/DM tests in further text) or excavation (hydraulic jack testing, or similar). Nevertheless, the best results are obtained when these two groups of method are combined, in a way that spatial distribution of different deformability properties of rock mass could be estimated based on the derived correlation between the rock mechanics testing results and results of geophysical methods. Results of the previous investigation works confirmed

the successful determination of deformability properties of rock masses by using this approach. However, if construction is designed in a limited surrounding (urban environment) or hardly accessible terrain (on the water), then possibilities for application of different testing methods are very narrowed. In such cases, one is usually limited to application of PM/DM testing in a borehole. These testing methods are considered to be somewhere between the laboratory testing and the real in-situ large scale testing, because the quality of testing and, consequently the obtained results depend primarily on the method, precision and quality of drilling, and on the frequency of testing and are depth-limited (testing at depths > 50m is strongly affected by the quality of cable transmission). Apart from this, PM/DM testing in boreholes are expensive and time-consuming. Therefore, it is of practical interest to try to establish a correlation between the results of PM/DM testing with the results of application of other methods. In present paper, we suggest possible correlation between the results of laboratory analyzes and PM/DM testing. There have been some attempts in correlating the in situ and laboratory testing results. Zhao et al (2023) developed models for predicting rock strength parameters including Poisson's ratio, elastic modulus, and uniaxial compressive strength based on computer drilling jumbo measurement while drilling (MWD) data. Yoon et al (2023) developed the relationship between granite rock properties and longitudinal wave velocity in rock bolts to predict rock properties using the longitudinal wave velocity in a rock bolt. Khan et al (2022) proposed a model for uniaxial compressive strength and elasticity modulus using temperature, p-wave velocity, porosity, density and dynamics Young's modulus as input parameters. Hua et al (2022) derived correlations between some easily measured geotechnical parameters and rock mass deformation modulus, based on 84 data sets from Yalong River Jinping-I hydropower station.

In present paper, we establish statistically significant correlation between the values of elasticity modulus  $E$  and deformability modulus  $D$  (output parameters, determined in situ), and minimum, maximum load stress (during in situ testing) and laboratory determined values: unit weight, uniaxial compression strength, cohesion and friction angle and moisture content. As we are aware this is the first time such extensive correlations are established between the field determined modulus  $E$  and  $D$  and common geomechanical parameters determined in laboratory conditions.

## 2 APPLIED METHODOLOGY

In order to establish reliable and statistically significant estimation models for rock mass elasticity and deformability modulus,  $E$  and  $D$ , respectively, in present paper we examine field and laboratory data on geomechanical properties of Neogene marlstones in Serbia at two locations: Belgrade marlstones in Višnjica region (Kostić, 2021) and marlstones underlying the riverbed sediments of the Danube River in Donji Milanovac (Kostić, 2022) (see Table 1). Field data on modulus  $E$  and  $D$  were collected using field pressuremeter test in boreholes. Data are analyzed using the multiple linear regression method, while the results obtained are evaluated using ANOVA test and common descriptive statistics.

Table 1. Analyzed input and output parameters of the rock mass.

Parameters	min	max
Minimum load stress $P_{min}$ (MPa)	0.9	2.2
Maximum load stress $P_{max}$ (MPa)	3.2	8.2
Unit weight $\gamma$ (g/cm <sup>3</sup> )	1.821	2.184
Uniaxial compression strength $\sigma$ (MPa)	0.5	3.86
Cohesion $c$ (kPa)	28	100
Angle of internal friction $\varphi$ (°)	17	30
Moisture content $\omega$ (%)	18.5	24.2
Elasticity modulus $E$ (MPa)	180	380
Deformability modulus $D$ (MPa)	119	333

### 3 RESULTS

#### 3.1 Dependence of elasticity modulus on geomechanical parameters

Results of the applied multiple linear regression resulted in the following statistically significant and physically meaningful dependence (see Table 2 and Table 3).

$$E = 1535.2 + 63.6 \cdot P_{min} + 50.2 \cdot P_{max} - 106.5 \cdot \gamma - 159.1 \cdot \sigma - 5.9 \cdot c + 6.1 \cdot \varphi - 28.1 \cdot \omega - 69.4 \cdot P_{max} \cdot \gamma + 22.8 \cdot P_{max} \cdot \sigma + 0.7 \cdot P_{max} \cdot c \quad (1)$$

As one can see from Figure 1 (a), residuals of model (1) follow normal distribution, which confirms the reliability of the derived model (1). Moreover, as one can see from Figure 1(b), there is a good correlation between the estimated and determined values of E.

Table 2. ANOVA for Response Surface model (1).

Parameters	Sum of Squares	Mean Square	F value	p-value
Pmin	2185.858	2185.858	39.610	< 0.0001
Pmax	296.364	296.364	5.370	0.0430
$\gamma$	7838.037	7838.037	142.034	< 0.0001
$\sigma$	2401.082	2401.082	43.510	< 0.0001
c	5081.285	5081.285	92.078	< 0.0001
$\varphi$	3108.912	3108.912	56.337	< 0.0001
$\omega$	7937.803	7937.803	143.842	< 0.0001
$P_{max} \cdot \gamma$	462.527	462.5273	8.381	0.0160
$P_{max} \cdot \sigma$	2991.973	2991.973	54.218	< 0.0001
$P_{max} \cdot c$	2740.437	2740.437	49.660	< 0.0001

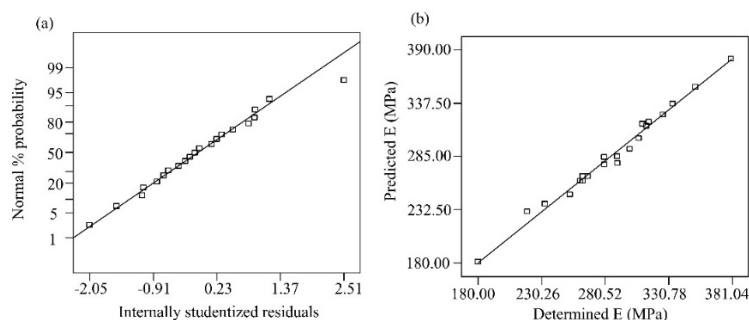


Figure 1. (a) Normal probability plot for model (1) and (b) Predicted (model 1) vs determined values of modulus E ( $R^2=0.986749$ ,  $SE=7.43$ ).

Table 3. Error parameters for model (1).

Parameter	Coefficient estimate	Standard error	95% CI Low	95% CI High	VIF
Pmin	41.340	6.568	26.704	55.975	3.241
Pmax	37.965	16.382	1.462	74.469	28.256
$\gamma$	-91.124	7.646	-108.161	-74.088	8.889
$\sigma$	-36.791	5.577	-49.219	-24.363	5.102
c	-70.003	7.295	-86.258	-53.748	6.612
$\varphi$	39.165	5.218	27.539	50.792	5.959

$\omega$	-80.188	6.685	-95.085	-65.291	8.915
$P_{max}*\gamma$	-31.491	10.877	-55.728	-7.255	7.797
$P_{max}*\sigma$	72.096	9.791	50.279	93.913	9.155
$P_{max}*c$	62.983	8.938	43.069	82.898	7.045

Effects of statistically significant individual factors and two-factor interactions are shown in Figure 2 and Figure 3, respectively.

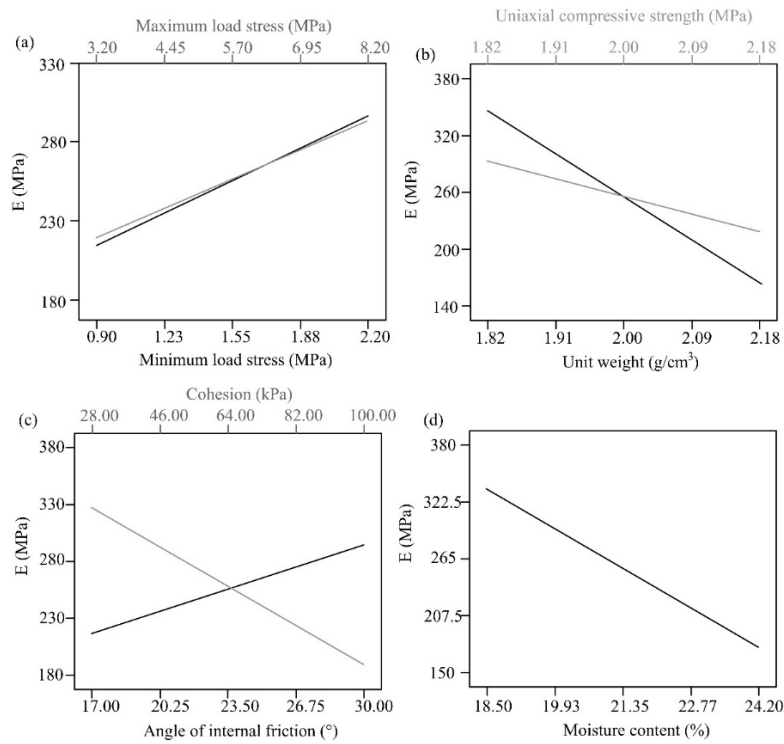


Figure 2. Dependence of elasticity modulus E on (a)  $P_{max}$ ,  $P_{min}$  (MPa), (b) unit weight and uniaxial compressive strength, (c) friction angle and cohesion, (d) moisture content.

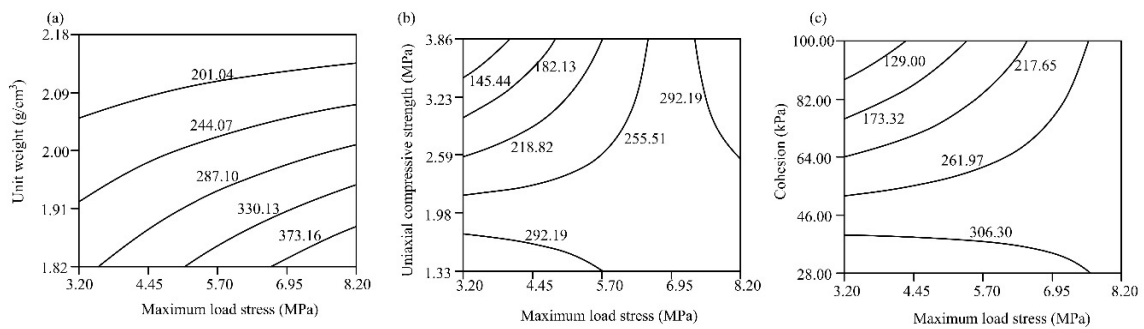


Figure 3. Significant two-factor interactions: dependence of E on (a) unit weight and maximum load stress, (b) uniaxial compressive strength and maximum load stress and (c) cohesion and maximum load stress.

### 3.2 Dependence of deformability modulus D on geomechanical parameters

Results of the applied multiple linear regression resulted in the following statistically significant and physically meaningful dependence (see Table 4 and Table 5).

$$D = 3361.6 - 93.5 \cdot P_{max} - 820.1 \cdot \gamma - 2.1 \cdot c - 11.1 \cdot \varphi - 41.4 \cdot \omega + 2.4 \cdot P_{max} \cdot \varphi \quad (2)$$

Table 4. Error parameters for model (1) ANOVA for Response Surface model (2).

Parameters	Sum of Squares	Mean Square	F value	p-value
Pmax	37402.808	37402.808	161.146	< 0.0001
$\gamma$	33860.847	33860.847	145.886	< 0.0001
c	19577.213	19577.214	84.346	< 0.0001
$\varphi$	886.877	886.877	3.821	0.0709
$\omega$	20059.177	20059.177	86.423	< 0.0001
Pmax* $\varphi$	3440.464	3440.464	14.823	0.0018

Table 5. Error parameters for model (2).

Parameter	Coefficient estimate	Standard error	95% CI Low	95% CI High	VIF
Pmax	-95.680	7.537	-111.846	-79.515	1.422
$\gamma$	-148.847	12.323	-175.279	-122.416	5.490
c	-75.789	8.252	-93.488	-58.089	2.011
$\varphi$	14.742	7.542	-1.433	30.917	2.960
$\omega$	-118.032	12.696	-145.263	-90.800	7.643
Pmax* $\varphi$	38.193	9.920	16.917	59.470	1.433

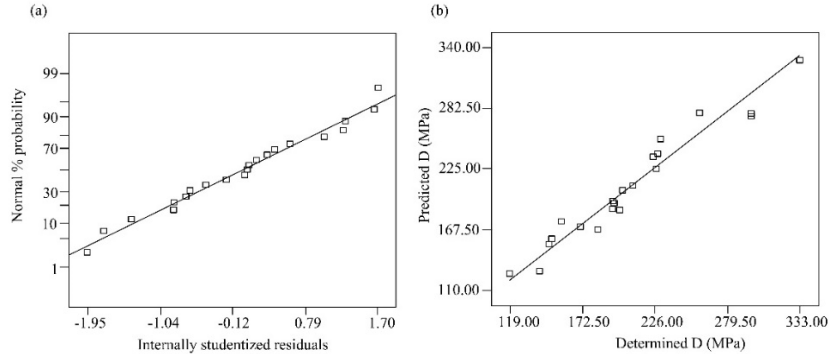


Figure 4. (a) Normal probability plot for model (2) and (b) Predicted (model 2) vs determined values of modulus D ( $R^2=0.95$ ;  $SD=15.2$ ).

As one can see from Figure 4 (a), residuals of model (2) follow normal distribution, which confirms the reliability of the derived model (2). Moreover, as one can see from Figure 4 (b), there is a good correlation between the estimated and determined values of D. Effects of statistically significant individual factors and two-factor interactions are shown in Figure 5.

#### 4 DISCUSSION AND CONCLUSIONS

Results obtained indicated the following influence of the examined input factors on the modulus E and D. As one can see from Figure 2, there is a strong positive effect of maximum and minimum load stress and friction angle on the modulus E. This is expected, since with the increase of load stress (both minimum and maximum) and friction angle, value of modulus E should also increase. On the other hand, uniaxial compressive strength, unit weight, cohesion and moisture content show evident

negative effect of modulus E. The effect of moisture content is expected: with the increase of moisture content there is a decrease in modulus E. However, negative effect of uniaxial compressive strength, unit weight and cohesion on E indicate the following: apparently, value of E is predetermined by the in situ properties of rock mass (where existence of joints and their properties play a crucial role), while properties of rock mass determined on undisturbed rock sample do not affect modulus E. In such case, effect of compressive strength, cohesion and unit weight could be considered as insignificant. This insight is also valid for statistically significant two-factor interactions in Figure 3.

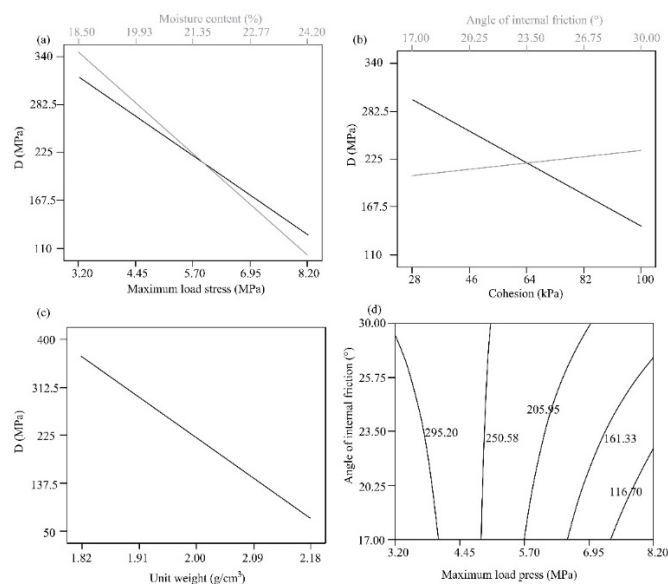


Figure 5. Dependence of deformation modulus D on (a) maximum load stress and moisture content, (b) cohesion and friction angle, (c) unit weight, (d) two-factor interaction: friction angle and maximum load stress.

As one can see from Figure 5, there is a strong positive effect of friction angle on D, as it was the case of dependence of E on friction angle. Also, cohesion and unit weight have negative impact on D, which is the same nature of the effect as it was the case of modulus E. However, maximum and minimum load stress have negative impact of D, which is the opposite effect when compared to modulus E. This means that with the increase of load stress modulus D decreases, which could be connected to the deterioration of rock mass with the increase of load range. Such effect is also revealed in statistically significant two-factor interaction in Figure 5 (d).

Results obtained in present study should be further verified by including more data, or one could try to establish the statistically significant and physically meaningful relationship between the modulus E and D and some other field data, e.g. CPT, SPT values or similar.

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